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Сборник лексических упражнений

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Сборник лексических упражнений предназначен для студентов I и II курсов механико-математического факультета МГУ. Его цель — освоение навыков работы с физико-математическими терминами и общенаучной лексикой в устной и письменной речи.

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Настоящий сборник лексических упражнений ставит своей задачей повторение и закрепление базовых слов и выражений лексического минимума, предусмотренного учебной программой по английскому языку для студентов I курса механико-математического факультета МГУ. Сборник состоит из упражнений и заданий, развивающих профессионально ориентированный «английский язык для специальных целей» (English for Specific Purposes, or ESP), в который входят как слова терминологического слоя лексики — в данном случае математические и физико-математические термины, — так и базовые выражения и обороты общенаучной лексики.

Сборник состоит из двух частей (в части I *Vocabulary Practice* отрабатываются основные термины математики и механики; часть II *Revision* нацелена на повторение пройденного материала) и поделен на 12 разделов (sections). Различные виды упражнений и тексты по специальности, представленные в сборнике, объединяются общей тематической направленностью, что позволяет обеспечить высокую естественную повторяемость специальной лексики, характерной для научного стиля речи. Большое внимание уделяется как заданиям, в которых представлены дефиниции основных физико-математических терминов, так и упражнениям, вырабатывающим навыки перевода с русского на английский язык. Значительный упор в данном случае делается на умение владеть специальной лексикой и использовать клишированные выражения и обороты речи при переводе научных текстов.

Материалом сборника послужили оригинальные источники (английские и американские словари, энциклопедии, статьи). Однако в ряде случаев взгляды англо-американских ученых на вопросы математики и механики отличаются от взглядов их российских коллег, что следует учитывать при работе с предложенными в сборнике упражнениями и текстами.

Данный сборник лексических упражнений может быть использован как приложение к учебным пособиям Е.Н. Егоровой и Е.И. Миндели *English for Students of Mathematics and Mechanics* для студентов I курса, а также студентами II курса в качестве повторительного материла.

C O N T E N T S

Part I (Vocabulary Practice)
Section 1

1. Study these words and expressions:

- science / scientist / scientific — наука / ученый / научный
  - natural / computer science — естественная наука / информатика
  - scientific research / approach / method — научное исследование / подход / метод

  branch of science — раздел науки
- research / to research — исследование / исследовать
  - to do / to carry out / to conduct research into (on) sth. — проводить исследование чего-либо
  - to research the problem — изучать проблему
- study / to study — изучение / изучать
  - the study of mathematics — изучение математики
  - to study mathematics — изучать математику
- mathematics / mathematic(al) /mathematician [ˌmæθəməˈtɪʃn] — математика / математический /математик
  - applied mathematics — прикладная математика
  - pure (abstract) mathematics — чистая математика
  - mathematical operation — математическая операция
  - mathematical procedure — математическая процедура
  - mathematical / numerical expression — математическое / числовое выражение
- arithmetic [əˈrɪθmətɪk]/ arithmetic(al) [ˌærɪθˈmetɪk(əl)]/ arithmetician [əˌrɪθməˈtɪʃn]— арифметика / арифметический / арифметик
- algebra / algebraic(al) /algebraist— алгебра / алгебраический / алгебраист
  - algebraic equation / number /function — алгебраическое уравнение / алгебраическое число / алгебраическая функция
- geometry/geometer(geometrician)/geometric(al) — геометрия / геометр / геометрический
  - geometric object / figure / body — геометрический объект / фигура / тело
  - geometrical progression — геометрическая прогрессия
- to calculate / calculation — вычислять / вычисление

- to do / make / perform calculations — производить вычисления

- to measure /measurement — измерять / измерение, замер
  - square measure — квадратная мера (мера площади)
  - linear measure — линейная мера, мера длины
  - metrical measure — метрическая мера
  - dry (liquid) measure — мера сыпучих тел (жидкостей)
  - to make / to take measurements — производить измерения, делать замер
- object — объект, предмет; цель (исследования и т.д.)
- number — число
- digit — цифра
- unit — единица; единица измерения
  - theory of units — теория единиц
  - unit of length — единица длины
  - central processing unit — центральный процессор
  - power supply unit — блок питания
- unity — единица, число один
  - root of unity — корень из единицы
- sign — знак
  - change the sign (sign change) — изменить знак (перемена знака)
- symbol — символ
  - algebraic symbol — алгебраический символ
- property — свойство
  - continuity property — свойство непрерывности

2. Translate the following sentences into English:

1. Ферма (Fermat) был известным французским математиком.
2. Это очень сложная математическая процедура.
3. В науке, например, в математике, мы часто используем символы.
4. Результаты данного исследования оказались очень важными для дальнейшего развития этого раздела науки.
5. Давайте произведем следующие вычисления.
6. Вычисления этого математика неверны.
7. Какие геометрические объекты вы знаете?
8. Два основных раздела математики — это прикладная и чистая математика.
9. В математике используются знаки и символы.
10. Метр — это мера длины.

3. Study these basic mathematical terms:

- quantity — величина, количество
  - incommensurable quantities — несоизмеримые величины
  - like quantities — однородные величины
  - known quantity — данная величина, известное
  - unknown quantity — неизвестное
  - negligible quantity — ничтожно малая величина
- magnitude — величина, протяженность, размер
  - magnitude of a vector — длина (модуль) вектора
- value — значение, величина
  - absolute value (modulus) — абсолютная величина, модуль
  - negative / positive value — отрицательное / положительное значение
- addition / to add / additive — сложение / складывать / аддитивный
- subtraction / to subtract — вычитание / вычитать
- multiplication / to multiply (by sth.) — умножение / умножать (на что-то)
  - multiplication table — таблица умножения
- division / to divide (by sth.) — деление / делить (на что-то)
- product — произведение
- sum — сумма
  - partial sum — частичная сумма
- difference — разность
- equation — уравнение
  - to solve an equation — решать уравнение
  - to satisfy an equation — удовлетворять уравнению
  - algebraic / differential / simultaneous equation — алгебраическое / дифференциальное уравнение / система уравнений
  - linear / quadratic equation — линейное / квадратное уравнение
- (in)equality — (не)равенство
- power — степень
  - to raise to a power — возводить в степень
  - power series — степенной ряд
- root — корень
  - to extract a root — извлекать корень

- perfect square — полный квадрат
- variable — переменная
  - (in)dependent variable — (не)зависимая переменная
- invariant — инвариант
- unknown — неизвестное
  - an equation in two unknowns — уравнение с двумя неизвестными
- term — член, терм; термин
  - general term of an expression — общий член выражения
  - like terms — подобные члены
- polynomial — полином, многочлен; полиномиальный
  - degree of a polynomial — степень многочлена
  - differential polynomials — дифференциальные многочлены

4. Translate the following definitions from English into Russian paying special attention to the underlined words and expressions:

1. *Mathematics* is the study or use of numbers and shapes to calculate, represent, or describe things. Mathematics includes arithmetic, geometry, and algebra.
2. *Arithmetic* is the part of mathematics that involves basic calculations such as adding or multiplying numbers.
3. *Algebra* is a type of mathematics that uses letters and symbols in place of numbers.
4. *Arithmetic progression* is a series of numbers in which the same number is added to each number to produce the next, for example 3, 6, 9, 12.
5. A *formula* in mathematics or physics is a general relationship, principle, or rule stated, often as an equation, in the form of symbols.
6. A *magnitude* is a number assigned to a quantity, such as weight, and used as a basis of comparison for the measurement of similar quantities.
7. A *quantity* is an entity having a magnitude that may be denoted by a numerical expression.
8. A *value* is a number represented by a figure, symbol, or the like.
9. A *function* is a value which depends on and varies with another value.

10. A *number* is a concept of quantity that is or can be derived from a single unit, the sum of a collection of units, or zero.

5. Match the terms in the left column with the definitions in the right column and translate the sentences into Russian paying special attention to the underlined words and expressions:

1. equation	1. a letter representing a number that can change depending upon the other numbers in an equation
2. power	2. a variable, or the quantity it represents, the value of which is to be discovered by solving an equation
3. root	3. a quantity or expression that is constant throughout a certain range of conditions
4. variable	4. a mathematical statement that two expressions are equal
5. polynomial	5. a number that is the result of multiplying two other numbers
6. product	6. a total amount made by adding several numbers or amounts together
7. sum	7. used for describing an expression in algebra that contains two or more terms
8. sign	8. used in mathematics for saying how many times you multiply a number by itself
9. invariant	9. a number or quantity that when multiplied by itself a certain number of times equals a given number or quantity
10. unknown	10. the positivity or negativity of a number, quantity, or expression

6. Supply the following words and expressions with their English equivalents from ex. 4-5 and give their derivatives (part *a*):

a) - изучение чего-то — изучать
- использование (польза) чего-либо — использовать — полезный
- представлять (символ, значение) — представление
- описывать — описание — описательный

- включать (разделы науки и т.д.) — включение
- вовлекать (влечь за собой) — вовлечение
- основной — основа чего-либо — основываться на чем-то
- сравнение — сравнивать — сравнительный
- отношение — относится к чему-то (быть связанным с чем-то) — относительный — относительность
- зависеть от чего-то — зависимость — (не)зависимый
- различаться — различный
- получать (выводить) из чего-то — производная
- условие — условный

b)
- такой, как
- последовательность цифр
- например
- устанавливать правило
- приписывать чему-то (ставить в соответствие, назначать, определять)
- единица (целостность)
- обозначать (величину и т.д.)
- понятие чего-то
- общий принцип
- математическое утверждение
- постоянный
- результат чего-то
- данное число

7. Translate the following sentences into English:

1. Значение переменной равно 7.
2. 2 является квадратным корнем из 4, кубическим корнем из 8 и корнем четвертой степени из 16.
3. Вычитание из нуля меняет знак выражения.
4. Сложение, вычитание, деление и умножение — основные математические операции.
5. Решите уравнение 5x-3=27.
6. x, y, z являются неизвестными величинами.
7. Если x и y – два числа, то их сумма обозначается x+y, а разность x–y.
8. Если два числа равны, будут равны также результаты их умножения на одно и то же число.

9. Продемонстрируем другой метод решения систем уравнений.
10. Неравенства, содержащие неизвестную величину, решают методами, похожими на используемые при решении уравнений.

Section 2

1. Study these mathematical terms and expressions:

- geometry — геометрия
  - analytic geometry — аналитическая геометрия
  - descriptive geometry — начертательная геометрия
  - projective geometry — проективная геометрия
  - Euclidian / non-Euclidian geometry — евклидова / неевклидова геометрия
  - plane geometry — планиметрия
  - solid geometry — стереометрия
- space — пространство
  - Cartesian space — декартово пространство
- direction — направление
  - p-direction — р-мерное направление
- size — размер
  - volume — объем
  - area — площадь
- dimension — измерение, размерность
  - two-dimensional / three-dimensional (tridimensional) — двухмерный / трехмерный
  - length — длина
  - width — ширина
  - height — высота
- shape — форма
  - solid — объемный
  - flat — плоский
  - round — круглый
- distance — расстояние
  - distance axioms — аксиомы расстояния
  - distance scale — линейный масштаб
  - equidistant — эквидистантный, равноотстоящий, равноудаленный

- surface — поверхность
  - surface area — площадь поверхности
  - algebraic surface — алгебраическая поверхность
  - closed surface — замкнутая поверхность
  - surface of revolution — поверхность вращения
- plane / planar / coplanar — плоскость / плоскостной, плоский / компланарный
  - projection plane — плоскость проекции
  - tangent plane — касательная плоскость
  - plane curve — плоская кривая
  - plane domain — плоская область
  - plane topology — плоская топология
  - planar graph — плоский граф
- tangent — касательная, тангенс
- point — точка
  point of intersection — точка пересечения
- line — линия
  - straight line — прямая линия
  - vertical line — вертикальная линия
  - parallel line — параллельная линия
  - horizontal line — горизонтальная линия
  - broken line — ломаная линия
  - curve — кривая
- segment — отрезок
- parabola — парабола
- hyperbola — гипербола
- circle / circumference — круг / окружность, длина окружности
- ellipse [ɪˈlɪps] — эллипс
- angle — угол
  - right (straight) angle — прямой угол
  - acute angle — острый угол
  - obtuse angle — тупой угол
  - adjacent angle — прилежащий (смежный) угол
- to inscribe — вписывать
- to circumscribe — описывать

- to intersect / intersection (meet, product) — пересекать(ся) / пересечение
- to bisect [baɪˈsekt] / bisection / bisector (bisectrix [baɪˈsektrɪks]) / bisecant [baɪˈsiːknt] — делить пополам / деление пополам / биссектриса / хорда
- to trisect [traɪˈsekt] / trisection — делить на три равные части / трисекция
- graph (linear graph) — граф, график
  - planar graph — плоский граф
- node — вершина графа
- vertex [ˈvəːteks] (pl. vertices [ˈvəːtɪsiːz]) — вершина (мн.ч. вершины)
  - vertex figure — вершинная фигура
  - vertex of an angle — вершина угла
  - vertex of a cone — вершина конуса
- arc — дуга, арка
  - arc length — длина дуги
  - arc cosine — арккосинус
  - arc sine — арксинус
  - arc tangent — арктангенс
- locus [ˈləʊkəs] (pl. loci [ˈləʊsaɪ]) — геометрическое место точек
- vicinity — окрестность
- coordinate — координата
  - Cartesian coordinates — декартовы координаты
- axis [ˈæksɪs] (pl. axes [ˈæksiːz]) — ось (оси)
  - coordinate axis — координатная ось
- origin — начало координат
- function / functional— функция / функциональный
  - rate of change of a function — скорость изменения функции
  - domain of a function — область определения функции
- manifold — многообразие
  analytic manifold — аналитическое многообразие
- base — основание, основа, база
  - base angles — углы при основании
- edge — ребро
  - edge of a cube — ребро куба
- face — грань, (плоская) поверхность

- topology / topological — топология / топологический
  - topology of space — топология пространства
  - topological group — топологическая группа
  - topological product — топологическое произведение
  - topological isomorphism — топологический изоморфизм
  - topological transformation — топологическое преобразование
- geometric figures — геометрические фигуры
  - polygon — многоугольник
  - polyhedron [ˌpɔlɪˈhi:drən]— многогранник, полиэдр
  - regular polyhedron — правильный многогранник
  - triangle — треугольник
  - right triangle — прямоугольный треугольник
  - equilateral triangle — равносторонний треугольник
  - similar triangles — подобные треугольники
  - isosceles [aɪˈsɔsəli:z] triangle — равнобедренный треугольник
  - square — квадрат
  - rectangle — прямоугольник
  - parallelogram — параллелограмм
  - cube — куб
  - cylinder — цилиндр
  - pyramid — пирамида
  - parallelepiped [ˌpærəˌleləˈpɪped]— параллелепипед
  - prism — призма
  - cone — конус
  - sphere — сфера
  - rhomb(us) — ромб

2. Translate the following definitions from English into Russian paying special attention to the underlined words and expressions:

1. *Geometry* is the part of mathematics that deals with the relationships between lines, angles, and surfaces.
2. *Topology* is the study of those properties of geometric forms that remain invariant under certain transformations, as bending or stretching.
3. *Dimension* is a property of space; it is extension in a given direction.
4. A *line* is any straight one-dimensional geometrical element whose identity is determined by two points.
5. A *segment* is a part of a line or curve between two points.

6. An *angle* is the space within two lines or three or more planes diverging from a common point, or within two planes diverging from a common line.
7. A *curve* is a continuously bending line, without angles.
8. A *point* is a geometric element having no dimensions and whose position in space is located by means of its coordinates.
9. A *coordinate* is any of a set of numbers that defines the location of a point in space with reference to a system of axes.
10. An *axis* is a central line that bisects a two-dimensional body or figure or it is a line about which a three-dimensional body or figure is symmetrical.
11. A *volume* is the amount of space, measured in cubic units, that an object or substance occupies.
12. An *area* is the quantitative measure of a plane or curved surface.

3. Supply the following words and expressions with their English equivalents from ex.2 and give their derivatives (part *a*):

a)
- симметричный — симметрия
- расходиться (отклоняться от чего-то) — расхождение — расходящийся
- располагаться — расположение
- определять (давать определение) — определение
- непрерывно — непрерывный — непрерывность — продолжаться

b)
- расширение (продолжение)
- общая точка
- иметь дело с чем-то (заниматься чем-то)
- определяться чем-то
- положение в пространстве
- посредством чего-либо
- множество чисел
- по отношению к чему-либо
- вещество
- занимать (место)
- количественная мера
- оставаться неизменным
- при определенных преобразованиях

4. Match the given geometric figures in the left column with their definitions in the right column and translate the sentences into Russian:

1. pyramid
2. polygon
3. right triangle
4. equilateral triangle
5. isosceles triangle
6. square
7. rectangle
8. cube
9. cylinder
10. cone
11. sphere
12. circumference

1. a parallelogram having four right angles
2. a flat shape with three or more sides and angles
3. a solid having a polygonal base and triangular sides that meet in a common vertex
4. the outer boundary, especially of a circular area; perimeter
5. an object like a box with six square sides that are all the same size
6. an object shaped like a wide tube
7. a shape with four straight sides of equal length and four corners called right angles
8. a round body whose surface is at all points equidistant from the centre
9. a geometric solid consisting of a plane base bounded by a closed curve, usually a circle or an ellipse, every point of which is joined to a fixed point, the vertex, lying outside the plane of the base
10. a triangle that has three sides that are the same length
11. a triangle in which two sides are the same length
12. a triangle one angle of which is a right angle

5. Fill in the gaps with the words below and translate the text into Russian:

The word graph may refer to the familiar —**1**— of analytic geometry and function theory, or it may refer to simple —**2**— consisting of points and lines connecting some of these points; the latter are sometimes called linear graphs, although there is little confusion within a given context. Such graphs have long been associated with puzzles.

If a finite number of —**3**— are connected by lines, the resulting —**4**—is a graph; the points, or corners, are called the —**5**—, and the lines are called the edges. If every pair of vertices is connected by an **—6—,** the graph is called a complete graph.

A planar graph is one in which the edges have no **—7—** or common points except at the edges. (It should be noted that the edges of a graph need not be straight lines.) Thus a nonplanar graph can be transformed into an equivalent, or isomorphic, **—8—.** Planar graphs have proved useful in the design of electrical networks.

A connected graph is one in which every vertex, or point (or, in the case of **—9—,** a corner), is connected to every other point by an **—10—;** an arc denotes an unbroken succession of edges. A route that never passes over an edge more than once, although it may pass through a point any number of times, is sometimes called a path.

1. edge
2. intersection
3. points
4. figure
5. curves
6. arc
7. geometric figures
8. vertices
9. planar graph
10. a solid

6. Supply English equivalents and translate the text into Russian:

A graph is a pictorial representation of statistical data or of a (1) *функционального отношения между переменными*. Most graphs employ two axes, in which the (2) *горизонтальная ось представляет* a group of (3) *независимых переменных*, and the vertical axis represents a group of (4) *зависимых переменных*. (5) *В аналитической геометрии*, graphs are used to map out functions of two variables on a (6) *декартовой системе координат*, which is composed of a horizontal x-axis, or abscissa, and a vertical y-axis, or ordinate. Each axis is a real number line, and their (7) *пересечение в нулевой точке* of each is called (8) *началом координат*. A graph in this sense is (9) *геометрическое место всех точек* (x,y) that (10) *удовлетворяют определенной функции*.

7. Translate the following sentences into English:

1. У прямой линии есть одно измерение, у параллелограмма — два, а у параллелепипеда — три измерения.
2. Две линии пересекаются в точке *A*.
3. Впишите круг в квадрат.

4. Две основные оси — это вертикальная ось *y* и горизонтальная ось *x*.
5. Какими свойствами обладают трехмерные фигуры?
6. У куба есть шесть поверхностей.
7. Все точки находятся в одной плоскости.
8. Какова длина данного отрезка?
9. Объем измеряется в кубических единицах, а площадь — в квадратных.
10. Числовые данные могут быть представлены с помощью разнообразных графических средств – диаграмм, графиков и т.д.

Section 3

1. Study these mathematical terms and expressions:

- numeration (numeral) system — система счисления
  - decimal (base-ten) system — десятичная система
  - binary (base-two) system — бинарная (двоичная) система
  - duodecimal (base-twelve) system — двенадцатеричная система
- place — разряд
- place value — разрядное значение (цифры)
- fraction — дробь
  - decimal fraction — десятичная дробь
  - terminating (finite) decimal — конечная дробь
  - nonterminating (infinite) decimal — бесконечная десятичная дробь
  - repeating decimal — периодическая десятичная дробь
  - continued fraction — цепная (непрерывная) дробь
  - common (vulgar, simple) fraction — простая дробь
  - proper / improper fraction — правильная / неправильная дробь
  - mixed number — смешанная десятичная дробь
- denominator — знаменатель
  - common denominator — общий знаменатель
  - to reduce (to reduce fractions to a common denominator) — сводить, приводить, сокращать (приводить дроби к общему знаменателю)
- numerator — числитель
- decimal point — десятичная точка
- logarithm — логарифм

- quotient [ˈkwəʊʃnt] — частное, отношение
- ratio (proportion) — соотношение, коэффициент, пропорция
- dividend — делимое
- divisor — делитель
- multiple — кратное
- factor — множитель
  - to factorise / to be decomposed into factors — разлагать на множители
  - monomial factor — общий (алгебраический) множитель
- remainder — остаток
- exponent / exponential— экспонент, показатель степени / показательный, экспоненциальный
  - fractional exponent — дробный показатель степени
  - exponential curve — показательная кривая (экспонента)
  - exponential distribution — показательное (экспоненциальное) распределение
  - exponential equation — показательное уравнение
  - exponential function — показательная функция
  - exponential notation — экспоненциальное представление чисел
  - exponential series — показательный (экспоненциальный) ряд
- coefficient — коэффициент, индекс
  - coefficient field — поле коэффициентов
- index — индекс, показатель степени
  - index of a radical — показатель корня

2. Match the terms in the left column with their definitions in the right column and translate the sentences into Russian paying special attention to the underlined words and expressions:

1. dividend
2. common (vulgar) fraction
3. factor
4. ratio (proportion)
5. exponent
6. denominator (divisor)
7. logarithm
8. numerator

1. that term of a fraction, usually written under the line, that indicates the number of equal parts into which the unit is divided
2. a symbol written above and to the right of a mathematical expression to indicate the operation of raising to a power
3. the term of a fraction, usually above the line, that indicates the number of equal parts that are to be added together

9. decimal fraction	4. a number that is to be divided by a divisor
10. terminating (finite) decimal	5. quotient of two numbers or quantities
11. nonterminating (infinite) decimal	6. a fraction whose denominator is some power of 10, usually indicated by a dot (decimal point or point) written before the numerator
12. mixed decimal (number)	7. a decimal numeral in which, after a finite number of decimal places, all succeeding place values are 0, as 1/8 = 0.125
	8. a decimal numeral that does not end in an infinite sequence of zeros
	9. a fraction represented as a numerator above and a denominator below a horizontal or diagonal line
	10. a number consisting of a whole number and a fraction or decimal, as $4^{1/2}$ or 4.5
	11. the exponent indicating the power to which a fixed number, the base, must be raised to obtain a given number or variable
	12. one of two or more numbers, algebraic expressions, or the like, that when multiplied together produce a given product; a divisor

3. Place the sentences in the correct order to make a coherent text. Translate it into Russian paying special attention to the underlined words and expressions:

Numeration system

1. For example, in 333, the 3 on the right means three, but the 3 in the middle means three tens and the 3 on the left means three hundreds.
2. The numeration system (or system for writing number symbols) widely used throughout the world today is a place-value system based on the number 10 and usually called the Arabic, or Hindu-Arabic, numeration system.
3. Only the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 and the decimal point are needed to write numbers of any size.

4. There are some rules regarding the order of symbols in the Roman numeral system, however (for example, IX means 9, while XI means 11), though generally position is not as important as in place-value systems.
5. In this system the position a symbol occupies helps determine the value of the symbol.
6. In modern Roman numerals (where I, V, X, L, C, D, and M are 1, 5, 10, 50, 100, 500, and 1,000, respectively), on the other hand, CCC means 300 — each C stands for one hundred, and the relative position of the C's is of no importance.
7. A place-value system, such as the Arabic numeral system, has clear advantages in economy of symbolism and in efficiency of computation.

4. Supply the following words and expressions with their equivalents from ex.2-3 and give their derivatives (part *a*):

a)
- операция (возведения в степень) — оперировать чем-то
- состоящий из чего-либо — состоять
- представленный в виде чего-либо — представление
- математическое выражение — выражать что-либо — выраженный чем-либо

b)
- конечное число
- бесконечная последовательность
- последующий
- обозначать (указывать на) число
- получать данное число
- порядок символов
- не представлять важности (значения, интереса)
- справа / слева / в середине
- занимать положение
- определять значение (символа)
- иметь явные преимущества
- с другой стороны
- касающийся чего-либо (относящийся к чему-либо)
- соответственно

5. Supply English equivalents and translate the text into Russian:

a) Adding and subtracting fractions

From the definition of (1) *дроби* it follows that (2) *сумма* (or (3) *разность*) of two fractions having the same (4) *знаменатель* is another fraction with this denominator, (5) *числитель* of which is the sum (or difference) of the numerators of the given fractions. Two fractions having different denominators (6) *можно складывать или вычитать* by first (7) *приводя их к дробям* with the same denominator.

b) Multiplying and dividing fractions

(1) *Произведение* of two fractions is a fraction the numerator of which is (2) *произведение числителей* of the factors, and the denominator of which is (3) *произведение знаменателей дробей*. (4) *Соотношение* of two fractions (5) *равняется произведению делимого на делитель* inverted; that is, the divisor with its terms interchanged.

6. Translate the following sentences into English:

1. Числитель дроби 2/3 — это 2, а знаменатель — 3.
2. Отношение 5 к 2 записывается как 5/2 или 5:2.
3. Показатели степени величин x^n и 2^m — это n и m.
4. Какие системы счисления вы знаете?
5. Три пятых, выраженные как десятичная дробь, — это 0,6.
6. Процентом называется дробь, у которой знаменатель равен 100.
7. В некоторых странах вместо десятичной точки используется запятая.
8. Например, логарифм числа 100 по основанию 10 равен 2, т.е. 10 нужно возвести в квадрат, чтобы получить число 100.
9. Любое положительное число, кроме единицы, может служить основанием логарифмов.
10. Архимед обратил внимание на свойство показателей степеней, лежащее в основе эффективности логарифмов: произведение степеней соответствует сумме показателей степеней.

Section 4

1. Study these mathematical terms and expressions:

- number — число
  - arbitrary number — произвольное число
  - number line — числовая прямая
  - even number — четное число

- odd number — нечетное число
- positive number — положительное число
- negative number — отрицательное число
- natural number — натуральное число
- prime (number)— простое число; штрих
- twin primes — сдвоенные простые числа; простые числа близнецы
- the sequence of primes — последовательность простых чисел
- coprime (relatively prime) — взаимно простое число
- composite number — составное число
- complex number — комплексное число
- imaginary number — мнимое число
- real number — действительное (вещественное) число
- integer (number) — целое число
- density of integers — плотность целых чисел
- rational number — рациональное число
- irrational number — иррациональное число

- reciprocal — обратный; обратная величина
  - reciprocal correspondence — взаимно обратное соответствие
  - reciprocal difference — обратная разность
  - reciprocal equation — возвратное уравнение
  - reciprocal (inverse) ratio — отношение обратных величин
- inverse — обратный, противоположный
  - inverse operation — обратная операция
  - inverse logarithm — антилогарифм
  - inverse matrix — обратная матрица
  - inverse semigroup — инверсная подгруппа
- invertible — обратимый
  - invertible matrix — обратимая (неособенная) матрица
- set — множество
  - set theory — теория множеств
  - set membership — принадлежность множеству
  - union of sets — объединение множеств
  - intersection of sets — пересечение множеств
  - finite set — конечное множество
  - infinite set — бесконечное множество
  - empty set — пустое множество

- closed set — замкнутое множество
- series (pl. series) — ряд (мн.ч. ряды)
  - infinite series — бесконечный ряд
  - convergent series / to converge — сходящийся ряд / сходиться
  - to extract the convergent series — извлекать сходящийся ряд
  - divergent series / to diverge — расходящийся ряд / расходиться
  - harmonic series — гармонический ряд
  - trigonometric series — тригонометрический ряд
  - binomial series — биномиальный ряд
- ordered pair — упорядоченная пара
- infinitesimal [ˌɪnfɪnɪˈtesɪməl] — бесконечно малое

2. Match the terms in the left column with their definitions in the right column and translate the sentences into Russian:

1. real number
2. natural number
3. integer
4. rational number
5. irrational number
6. prime
7. complex number
8. coprime
9. series
10. twin primes
11. even number
12. odd number
13. infinitesimal
14. imaginary number

1. any rational number that can be expressed as the sum or difference of a finite number of units, being a member of the set: -3, -2, -1, 0, 1, 2, 3
2. any real number that cannot be expressed as the ratio of two integers, such as π
3. quantity less than any finite quantity, yet not zero
4. a positive integer or zero
5. any real number of the form a /b, where a and b are integers and b is not zero
6. a complex number having its real part equal to zero
7. an integer that cannot be factorized into other integers but is only divisible by itself or 1, such as 2, 3, 7, and 11
8. a number consisting of a real and an imaginary part, either of which can be zero
9. a number leaving a remainder of 1 when divided by 2
10. any rational or irrational number
11. the sum of a finite or infinite sequence of

numbers or quantities
12. pairs of primes differing only by 2
13. a number divisible by two
14. any two positive integers whose only common positive divisor is 1, such as $8 = 2^3$ and $9 = 3^2$

3. Fill in the gaps with the words below and translate the text into Russian:

It is clear that for any complex number α there are — **1**— $z^n = \alpha$, and this shows one of — **2**— complex numbers over the other kinds of numbers considered. —**3**— of integers, or rational numbers or real numbers, one notes that each of these number systems is —**4**— the succeeding one, and that they are all subsystems of —**5**—. To find z such that $z^n = \alpha$, when α is, say, a rational number, is to consider α as a complex number that just happens to belong to the subsystem of real numbers and then —**6**— for complex numbers. This process is usually referred to as —**7**— of rational numbers to the complex field.

1. in order to extract roots
2. extending the field
3. the advantages of
4. the complex numbers
5. a subsystem of
6. *n* solutions of
7. to consider the problem

4. Place the sentences in the correct order to make a coherent text. Translate it into Russian paying special attention to the underlined words and expressions:

1. Sets may be finite or infinite.
2. A set is commonly represented by a list of its members, or elements, enclosed within braces; the statement that a set called A comprises the numbers 1, 2, and 3 is made by the expression A = {1,2,3}.
3. An infinite set has an endless number of members; all the positive integers or all points along a given line compose infinite sets.
4. Set theory is a branch of mathematics that deals with the properties of well-defined collections of objects, which may be of a mathematical nature, such as numbers or functions, or not.
5. A set that has no members is called an empty set (or a null or void set) and is denoted by the symbol Ø.

6. A finite set has <u>a definite number of members</u>; such a set might consist of all the integers from 1 to 1,000 or all marked bus stops along a given route.

5. Supply the following words and expressions with their English equivalents from ex. 4:

- определенное число членов
- бесконечное число членов
- образовывать бесконечные множества
- четко определенный (термин и т.д.)
- обычно представляют чем-либо (символом и.т.д.)
- математическая природа (сущность, характер)
- фигурные скобки
- список (перечень) элементов
- вдоль (на) данной прямой

6. Supply English equivalents and translate the text into Russian:

(1) *Операции* so far considered — namely, addition, subtraction, multiplication, and division — are known as (2) *элементарные алгебраические операции*. In another section the extension of these operations to more complex systems, such as polynomials, will be considered. One other (3) *операция над действительными или комплексными числами* must be considered here. (4) *Теорема утверждает, что:* If *a* is any positive real number and *n* is (5) *любое положительное целое число*, there exists a unique positive real number *x* such that $x^n = a$, in which x^n is (6) *произведение* of *n* factors each (7) *равное x*. (8) *Важно отметить, что* if *a* is an integer ((9) *рациональное число, отрицательное действительное число)* there is, in general, no integer (rational number, real number) such that $x^n = a$. (10) *Таким образом*, this operation (11) *называющаяся извлечением корня n-й степени* is not a satisfactory algebraic operation as it stands. To obtain a satisfactory operation, (12) *необходимо использовать комплексные числа*.

7. Translate the following sentences into English:

1. 6 и 3 являются множителями 18.
2. Давайте разложим это число на простые множители.

3. Рациональные и иррациональные числа вместе называются действительными или вещественными числами.
4. Дроби принято также называть рациональными числами, так как они представимы в виде отношений двух целых чисел.
5. «Число» i и его комбинации с обычными числами (типа $2 + 3i$) стали называться мнимыми, но современные математики предпочитают называть такие числа «компле́ксными».
6. Если a, b, c – целые числа и $a \times b = c$, то a и b являются делителями числа c.
7. Любое целое число, отличное от 1 и не имеющее собственных делителей, называется простым числом.
8. Если наибольший общий делитель двух целых чисел a и b равен 1, то числа a и b называются взаимно простыми.
9. Целое число, имеющее собственные делители, называется составным числом.
10. В качестве примера бесконечных рядов можно рассматривать бесконечные десятичные дроби.
11. О бесконечном ряде, который не сходится, говорят, что он расходится (такой ряд называют расходящимся).
12. Приведем еще несколько примеров сходящихся рядов с положительными членами.

Section 5

1. Study these mathematical terms and expressions:

- law — закон
  - commutative law — коммутативный закон
  - associative law — ассоциативный закон
  - distributive law — дистрибутивный закон
  - a law of cancellation — закон сокращения
  - law of contradiction — закон противоречия
  - law of identity — закон тождества
- an equivalence relation — отношение эквивалентности
  - symmetry — симметрия
  - reflexivity — рефлексивность, возвратность
  - transitivity — транзитивность
- scalar (dot) product — скалярное произведение
- cross product — векторное произведение
- unit vector — единичный вектор (орт)

- field — поле, тело, область
  - algebraic number field — поле алгебраических чисел
  - field of constants — поле констант
  - field of definition — поле определения
  - field of sets — тело множеств
- ring — кольцо
  - ring domain — кольцевая область
  - ordered ring — упорядоченное кольцо
  - density ring — плотное кольцо
- group — группа
  - group of extensions — группа расширений
  - group property — групповое свойство
  - group variety — групповое многообразие
  - permutation group — группа подстановок
  - continuous group — непрерывная (топологическая) группа
  - solvable group — разрешимая группа
  - Abelian group — коммутативная (абелева) группа
- matrix (pl. matrices) — матрица (мн.ч. матрицы)
  - matrix algebra — матричная алгебра
  - matrix unit — матричная единица
  - matrix represenration — матричное представление
  - matrix inversion — обращение матриц
- ideal — идеал
  - additive ideal theory — аддитивная теория идеалов
  - radical ideal — радикальный идеал
- elimination — элиминация, устранение, исключение (неизвестного)
- probability — вероятность
  - the theory of probability — теория вероятности
  - probability density — плотность вероятности
  - probability element — элемент вероятности
  - probability measure — вероятностная мера
  - absolute probability — безусловная вероятность
  - a priori (prior) probability — априорная вероятность

2. Fill in the gaps with the words below and translate the texts into Russian:

a) If it be assumed that division (except by zero) and subtraction are always possible, **—1—** positive integers then leads to a system of rational numbers (positive and negative fractions and integers, as well as zero). The **—2—** that every equation *by = a* in which *a* and *b* are positive integral coefficients has a solution **—3—** the idea that a **—4—,** *a*, can be divided into any positive whole number, *b,* of equal parts. The fundamental laws of arithmetic and **—5—** for multiplication, which states that, if *a x m = a x n*, then *m = n*, can all be preserved.

By the cancellation law for multiplication, *by = a* can have only one **—6—,** which is written *a/b*. It may be shown that the —7— for adding and multiplying fractions must hold. For example, if *by = a* and *dz = c*, then *bdy = ad, bdz = bc;* and so *bd(y + z) = bdy + bdz = ad + bc*, which proves the rule for adding fractions having different **—8—.** It can also —**9**— that division by fractions as well as by integers is possible in the new system.

Conversely, it can be proved that the rules for adding and multiplying fractions do give a system in which the laws of arithmetic**,** as well as the cancellation law for multiplication and certain other basic laws, are —**10—.**

A more thorough **—11—** would reveal that a considerable reduction in the number of postulates (fundamental laws needed to imply the others) is possible. All the laws for positive fractions thus can **—12—** the associative law for addition; the distributive laws, *a x (b + c) = (a x b) + (a x c)* and *(a + b) x c = (a x c) + (b x c)*; and the unit laws, *a x 1 = 1x a = a* and *1 + 1 x 1*.

1. be deduced from
2. the cancellation law
3. assumption
4. the system of
5. denominators
6. quantity
7. solution
8. usual rules
9. corresponds to
10. be proved
11. valid
12. study

b) Groups, in mathematics, are systems of elements with a composition **—1—** certain laws. The elements may be operations; for example, the rotations of a **—2—**. The symmetries of **—3—** are best described as a group. The ornamental wall designs of the ancient

Egyptians exhibit all possible **—4—.** Euclid studied the **—5—** of the regular **—6—** and the five regular solids. Not until the late 18th and 19th centuries, however, were groups recognized as **—7—.** The French mathematician Joseph-Louis Lagrange was one of the first **—8—** them. Another French mathematician, Augustin-Louis Cauchy, began a study of permutation groups. In studying the solution of **—9—,** a Norwegian mathematician, Niels Henrik Abel, showed that in general the equation of fifth degree cannot be solved by radicals. Then the French mathematician Évariste Galois, using groups systematically, showed that the solution of an equation by radicals is possible only if a group associated with the equation has certain specific properties; these groups are now called solvable groups.

The group concept is now recognized as one of the most fundamental in all of mathematics and in many of its **—10—.** The German mathematician Felix Klein considered geometry to be those properties of a **—11—** left unchanged by a certain specific group of transformations. In topology geometric entities are considered **—12—** if one can be transformed into another by an element of a continuous group.

1. to consider
2. space
3. a geometrical figure
4. equivalent
5. combinations of symmetries
6. mathematical systems
7. polynomial equations
8. sphere
9. satisfying
10. applications
11. properties
12. polygons

3. Read the following text and insert the missing sentences given below in the right places. Translate the text into Russian:

Relations involved in set theory

(1) The relations between sets can be of many sorts; e.g., "is a subset of " ($\subseteq$), "is equivalent to" (~), "is a complement of " ('), "is in one-to-one correspondence with," and "has the same cardinal number as". (2) Examples of pairing relations are: "is parallel to" (||), "is equal to" (=), "is less than" (<), and "is the same colour as." (3) More broadly conceived, pairing can include the relations depicted on charts and graphs, on which, for example, calendar years may be paired with automobile production figures, weeks with Dow-Jones averages, and degrees of angular rotation

with the lift accomplished by a cam. (4) If A = {x, z, w}, for example, and B = {4, 3, 9}, then A is in one-to-one correspondence with B if and only if a matching such as 4 with x, 3 with z, and 9 with w obtains without any element in either set left unmatched as a remainder. (5) The relations "is parallel to," "is the same colour as," and "is in one-to-one correspondence with," for example, all bear the stated relation to themselves as well as to other elements; thus, these relations are said to be reflexive. (6) These same relations share, in addition, the property that, if an element bears the stated relation to a second element, then the second also bears that relation to the first — a property known as symmetry. (7) Relations also have the property that, if two elements bear the stated relation to a third element, then they bear it to one another as well — a property known as transitivity. (8) In an equivalence relation, all elements related to a particular element are related to each other, thus forming what is called an equivalence class. (9) For each of the equivalence classes of sets, it is possible to construct an ordered set — for which not only the membership but also the sequence of its elements is significant — that can be used to name the class.

The sentences missing:

a. Those relations that have all three properties — reflexivity, symmetry, and transitivity — are called equivalence relations.
b. The relation of one-to-one correspondence between two sets can be conceived as one in which each element of a set A is matched with an element of another set B.
c. Many relations display identifiable similarities.
d. In addition, pairing relations, defined in terms of some specific criterion, can exist between the individual elements of a set.

4. Supply English equivalents and translate the texts into Russin:

I. Rings

A ring is a set of elements that (1) *можно складывать, вычитать и умножать* and in which these three operations obey (more or less) the usual rules. In particular, if R is a (2) *множество действительных или комплексных чисел* such that (3) *сумма, разность и произведение* of any two elements of R is in R, then R is a ring for the usual operations. It is not required that (4) *деление на ненулевой элемент кольца* be always possible; thus the divisibility problem for elements of a ring is important, as it is for the ordinary (5) *целых чисел*.

The rings introduced by the German arithmeticians of the 19th century (6) *для изучения задач* involving divisibility and factorization were, in fact, mostly (7) *множества комплексных чисел* (more precisely of algebraic integers) as above. The main impetus came from the intensive study of Fermat's *уравнения* $x^n + y^n = z^n$, stated (8) *без доказательства* by the French mathematician Pierre de Fermat to have no solution (x, y, z) in integers greater than 1 for (9) *показателей степеней n* greater than 2.

An early fruitful (10) *подход к задаче* introduced a complex *n*th root of unity and the numbers that are polynomials in such an *n*th root, coefficients a_i being ordinary integers; these numbers formed a ring R. Fermat's equation could be written in a way that (11) *был связан с задачей* of decomposing x^n in the ring R. If the analogue of the unique (12) *разложения целого на простые множители* were true in R, then Fermat's statement could be proved. This method (or a variant) worked for some (13) *значений* of *n* (e.g., *n* = 3, 5, 14), but the German mathematicians Peter Gustav Lejeune Dirichlet and Ernst Eduard Kummer noticed that the analogue of the unique decomposition was not true in general. As a remedy, Kummer and Richard Dedekind (14) *ввели понятие идеала*, which proved to be of fundamental importance for ring theory.

II. Fields

Broadly speaking, a field is an algebraic system (1) *состоящая из элементов* that are commonly called numbers, in which the four familiar (2) *операции сложения, вычитания, умножения и деления* are universally defined (except for division by zero) and have all their (3) *обычные свойства*. Much of the general theory of vectors and matrices can be developed over an arbitrary field. In particular, this is true of the general theory of (4) *системы линейных уравнений* and of (5) *их решения* by a method known as Gaussian elimination. The study of various special fields also explains which (6) *геометрические построения, таких как* angle bisection and angle trisection, can be made with ruler and compass and which cannot. Moreover, (7) *конечные поля* of 2 elements have been used to construct the best known error-correcting codes.

Although many particular fields — including especially the rational, real, and complex fields, (8) *конечные поля и поля алгебраических чисел*

— were intensively studied in the 17th, 18th, and early 19th centuries, the idea of investigating all possible fields seems not to have been conceived until 1910, when (9) *математик* Steinitz предложил (10) *систематическую схему* for classifying them.

5. Translate the following sentences into English:

1. Используя закон дистрибутивности, можно раскрыть скобки: 2(3 + 4) = 2·3 + 2·4.
2. Скалярное умножение также обладает свойством дистрибутивности.
3. Зная скалярное поле, можно определить связанное с ним векторное поле.
4. Сложение чисел подчиняется закону ассоциативности.
5. Теория вероятностей занимается изучением событий и их вероятностей, представляемых числами, заключенными в интервале от 0 до 1.
6. Если к аксиомам группы добавить в качестве четвертой аксиомы свойство коммутативности $a \cdot b = b \cdot a$, то мы получим структуру коммутативной (или абелевой) группы.
7. Матрицы используются и при решении систем дифференциальных уравнений, которые возникают в большинстве наук.
8. Значительная часть теории колец возникла в результате попыток доказать теорему Ферма.
9. Одна из основных задач теории групп – более явное описание структуры некоторых классов групп.
10. Два многочлена являются одним и тем же элементом кольца в том и только в том случае, когда коэффициенты при одинаковых степенях переменной x равны.

Section 6

1. Study these terms and expressions from the field of physics and mechanics:

- physics /physicist / physical — физика / физик / физический, телесный, материальный
  - physical magnitudes — физические величины
  - physical world — материальный мир

- physical state — физическое состояние
- physical space — физическое пространство
- physical body — физическое тело
- physical properties — физические свойства
- physical laws — физические законы

- mechanics / mechanic/ mechanical — механика / механик / механический
  - solid mechanics — механика твердого тела
  - classic mechanics — классическая (теоретическая) механика
  - continuum mechanics — механика сплошной среды
  - fluid mechanics — механика жидкостей и газов
  - analytical mechanics — аналитическая механика
  - statistical mechanics — статистическая механика
  - celestial mechanics — небесная механика
  - mechanical engineering — машиностроение
- body — тело
  - body in motion — движущееся тело
  - body at rest — покоящееся тело
  - solid body — твердое тело
  - celestial (heavenly) body — небесное тело
  - terrestrial body — земное тело
  - black body (full radiator) — абсолютно черное тело
- matter — материя, вещество
- substance — вещество, материя, субстанция
- particle — частица, материальная точка
- force — сила
  - external force — внешняя сила
  - impressed force — приложенная сила
  - centrifugal force — центробежная сила
  - centripetal force — центростремительная сила
  - resulting force — результирующая сила
  - composition / resolution of forces — сложение / разложение сил
- motion — движение
  - laws of motion — законы движения
  - uniform (constant) motion — равномерное прямолинейное движение

- simple harmonic motion — гармонические колебания

- medium — среда
  - elastic medium — упругая среда
  - magnetic medium — магнитная среда
  - fluid — текучая среда (жидкость, газ)
  - fluid dynamics — динамика жидкостей, гидродинамика
  - liquid — жидкость
- equilibrium — равновесие
  - stable equilibrium — устойчивое равновесие
  - unstable equilibrium — неустойчивое равновесие
  - neutral (indifferent) equilibrium — безразличное равновесие
  - equilibrium condition — условие равновесия
- gravity — притяжение
  - pull (force) of gravity — сила притяжения
  - centre of gravity — центр тяжести
  - gravitational field — гравитационное поле
- heat — тепло
  - heat capacity — теплоемкость
  - heat conduction — теплопроводность
- energy — энергия
  - energy flow — поток энергии
  - energy level — энергетический уровень
  - kinetic (potential, total) energy — кинетическая потенциальная, полная) энергия
- power — сила, энергия, мощность, способность производить работу
  - power engineering — энергетика
  - power output — выходная мощность
  - power supply — энергопитание
- work — работа
  - units of work — единицы работы
- relativity / relative — относительность / относительный
  - relativity theory — теория относительности

2. Translate the following definitions from English into Russian paying special attention to the underlined words and expressions:

1. *Physics* is the science that deals with heat, light, and other forms of energy and how they affect objects.
2. *Mechanics* is the area of physics that deals with the forces such as gravity that affect all objects.
3. *Fluid* is a substance, as a liquid or gas, that is capable of flowing and that changes its shape at a steady rate when acted upon by a force tending to change its shape.
4. *Liquid* is a substance in a physical state in which it does not resist change of shape but does resist change of size.
5. *Equilibrium* is a state of rest or balance due to the equal action of opposing forces.
6. *Medium* is an intervening substance, as air, through which a force acts or an effect is produced.
7. *Power* is work done or energy transferred per unit of time.

3. Match the terms in the left column with their definitions in the right column and translate the sentences into Russian:

1. particle	1. the force of attraction by which terrestrial bodies tend to fall toward the centre of the earth
2. gravity	2. the capacity of a body or system to do work
3. heat	3. the energy transferred as a result of a difference in temperature
4. matter	4. the transfer of energy expressed as the product of a force and the distance through which its point of application moves in the direction of the force
5. work	5. a force moving or directed outward from the centre
6. force	6. a force directed toward the centre
7. centrifugal force	7. the substance or substances of which any physical object consists or is composed
8. centripetal force	8. one of the extremely small constituents of matter, as an atom or nucleus
9. body	9. an object or substance that has three dimensions, a mass, and is distinguishable from surrounding objects
10. energy	10. a dynamic influence that changes a body from a state of rest to one of motion or changes its rate of motion

4. Supply the following words and expressions with their equivalents from ex.2-3 and give their derivatives (part *a*):

a) - вместимость, емкость, производительность — вместительный, емкий
- быть способным (сделать что-то) — способность
- притяжение — притягивать
- передавать (энергию и т.д.) — передача
- стремиться сделать что-то — стремление, тенденция к чему-то
- сопротивляться — сопротивление
- составляющая (материи и т.д.) — составлять что-либо
- отличающийся от чего-либо — отличать что-либо от чего-либо
- по направлению к чему-либо — направленный к чему-либо / от чего-либо

b) - на постоянной скорости
- благодаря чему-либо
- влиять (оказывать влияние) на объекты (предметы)
- на единицу времени
- производить результат (действие)
- противоположные силы
- состояние покоя

5. Supply English equivalents and translate the text into Russian:

(1) *Теория относительности* is a theory, formulated essentially by Albert Einstein, that all (2) *движение* must be defined relative to a frame of reference and that space and time are relative, rather than absolute (3) *понятия*: it (4) *состоит из* two principal parts. The theory (5) *в которой рассматривается равномерное прямолинейное движение* (special theory of relativity or special relativity) (6) *основана на* the two postulates that (7) *физические законы* have the same (8) *математическую форму* when expressed in any inertial system, and the velocity of light (9) *не зависит от* the motion of its source and will have the same (10) *значение* when measured by observers moving with constant velocity with respect to each other. Derivable from these postulates are the conclusions that there can be no motion at a speed greater than that of light in a vacuum, mass increases as velocity increases, mass and energy are equivalent, and time is dependent on (11) *относительного движения* of an observer (12) *измеряющего время*.

6. Translate the following sentences into English:

1. Физики изучают явления природы.
2. Жидкость может превратиться в газ.
3. Химические процессы производят тепло.
4. Механика — раздел физики, в котором изучается движение тел под действием сил.
5. Солнце является главным источником энергии, поступающей на Землю.
6. Сила заставляет тело двигаться или изменять способ его движения.
7. Механические свойства газов и жидкостей в какой-то мере сходны.
8. Эйнштейн (Einstein) известен прежде всего как автор теории относительности.
9. Физика твердого тела — раздел физики, изучающий структуру и свойства твердых тел.
10. Сложение и разложение сил — одни из основных понятий механики.
11. Тяготение, или гравитация, — свойство материи, которое состоит в том, что между любыми двумя частицами существуют силы притяжения.
12. Твердые тела обладают разнообразными механическими, электрическими и магнитными свойствами.

Section 7

1. Study these terms and expressions from the field of physics and mechanics:

- weight / to weigh — вес / взвешивать
  - a table (a system) of weights — таблица (система) мер веса
- mass — масса
  - rest mass — масса покоя
  - centre of mass — центр массы, центр тяжести
  - mass point — материальная точка
- speed / velocity — скорость

- terminal velocity — предельная скорость
- zero velocity — нулевая скорость
- angular velocity — угловая скорость
- relative velocity — относительная скорость
- constant velocity — скорость равномерного прямолинейного движения
- constant speed — скорость, постоянная по величине
- critical speed — критическая скорость
- at high /low speed — на высокой/низкой скорости

- acceleration [əkˌseləˈreɪʃn]/ to accelerate — ускорение / ускоряться
- acceleration of gravity — ускорение силы тяжести
- deceleration [diːˌseləˈreɪʃn] / to decelerate — замедление / замедляться
- free fall — свободное падение
- altitude — высота
- momentum — количество движения, импульс
- angular (moment of) momentum — момент количества движения, кинетический момент
- axle — ось, вал
- rotation — вращение
  - rotation of axes — поворот осей
  - rotating body — вращающееся тело
- action / reaction — действие/противодействие
- inertia — инерция
  - inertia force — сила инерции
  - inertia field — поле инерции
  - moment of inertia — (осевой, полярный) момент инерции
  - product of inertia — центробежный момент инерции
  - principle axes of inertia — главные оси инерции
- torque [tɔːk] — момент силы
- pressure (upon sth.) — давление (на что-либо)
  - impact pressure — полное давление
  - static pressure — статическое давление
- inclined plane — наклонная плоскость
- path — путь, траектория
- lever — рычаг
  - lever arm — плечо рычага

2. Match the terms in the left column with their definitions in the right column:

1. weight
2. axle
3. torque
4. acceleration
5. inertia
6. pressure
7. momentum
8. inclined plane
9. terminal velocity
10. altitude

1. the force applied to a unit area of a surface, usually measured in pascals (newtons per square metre), millibars, torr, or atmospheres
2. the vertical height of an object above some chosen level, esp. above sea level
3. a bar or shaft on which a wheel, pair of wheels, or other rotating member revolves
4. the rate of increase of speed or the rate of change of velocity
5. the constant maximum velocity reached by a body falling under gravity through a fluid, esp. the atmosphere
6. a plane whose angle to the horizontal is less than a right angle
7. the vertical force experienced by a mass as a result of gravitation
8. the tendency of a body to preserve its state of rest or uniform motion unless acted upon by an external force
9. a quantity expressing the motion of a body or system, equal to the product of the mass of a body and its velocity
10. any force or system of forces that causes or tends to cause rotation

3. Fill in the gaps with the words below and translate the text into Russian:

Acceleration is the time rate at which a —**1**— is changing. Because velocity has both —**2**— and direction, it is called —**3**—**;** acceleration is also a vector quantity and must account for changes in both the magnitude and direction of a velocity. The velocity of a point or an object moving on a —**4**— path can change in magnitude only; on a —**5**— path, it may or may not change in magnitude, but it will always change in direction. This —**6**— means that the acceleration of a point moving on a curved path can never be zero.

If the velocity of a point moving on a straight path is increasing (i.e., if the speed, which is the magnitude of the velocity, is increasing), the —**7**— will have the same —**8**— as the velocity vector. If the velocity is decreasing (that is, the point or object is decelerating), the acceleration vector will point in the —**9**— direction. The average acceleration during a time interval —**10**— the total change in the velocity during the interval —**11**— the time interval. The acceleration at any instant is equal to the limit of the —**12**— of the velocity change to the length of the time interval, as the time interval approaches zero.

1. ratio
2. velocity
3. condition
4. curved
5. acceleration vector
6. a vector quantity
7. direction
8. straight
9. divided by
10. is equal to
11. magnitude
12. opposite

4. Supply English equivalents and translate the text into Russian:

Weight is (1) *гравитационная сила* of attraction on an object, caused by the presence of a massive second object, such as the Earth or Moon. Weight is a consequence of (2) *всеобщего закона тяготения*: any two objects, because of their masses, attract each other with a force that is (3) *прямо пропорциональна произведению их масс* and inversely proportional to (4) *квадрату расстояния между ними*. Thus more massive objects, of course, weigh more in the same location; the farther an object is from the Earth, the smaller is its weight. (5) *Вес объекта* at the Earth's South Pole is slightly more than its weight at the Equator because the polar radius of the Earth is slightly less than the equatorial radius. Though the mass of an object (6) *остается постоянной*, its weight varies according to its location. The smaller mass and radius of the Moon (7) *по сравнению с* those of the Earth combine to make the same object (8) *на поверхности Луны* weigh one-sixth the value of its weight on Earth. Because of all the mass in the universe, (9) *каждая точка в пространстве обладает свойством* called the gravitational field at that point, numerically equal to (10) *ускорению силы тяжести* at that point. Alternatively, weight is the product of an object's mass and either the gravitational field or the acceleration of gravity at the point where the object is located.

5. Translate the following sentences into English:

1. Как можно измерить давление ветра на здание?
2. Принцип инерции был одним из самых значительных вкладов Галилея в физику.
3. Архимед показал, как находить центр тяжести различных геометрических фигур.
4. Вес тела равен его массе, умноженной на ускорение свободного падения.
5. Два основных простейших механизма — это рычаг и наклонная плоскость.
6. Под «силой тяжести» принято понимать силу, создаваемую тяготением массивного тела, а под «ускорением силы тяжести» – ускорение, создаваемое этой силой.
7. Момент силы равен произведению силы на ее плечо.
8. Чтобы изучать движение небесных тел, познакомимся с силой гравитации.
9. Количество теплоты, которым обладает тело при данной температуре, зависит от его массы.
10. Импульс тела *p* определяется как произведение его массы на скорость.
11. Чтобы можно было описывать движение материальной точки, нужно определить ее положение в данный момент.
12. Теоретически колесо можно представить как бесконечную последовательность рычагов.

Section 8

1. **Study these terms and expressions from the field of physics and mechanics:**
   - wave — волна
     - wave/quantum mechanics — волновая / квантовая механика
     - wave-particle theory — теория волновых частиц
     - wave drag — волновое сопротивление
     - wave equation — волновое уравнение
     - gravity wave — гравитационная волна
     - surface waves — поверхностные волны
     - plane / cylindrical / spherical wave — плоская / цилиндрическая / сферическая волна
     - theory of long waves — теория длинных волн (теория мелкой воды)
     - shock wave — ударная волна
     - wave length of light — длина волны света
   - light — свет
     - light quantum — световой квант
     - light year — световой год
     - a beam of light — луч света
     - the corpuscular theory of light — корпускулярная теория света
     - to absorb light — поглощать свет
   - spectrum (pl.n. spectra) — спектр, спектральная функция (мн.ч. спектры)
     - line spectrum — линейчатый спектр
     - continuous spectrum — непрерывный спектр
     - point spectrum — точечный (дискретный) спектр
   - diffraction / to diffract — дифракция / дифрагировать
     - diffraction grating — дифракционная решетка
   - refraction / to refract — рефракция, преломление / преломлять
     - refractive index — показатель преломления
   - dispersion / to disperse — дисперсия, рассеяние / рассеивать
     - dispersion index — индекс рассеяния
   - interference / to interfere — интерференция, помехи / интерферировать
   - optics / optical — оптика / оптический

- optical instrument (device) — оптический прибор
- optical illusion — оптический обман
- optical axis — оптическая ось
- optical effect — оптический эффект
- magnifying glass — увеличительное стекло
- reflecting / refracting telescope — телескоп-рефлектор / телескоп-рефрактор

- radiation / radiant / to radiate — излучение, радиация, источник излучения / лучистый, излучающий / излучать
  - radiation counter — индикатор излучения
  - radiation field — поле излучения
  - radiation pressure — световое давление
  - radiant / electromagnetic energy — энергия излучения / электромагнитная энергия
- sound — звук
  - sound intensity — интенсивность звука
  - sound wave — звуковая волна
- oscillation / to oscillate — колебания / колебаться
  - forced oscillations — вынужденные колебания
  - free oscillations — свободные колебания
  - oscillating circuit — колебательный контур
  - oscillator / harmonic oscillator — осциллятор, генератор колебаний / гармонический осциллятор
- friction — трение
  - rolling friction — трение качения
  - sliding friction — трение скольжения
  - static friction — трение покоя
  - angle of friction — угол трения
  - friction factor — коэффициент трения
- resistance — сопротивление (резистанс)
  - mechanical resistance — механический резистанс
- buoyancy — плавучесть
  - centre of buoyancy — центр плавучести
- gyroscope (gyro) — гироскоп
- density / dense — плотность / плотный
- current — ток
  - alternating current— переменный ток

- direct current — постоянный ток

2. Match the terms in the left column with the definitions in the right column. Translate the sentences into Russian paying special attention to the underlined words and expressions:

1. dispersion
2. radiation
3. friction
4. sound
5. diffraction
6. refraction
7. oscillation
8. optics
9. shock wave
10. magnifying glass

1. the change of direction of a ray of light, sound, heat, or the like, in passing obliquely from one medium into another in which its wave velocity is different
2. deviation in the direction of a wave at the edge of an obstacle in its path
3. a the separation of white or compound light into its respective colours, as in the formation of a spectrum by a prism
4. a lens that produces an enlarged image of an object
5. a region of abrupt change of pressure and density moving as a wave front at or above the velocity of sound, caused by an intense explosion or supersonic flow over a body
6. surface resistance to relative motion, as of a body sliding or rolling
7. an effect expressible as a quantity that repeatedly and regularly fluctuates above and below some mean value, as the pressure of a sound wave or the voltage of an alternating current
8. the process in which energy is emitted as particles or waves
9. the branch of physical science that deals with the properties and phenomena of both visible and invisible light and with vision
10. periodic disturbance in the pressure or density of a fluid or in the elastic strain of a solid, produced by a vibrating object

3. Read the following text and insert the missing sentences given below in the right places. Translate the text into Russian paying special attention to the underlined words and expressions:

(1) There are a number of ways in which spectra are produced in nature. (2) The primary rainbow is formed by reflection and refraction of light in raindrops. (3) The rays emerging from the drops are spread out, but for any given wavelength there is a minimum angle of deviation and there is a concentration of energy at this angle. (4) Because of the dispersion of water, the angles for different wavelengths are not exactly the same, and the red is seen on the outside and blue on the inside of the bow. (5) A weaker rainbow is formed by rays that have been twice reflected. (6) In this the colours are reversed. (7) A rainbow may be regarded as a spectrum of the Sun, but the purity is low.

The sentences missing:

a. The rainbow is the most striking of these.
b. For green light the minimum angle of deviation is about 138 and an observer with his back to the Sun sees the bow at an angle of 42 to the direction of the Sun's rays.
c. Still weaker supernumerary bows are caused by diffraction in droplets.

4. Supply the following words and expressions with their equivalents from ex.2-3 and give their derivatives (part *a*):

a) - наблюдатель — наблюдать — наблюдение
- быть вызванным чем-либо — причина чего-либо
- увеличенный — увеличивать
- концентрация (энергии и т.д.) — концентрировать, сосредотачивать
- косо (проходить)— косой, наклонный, скошенный
- обратный (процесс и т.д.) — обратно, противоположно — менять местами, менять (на противоположный)
- колебаться, быть неустойчивым — колебание, вариация
- разделение — разделять
- выражаемый (как что-либо) — выражать — выраженный — выражение (например, в математике)
- видимый — невидимый — видение, зрение
- явление (-я) (природы) — феноменальный
- распространяться — распространение чего-либо

b) - минимальный угол отклонения
- может считаться (рассматриваться) как что-либо
- появляться, возникать из чего-либо
- среднее значение
- резкая перемена в чем-либо
- выделять энергию
- первичный
- препятствие

5. Fill in the gaps with the words below and translate the text into Russian:

Archimedes' principle is a —**1**— of buoyancy, discovered by the ancient Greek mathematician and —**2**— Archimedes, stating that any body completely or partially submerged in a fluid (gas or liquid) at rest is acted upon by an upward, or buoyant, force the —**3**— of which is equal to the weight of the fluid displaced by the body. The —**4**— of displaced fluid is equivalent to the volume of an object fully immersed in a fluid or to that fraction of the volume below the —**5**— for an object partially submerged in a liquid. The —**6**— of the displaced portion of the fluid is equivalent to the magnitude of the buoyant force. The buoyant force on a body floating in a liquid or gas is also equivalent in magnitude to the weight of the floating object and is —**7**— in direction; the object neither rises nor sinks. A ship that is launched sinks into the ocean until the weight of the water it displaces is just —**8**— its own weight. As the ship is loaded, it sinks deeper, displacing more water, and so the magnitude of the buoyant force continuously matches the weight of the ship and its cargo.

If the weight of an object is less than that of the displaced fluid, the object rises, as in the case of a block of wood that is released beneath the surface of water or a helium-filled balloon that is let loose in air. An object —**9**— the amount of the fluid it displaces, though it sinks when released, has an apparent weight loss equal to the weight of the fluid displaced. In fact, in some accurate weighings, a correction must be made in order to compensate for the buoyancy effect of the surrounding air.

Buoyancy —**10**— the increase in fluid —**11**— at increasingly greater depths. The pressure on a submerged object, therefore, is greater on the parts more deeply submerged, and the buoyant force is always upward, or opposite to —**12**—; it is the net effect of all the forces exerted on the object by the fluid pressure.

1. is caused by
2. inventor
3. magnitude
4. heavier than
5. the gravitational force
6. equal to
7. surface
8. weight
9. volume
10. opposite
11. physical law
12. pressure

6. Supply English equivalents and translate the text into Russian:

Light is a (1) *основной* aspect of the human environment that (2) *нельзя определить* in terms of anything simpler or more directly appreciated by the senses than itself. Light, certainly, (3) *отвечает за* the sensation of sight. Light is propagated (4) *со скоростью* that is high but not infinitely high. (5) *Физики* are acquainted with two methods of propagation from one place to another, as 1) particles and as 2) waves, and for a long time they have sought to define light (6) *с точки зрения либо частиц, либо волн.*

(7) Два свойства света are, perhaps, more basic and fundamental than any others. The first of these is that light is (8) *вид энергии* conveyed through empty space (9) *на высокой скорости* (in contrast, many forms of energy, such as the chemical energy stored in coal or oil, (10) *могут передаваться* from one place to another only by transporting the *материю* in which the energy is stored). The unique property of light is, (11) *таким образом*, that energy in the form of light is always moving, and its movement is only in an indirect way affected by (12) *движением материи* through which it is moving. (When light energy (13) *прекращает двигаться,* because it has been absorbed by matter, it is no longer light.)

The second fundamental property is that (14) *луч света* can convey information from one place to another. This information concerns both the (15) *источника света* and also any objects that have partly absorbed or (16) *отразили или преломили* the light before it reaches the observer. More information (17) *достигает* the human brain through the eyes than through any other sense organ. Even so, the visual system extracts only a minute fraction of the information that is imprinted on the light that enters the eye. (18) *Оптические инструменты* extract much more information from the visual scene; spectroscopic instruments, for example, reveal far more about a source of light than the eye can discover by noting its colour, and telescopes and microscopes extract (19) *научную информацию* from the environment. Modern optical instruments produce, indeed, so much information that automatic (20) *методы записи и анализа* are needed to enable the brain to comprehend it.

7. Translate the following sentences into English:

1. Видимый свет – это лишь малая часть широкого спектра электромагнитного излучения.
2. Ньютон (Newton) высказал гипотезу о корпускулярной природе света.
3. Ньютон поставил перед собой задачу описать на языке математики процесс распространения звуковой волны в воздухе.
4. Типичный пример спектра – хорошо известная всем радуга.
5. Два основных вида телескопов — это рефлектор и рефрактор.
6. Плотность газа в фотосфере в тысячи раз меньше плотности воздуха у поверхности Земли.
7. Птолемей (Ptolemy) проводил опыты по преломлению света при переходе его из воздуха в воду или стекло.
8. Эйнштейн (Einstein) внес значительный вклад в создание квантовой механики, развитие статистической физики и космологии.
9. Галилей (Galileo) экспериментально установил, что в отсутствие сопротивления воздуха тяжелые и легкие тела падают на землю с одинаковым ускорением.
10. Квантовая механика — фундаментальная физическая теория динамического поведения всех элементарных форм вещества и излучения, а также их взаимодействий.

Part II (Revision)

Section 9 (Supplementary Exercises)

1. Fill in the gaps with the words below and translate the texts into Russian:

a) The Beginnings of Science in Greece

The first sciences in the modern sense were those —**1**— mathematics. They were begun in Mesopotamia and in Egypt and were passed on to the Greeks.

The Greek philosopher Archimedes was a great mathematician and an important early writer on —**2—.** Mathematics and mechanics —**3**— during the Golden Age of Greece, beginning about 600 BC. The knowledge of geometry —**4**— in Greek architecture. —**5**— physics was used in building as well as in war. The —**6**— made it possible to move huge stones for building. With the catapult soldiers were able to hurl heavy spears or large rocks at enemy fortifications.

Another science which the ancient Greeks —**7**— was astronomy. They could foretell to the day when a certain planet would be —**8**— and even where it would appear in the heavens. This kind of science was what would be known today as the "astronomy of position."

—**9**— began when the Greeks started to ask serious questions about the world around them. They wanted to know what things were made of and where they came from. They wished not only to make and build things but to know how and why things were as they were. Asking these questions and getting the first answers — many of which were later proved wrong – —**10**— of Western science. The Greeks —**11**— on to the Romans and the other people of Western Europe. For many centuries European science —**12**— for the most part on the early theories of the Greeks.

1. connected with
2. developed
3. laid the foundations
4. lever
5. passed their theories
6. the knowledge of
7. the science of mechanics
8. theoretical science
9. was applied widely
10. was based
11. were put to practical use
12. visible

b) It is important to be aware of the character of the sources for **—1—** the history of mathematics. The history of Mesopotamian and Egyptian mathematics **—2—** the many extant original documents written by scribes. Although **—3—** Egypt these documents are few, they are all of a type and leave little **—4—** that Egyptian mathematics was, on the whole, **—5—** and profoundly practical in its orientation. For Mesopotamian mathematics, **—6—,** there are a large number of clay tablets, which reveal mathematical **—7—** of a much higher order than those of the Egyptians. The tablets indicate that the Mesopotamians had a great deal of remarkable mathematical **—8—,** although they offer no **—9—** that this knowledge was organized into a deductive system. Future **—10—** may reveal more about the early **—11—** of mathematics in Mesopotamia or about its **—12—** on Greek mathematics, but it seems likely that this picture of Mesopotamian mathematics will stand.

1. achievements	7. influence
2. development	8. is based on
3. doubt	9. knowledge
4. evidence	10. on the other hand
5. elementary	11. research
6. in the case of	12. the study of

c) As was noted earlier, after the work of David Hilbert, at the turn of the 20th century, the **—1—** of geometry were generalized and the classical concepts of space and **—2—** in space, which derived from intuition, were replaced with abstract ideas. A step toward the generalization of classical geometry was taken when **—3—** geometry known as analytical geometry was created and first used in 1637 by Rene Descartes. Descartes applied algebra to geometry not just in the use of algebra to manipulate the dimensions of geometric figures but also in the representation of a **— 4—** by a pair of numbers and the representation of lines and curves by **— 5 —.** It was a powerful general method of **—6—** certain geometric problems and one that could be **— 7 —** to certain types of curves more readily than the geometry of the Greeks that was based on axioms. **— 8 —** analytic geometry is the idea that a point in **—9—** can be specified by numbers giving its position. The notion that any point, for example, can be **—10—** its latitude, longitude, and height above the Earth **—11—** Archimedes and to Apollonius of Perga, who lived in the 3rd century BC. It was Descartes and another 17th-century Frenchman, Pierre de Fermat, however, who **—12—** that notion systematically.

1. applied
2. foundations
3. developed
4. equations
5. goes back to
6. indicated by
7. objects
8. point
9. solving
10. space
11. the basis of
12. the branch of

d) A scientific — **1**— to the origin of —**2**— became possible only after the publication of Isaac Newton's —**3**— of motion and gravitation in 1687. Even after this breakthrough, many years elapsed while scientists struggled with applications of Newton's laws to explain the apparent —**4**— of planets, satellites, comets, and asteroids. Meanwhile, the first semblance of a modern theory for solar system —**5**— was proposed by the German philosopher Immanuel Kant in 1755. Kant's central — **6**— was that the system began as a cloud of dispersed —**7**—**.** He assumed that the mutual —**8**— of the particles caused them to start moving and colliding, at which point —**9**— kept them bonded together. As some of these aggregates became larger than others, they grew still more rapidly, ultimately forming the planets. Because Kant was not highly versed in either —**10**— or mathematics, he did not recognise the intrinsic limitations of his primitive approach. His model does not account for planets moving around the Sun in the same direction and in the same —**11**—, as they are observed to do, nor does it —**12**— the revolution of planetary satellites.

1. explain
2. approach
3. chemical forces
4. gravitational attractions
5. idea
6. laws
7. motions
8. plane
9. particles
10. the solar system
11. origin
12. physics

2. Supply English equivalents and translate the texts into Russian:

a) Linear algebra (1) *это раздел алгебры, в котором рассматриваются* primarily linear problems, that is to say, problems that depend for the most part (2) *от решения линейных уравнений*. An equation (3) *с двумя или более переменными, или неизвестными,* is linear if it contains no terms of the second degree or greater; (4) *то есть,* if it (5) *не содержит произведений или степеней* of the variables. The term linear (6) *происходит из того факта, что* the graph of a linear equation in x and y is (7) *прямая линия* in the Cartesian xy-plane. (8) *Таким образом,* a linear equation (9) *представляет линейное отношение между* the variables x and y (10) *в геометрическом смысле*. Similarly, a linear equation in three variables x, y, z represents (11) *плоскость в трехмерном пространстве*, and two such equations considered simultaneously (12) *представляют линию пересечения двух плоскостей,* provided they are not parallel. When the number of variables is greater than three, there is no longer a simple geometric interpretation because (13) *физическое пространство ограничено тремя измерениями.* (14) *Тем не менее,* it is customary to continue the geometric analogy and to think of the solutions of a linear equation in four variables as constituting a "hyperplane" in a four-dimensional space, and similarly for any finite higher dimension.

b) *Foundations of geometry*

Although the emphasis of mathematics after 1650 was increasingly on analysis, foundational questions in classical geometry (1) *продолжали вызывать интерес*. Attention centred on the fifth postulate of Book I of the Elements, which Euclid had used to (2) *доказать существование* of a unique parallel (3) *через точку к данной линии* to a given line. Since antiquity, Greek, Islamic, and European geometers (4) *безуспешно пытались показать* that the parallel postulate need not be a postulate (5) *но вместо этого его можно вывести из* the other postulates of Euclidean geometry. During the period 1600-1800 mathematicians (6) *продолжали эти попытки* by trying to show that the postulate was equivalent to some result that (7) *считался самоочевидным*. Although the decisive breakthrough to non-Euclidean geometry would not occur until he 19th century, (8) *исследователи достигли более глубокого и систематичного понимания* of the classical (9) *свойств пространства*.

Interest in the parallel postulate (10) *возник* in the 16th century after the recovery and Latin translation of Proclus' commentary on Euclid's Elements. The Italian researchers Christopher Clavius in 1574 and Giordano Vitale in 1680 showed that the postulate is equivalent to asserting that (11) *линия, равноудаленная от прямой линии есть* *прямая линия*. In 1693 John Wallis, Savilian professor of geometry at Oxford, attempted a different demonstration, proving that the axiom (12) *следует из предположения* that to every figure there exists (13) *такая же фигура произвольной величины*.

In 1733 the Italian Girolamo Saccheri (14) *опубликовал* his ‘Euclides ab Omni Naevo Vindicatus’ (Euclid Cleared of Every Flaw). This was an important work of synthesis in which he (15) *предоставил полный анализ* of the problem of parallels (16) *с точки зрения* Omar Khayyam's quadrilateral. Using the Euclidean assumption that straight lines do not enclose (17) *площадь*, he was able to exclude geometries that contain no parallels. (18) *Оставалось доказать существование* of a unique parallel through a point to a given line. To do this Saccheri adopted the (19) *процедуру доказательства от противного*; he assumed the existence of more than one parallel and attempted to derive a contradiction. (20) *После долгого и детального исследования*, he was able to convince himself (mistakenly) that he had found the desired contradiction.

c) The role of topology in mathematics

(1) *Применения* of topology to mathematics itself are much deeper and more far-reaching than the above examples would indicate. Most of those who work in topology are not searching for immediate applications. They study it because of the challenge it offers and because they wish (2) *узнать больше о свойствах действительного и абстрактного пространств*. They wish to know whether (3) *один ряд свойств* implies another set of properties. They wish to know what is true and what is possible. Topological (*4) понятия и методы лежат в основе* much of modern mathematics, and the topological (5) *подход* has illuminated and clarified (6) *самые основные структурные понятия в различных разделах этого предмета*.

In the same way that the Euclidean plane (7) *удовлетворяет некоторым аксиомам или постулатам*, it can be shown that certain abstract spaces (8) *имеют определенные свойства* without examining these spaces individually. By approaching topology (9) *с этой абстрактной точки зрения*, it is possible to use its methods to study things other than collections of points. Collections of entities that are of concern in analysis or algebra or collections of (*10) геометрических объектов* can be treated as spaces, and the elements in them as points.

d) Topological groups

(1) Взаимодействие of classical and modern mathematical methods has been most productive in that area of analysis in which topological methods play a role. The result has stimulated much work in topology and other (2) *разделов математики* and has (3) *важные применения в теоретической физике.*

It is advantageous (4) *рассматривать топологические методы* in analysis under three headings and in rather different manners. The first section, the theorems of Tikhonov and Ascoli, considers two basic theorems that (5) *можно доказать* in a quite general setting and the importance of which cannot be overstressed. After some preliminary remarks, the second section, continuous groups, starts from (6) *определения топологической группы и разрабатывает теорию* that is gradually seen to encompass a whole range of general results. The third section, (7) *анализ многообразий* — which (8) *в основном связан с* working from some important problems in classical analysis — discusses the methods topologists have employed in tackling them, and shows how the body of theory so developed has built itself into a coherent whole. In this way a broad view of the areas of topological groups is presented.

Before proceeding, however, (9) *некоторые элементарные и общие замечания* on the role of topology in analysis will be helpful. (10) *Отличительная черта* of topological arguments in analysis is that they are qualitative and nonnumerical. A simple instance of topological reasoning is (11) *следующее*. A ball is thrown vertically upward, and a few moments later it falls back to its starting point; one deduces that at some point on the way it must have been stationary. This (12) *наблюдение* depends, of course, on certain preconceptions about gravity and (13) *свойств непрерывности пространства и времени.* When suitably formulated, they (14) *образуют основу* of analysis and topology.

Now, in such a simple example as a falling ball, (15) *элементарное приложение* of the Newtonian (16) *исчисления* will give full information about the whole motion, while the topological argument merely (17) *утверждает* the obvious. If, however, a single ball is now replaced by a large number of balls moving in more complicated ways, then direct (18) *аналитическое вычисление* may become too lengthy or difficult. In such a situation one may settle for some qualitative aspects of the various motions, which may be derived by topological arguments but which may be far from self-evident because of the complexity of the situation. In fact, if the various balls are replaced by planets and one proceeds to study the

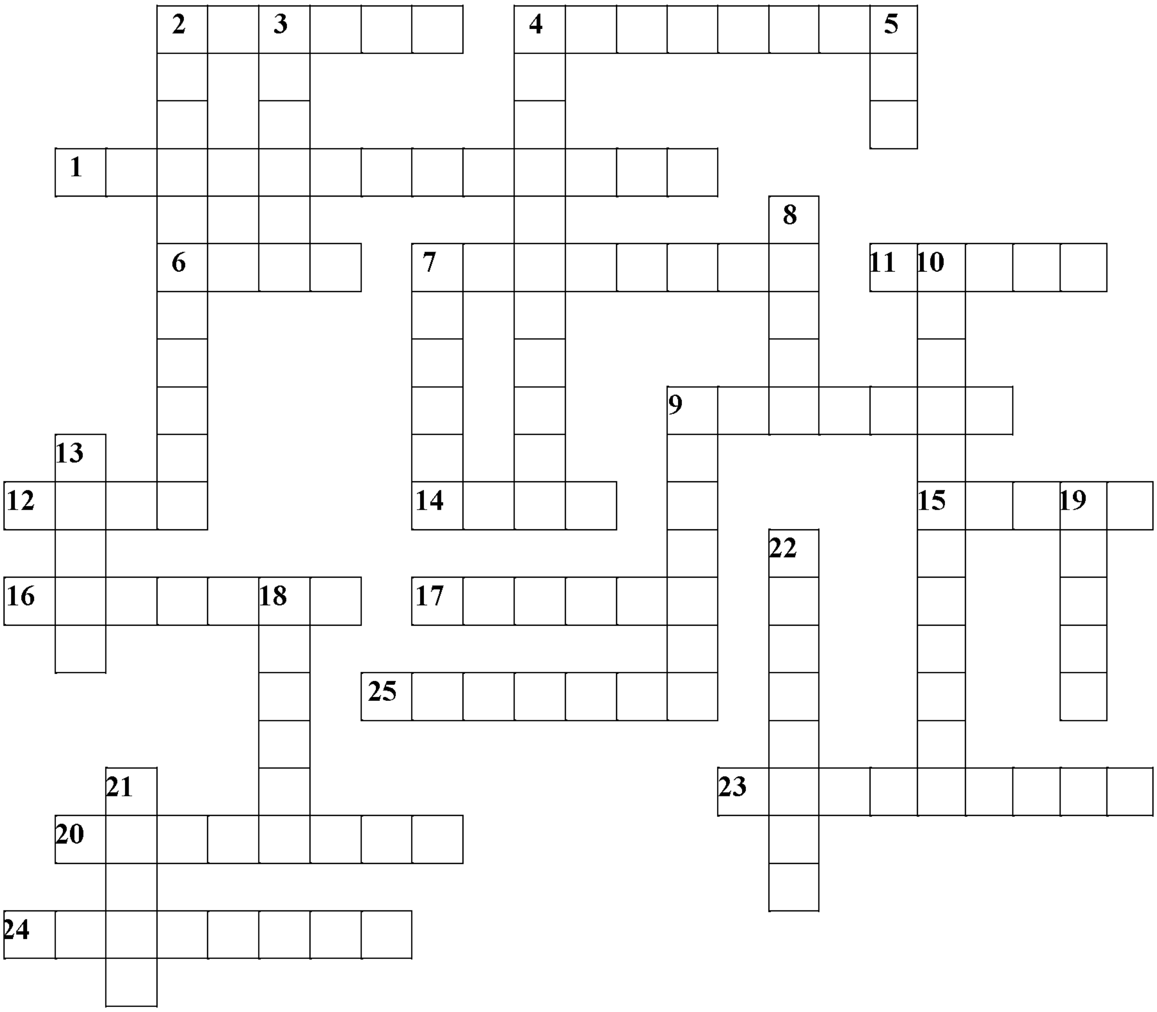
Newtonian (19) *движение нескольких тел* under their mutual gravitational attraction, extremely difficult and (20) *нерешенные математические проблемы* are met.

e) A computer is (1) *электронная машина* that (2) *выполняет вычисления* and processes data automatically at high speeds according to a prescribed (3) *ряд операций.* Broadly speaking, (4) *этот термин может относится не только к электронной машине, но и к* one of a mechanical, analog, or other variety. Although all these types are used, computers usually mean electronic digital machines because electronic digital machines (5) *имеют много преимуществ и широко используются*. Computers (6) *значительно различаются по размеру*; the smallest (7) *могут быть встроены в* a wristwatch, while the largest may fill an entire room. The electronic components of computers enable them (8) *выполнять операции на высоких скоростях*. Computers (9) *используются для решения многочисленных проблем, таких как* payroll calculations, inventory records, bank account transactions, airline reservations, and scientific and engineering computations. For each problem, the user must (10) *предоставить необходимую информацию и подготовить подходящую программу* by which the computer (11) *может обрабатывать информацию* to produce the desired output.

Personal computers (12) *разработаны для индивидуального использования*. Workstations (13) *широко используются для решения научных и технических проблем и применяются в бизнесе*. Computer networks are collections of computers and intelligent peripheral equipment, that are interconnected by telephone lines, microwave relays, and other high-speed communication links (14) *в целях обмена информацией* and sharing equipment. Networking (15) *развито на всех уровнях*, from local to international, in diverse sectors of society.

Section 10 (crosswords)

Crossword № 1 (mathematical terms)



1. an expert or specialist in mathematics
2. → numeration …
2. ↓a mathematical operation in which the difference between two numbers or quantities is calculated
3. a three-dimensional closed surface such that every point on the surface is equidistant from a given point, the centre

→ differential and integral …

4. ↓computation, counting
5. the result of the addition of numbers, quantities, objects, etc.
6. the quantitative measure of a plane or curved surface

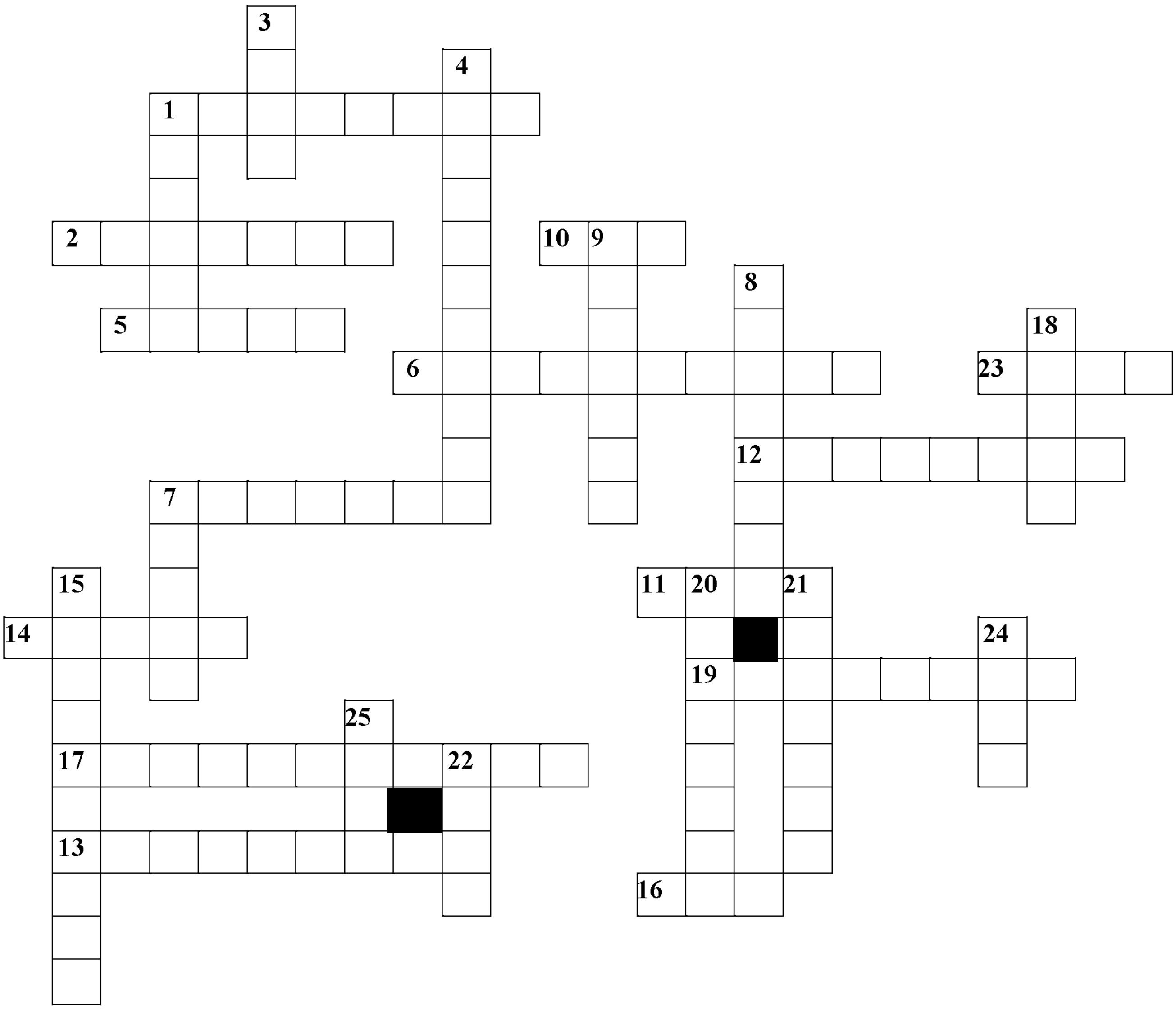
→the process of determining the answer to a problem

7. ↓ a letter, figure, or sign used in mathematics to represent a quantity, operation, function, etc.
8. right …

→a statement in mathematics that can be proved to be true by reasoning

9. ↓a straight line that touches the edge of a circle but does not pass through it
10. the symbol ∩, as in A ∩ B.
11. any of the ten Arabic numerals from 0 to 9
12. any symbol indicating an operation
13. the symbol denoting subtraction or a negative quantity
14. straight ...
15. three-dimensional …
16. flat ...
17. the second power of a quantity
18. a closed plane curve consisting of all points at a given distance from a point within it called the centre
19. a continuously bending line that has no straight parts
20. a characteristic or quality of a geometric object
21. linear …
22. unknown ...
23. a property of space; extension in a given direction
24. a symbol written above and to the right of a mathematical expression to indicate the operation of raising to a power
25. a part of a line or curve between two points

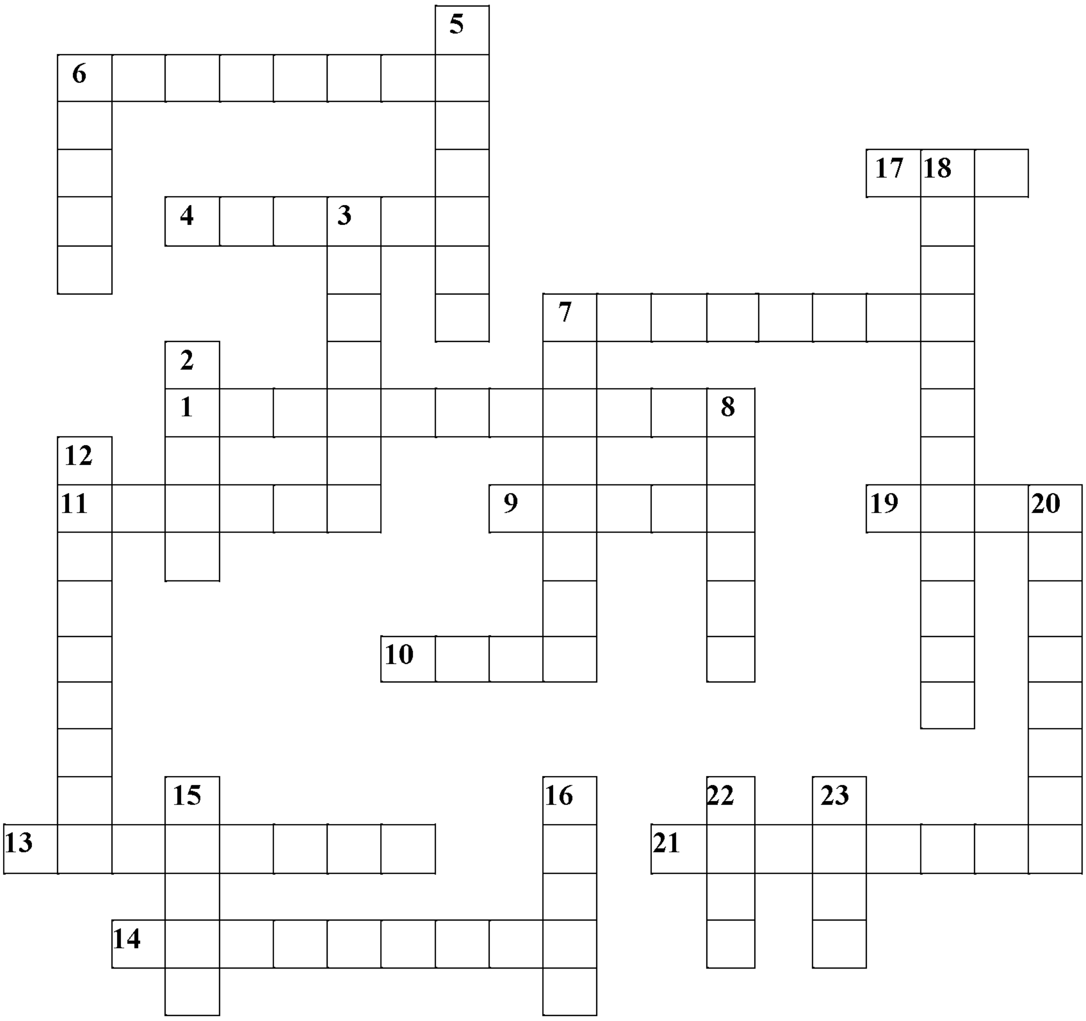
Crossword № 2 (mathematical terms)



1. →a ratio of two expressions or numbers other than zero; any rational number that can be expressed as the ratio of two integers
1. ↓a number that you can divide a larger number by exactly
2. a whole number
3. any rational or irrational number (adj.)
4. in mathematics, a number written before a variable that shows how much the variable is to be multiplied by
5. a number that can only be divided exactly by itself and one
6. remainder left after subtraction
7. →a number that is the result of multiplying two other numbers
7. ↓ a flat surface in which a straight line between any two points will lie completely on that surface

8. (of an infinite series) to approach a finite limit as the number of terms increases (verb)
9. (of a series) to have no limit (verb)
10. a whole number that cannot be divided exactly by two
11. a number that can be divided exactly by two
12. a statement in mathematics that two sets of numbers or expressions are equal
13. the number that appears above the line in a common fraction
14. a position on a drawing or map
15. one of a set of numbers that give the exact position of something on a map or graph
16. a group of numbers
17. the number that is below the line in a fraction
18. a statement that is generally believed to be obvious or true
19. any real number of the form a /b, where a and b are integers and b is not zero (adj.)
20. an expression that can be assigned any of a set of values; a symbol, esp. x, y, or z, representing an unspecified member of a class of objects, numbers, etc.
21. a positive integer or zero (adj.)
22. each of the members of which an expression, a series of quantities, or the like, is composed, as one of two or more parts of an algebraic expression
23. a central line that bisects a two-dimensional body or figure or a line about which a three-dimensional body or figure is symmetrical
24. decimal, or …-ten system
25. … vector

Crossword № 3 (terms of mechanics and physics)

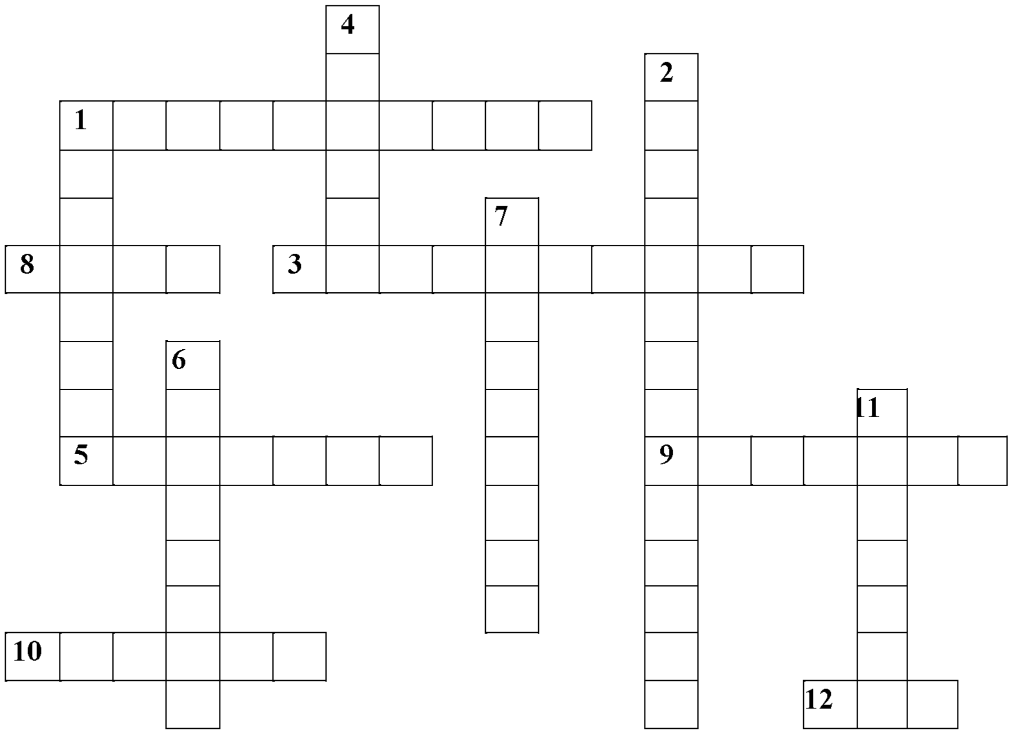


1. a state of rest or uniform motion in which there is no resultant force on a body
2. a bar used for prying something
3. the force of attraction by which terrestrial bodies tend to fall toward the centre of the earth
4. the vertical force experienced by a mass as a result of gravitation
5. the tendency of a body to preserve its state of rest or uniform motion unless acted upon by an external force
6. →a resistance encountered when one body moves relative to another body with which it is in contact
6. ↓an influence on a body or system, producing or tending to produce a change in movement or in shape or other effects
7. →force per unit area

7. ↓one of the extremely small constituents of matter, as an atom or nucleus
8. the action or process of moving or of changing place or position
9. an electromagnetic radiation in the wavelength range including infrared, visible, ultraviolet, and X rays
10. a disturbance or variation that transfers energy progressively from point to point in a medium
11. the capacity of a body or system to do work
12. an optical instrument for making distant objects appear closer by use of a combination of lenses or lenses and curved mirrors
13. terminal …
14. the branch of physics that deals with the action of forces on bodies and with motion, comprised of kinetics, statics, and kinematics
15. a measure of the rate of doing work expressed as the work done per unit time. It is measured in watts, horsepower, etc.
16. magnifying …
17. a general principle, formula, or rule describing a phenomenon in mathematics, science, philosophy
18. the rate of increase of speed or the rate of change of velocity
19. a physical quantity expressing the amount of matter in a body
20. often the band of colours produced when sunlight is passed through a prism, comprising red, orange, yellow, green, blue, indigo, and violet
21. the product of a body's mass and its velocity
22. force times the distance through which it acts
23. the energy transferred as a result of a difference in temperature

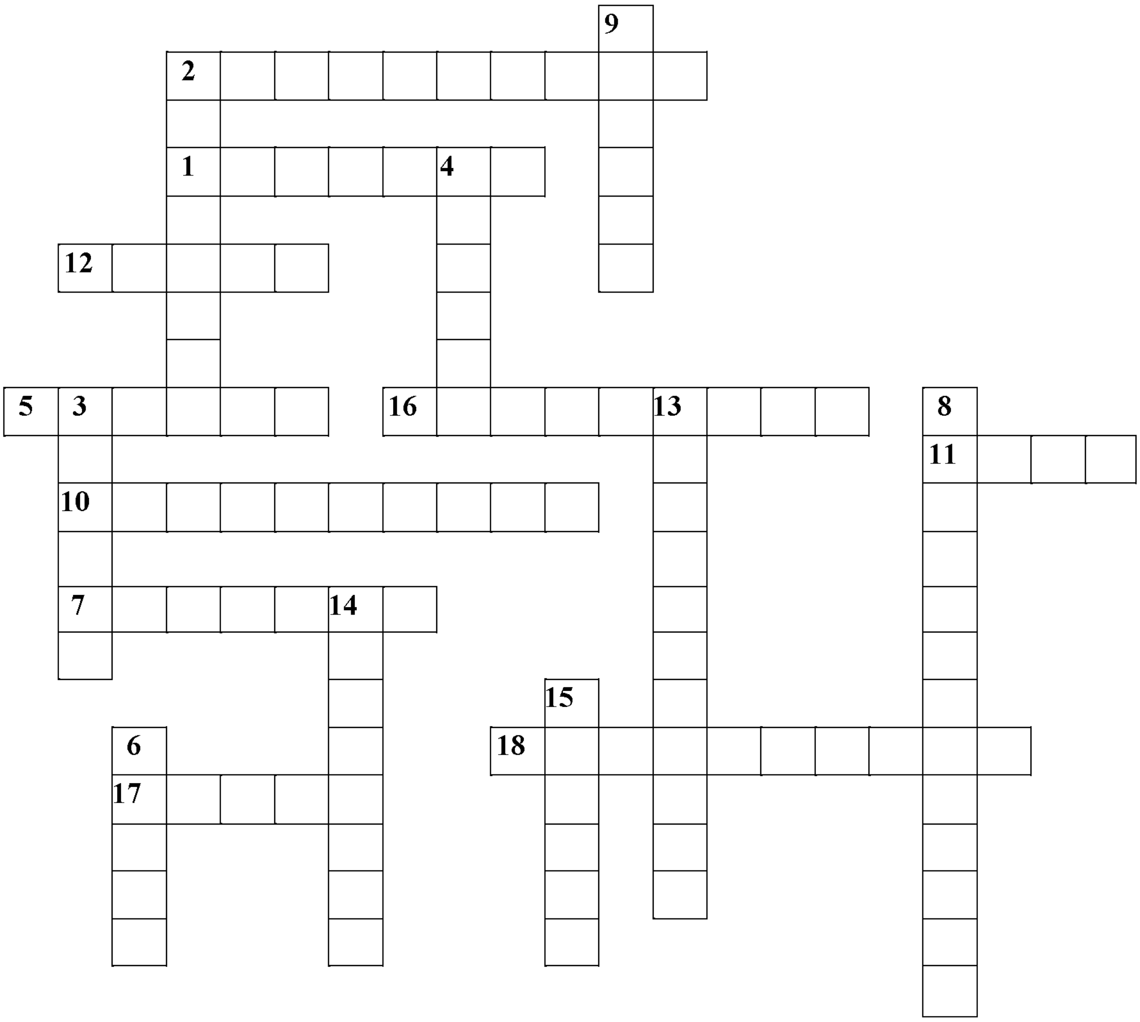
Crossword № 4 (mathematical terms)

Algebra may be described as a generalization and extension of (1→). Elementary arithmetic is concerned primarily with the effect of certain operations, such as (1↓) or multiplication, on specified numbers, hence, for instance, the (2) tables; elementary algebra deals with (3) of arbitrary numbers. Algebra is concerned with certain operations on numbers, and it is necessary to be precise about what a (4) is. The numbers dealt with are either the (5: adj.) numbers, 0, 1, 2, 3, 4, (some authors exclude 0); the rational numbers, which have the form p/q, in which p and q are (6: pl.n.) (natural numbers and their (7: pl.n.), with $q \neq 0$; the real numbers, which correspond to all the points on a (8); or the (9: adj.) numbers, which are constructed from the real numbers together with a number *i*, the (10) of which is -1. The essential property that these numbers have is that they can be added and multiplied by well-established rules. Arithmetic becomes (11) when general rules are stated regarding these operations, as, for example, the commutative (12) of addition.



Crossword № 5 (mathematical terms)

On the (1) of a sphere, such as a geographical globe or the Earth itself, the shortest (2↓) between two (3: pl.n) is an arc of a great (4). It is natural for navigators to regard such circles as the "lines" of a special kind of two-dimensional geometry, namely spherical geometry: the geometry of figures drawn on the surface of a (5) of radius 1. It will be seen later that this is almost the same as (6) elliptic geometry. Spherical geometry was studied by Menelaus of Alexandria about AD 100 and by the Arabs about 1000. Its most famous (7) (discovered by Albert Girard, a French (8) of the early 17th century) states that the three (9: pl.n.) of a spherical triangle (in radian measure) satisfy the (10) $A + B + C > \pi$ and that the (11) of a triangle is $A + B + C - \pi$. The gigantic step of extending this geometry from two (2→: pl.n.) to three (or more) was taken simultaneously (in the latter half of the 19th century) by Ludwig Schläfli in Switzerland and Bernhard Riemann in Germany. Schläfli regarded spherical three-dimensional (12) as the "surface" of the "sphere" in Euclidean four-dimensional space (that is, the hypersurface on which the four (13: pl.n.) satisfy the (14) $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$). If this three-dimensional continuum represents the astronomical space in which man lives, the unit of measurement (radius of the universe) must be very large; but in terms of this unit the total (15) of a line is 2 . This means, as Riemann remarked, that the unboundedness of space does not necessarily imply infinitely long lines. A sufficiently powerful (16) could theoretically enable an astronomer to observe the back of his own head, apart from the fact that the (17) reflected from his head would require thousands of millions of years to reach his eye. This idea, that space could be unbounded without being infinite, was adopted by Einstein in his general theory of (18).



Section 11(translation)[1]

1. Translate the following sentences from the field of mathematics into Russian:

1. Для представления чисел можно использовать любые символы, но чаще берут буквы латинского алфавита.
2. В некоторых задачах требуется найти одновременно несколько чисел, для чего необходимо решить несколько уравнений.
3. При умножении на неизвестную величину следует помнить, что эта величина может быть как положительной, так и отрицательной.
4. Самой древней математической деятельностью был счет.
5. Первым существенным успехом в арифметике стало изобретение четырех основных действий: сложения, вычитания, умножения и деления.
6. Первые достижения геометрии связаны с такими простыми понятиями, как прямая и окружность.
7. Геометрия у египтян сводилась к вычислениям площадей прямоугольников, треугольников, трапеций, круга, а также формулам вычисления объемов некоторых тел.
8. Египтяне имели дело только с простейшими типами квадратных уравнений, а также с арифметической и геометрической прогрессиями.
9. Дифференциальное исчисление дает удобный в вычислениях общий метод нахождения скорости изменения функции f (x) при любом значении x.
10. Более конкретным разделом математики является геометрия, основная задача которой – изучение размеров и форм объектов.
11. Сочетание алгебраических методов с геометрическими приводит, с одной стороны, к тригонометрии (первоначально посвященной изучению геометрических треугольников, а теперь охватывающей значительно больший круг вопросов), а с другой стороны – к аналитической геометрии, в которой геометрические тела и фигуры исследуются алгебраическими методами.
12. Первоначально математический анализ состоял из дифференциального и интегрального исчислений, но теперь включает в себя и другие разделы.

[1] The source of the sentences is www.krugosvet.ru

13. Геометрия – раздел математики, тесно связанный с понятием пространства.
14. Вычисления с дробями включают сложение, вычитание, умножение и деление.
15. Для извлечения кубического корня из положительного действительного числа существуют алгоритмы, аналогичные алгоритмам извлечения квадратного корня.
16. Графически векторы изображаются в виде направленных отрезков прямой определенной длины.
17. Чаще других приводится описание прямой, предложенное Архимедом: «Прямая – это кратчайшее расстояние между двумя точками».
18. Треугольником называется плоская фигура, ограниченная тремя прямыми.
19. Прямоугольным называется треугольник, у которого один из углов прямой.
20. Элементарная арифметика оперирует положительными целыми числами и нулем, дробями, в известной мере положительными действительными числами, такими, как $\sqrt{3}$, и иногда отрицательными действительными числами
21. Дроби принято также называть рациональными числами, так как они представимы в виде отношений двух целых чисел
22. Рациональные и иррациональные числа вместе называются действительными или вещественными числами.
23. Суммировать можно любое число векторов, причем векторы необязательно должны лежать в одной плоскости.
24. Вычитание векторов представляется как сложение с отрицательным вектором.
25. Многие хорошо известные нам предметы обладают симметрией относительно плоскости.
26. Число s называется суммой чисел n и m, и мы записываем это так: s = n + m. В этой записи числа n и m называются слагаемыми, а операция нахождения суммы – сложением.
27. В физике и математике вектор – это величина, которая характеризуется своим численным значением и направлением.
28. Теория вероятностей занимается изучением событий, наступление которых достоверно неизвестно.

29. Для детального анализа вероятностных задач, более сложных, чем простые азартные игры, необходима более строгая и абстрактная формулировка.
30. Трудность теории вероятностей заключается в том, что объекты, составляющие предмет ее изучения, носят гораздо более общий характер и поэтому не столь наглядны, как, например, объекты геометрии или механики.
31. Было бы ошибкой полагать, что решение любой вероятностной задачи всегда можно представить в виде простого отношения двух чисел вида P (A) = m/n.
32. Самой знаменитой теоретико-числовой проблемой, немало способствовавшей развитию теории колец, по праву следует считать так называемую великую теорему Ферма.
33. Теория групп находит применение почти во всех разделах математики, играя роль связующего звена между многими, на первый взгляд совсем разными, ее областями.
34. Одно из главных применений матриц в общественных науках связано с построением моделей различных ситуаций.
35. Математика всегда была для Ферма лишь увлечением, и тем не менее он заложил основы многих ее областей – аналитической геометрии, исчисления бесконечно малых, теории вероятностей.
36. Различают две основные области математики – чистую математику, в которой акцент делается на дедуктивные рассуждения, и прикладную математику.
37. Математика за последние сто лет претерпела огромные изменения, касающиеся как предмета, так и методов исследования.
38. Можно сказать, что наука начинается тогда, когда масса отдельных наблюдений объясняется одним общим законом; следовательно, открытие «теоремы Пифагора» можно рассматривать как один из первых известных примеров подлинно научного достижения.
39. У Евклида и его последователей аксиомы представлены лишь как исходные пункты для построения математики без всяких комментариев об их природе.
40. Прекрасный пример доказательства от противного, ставший одной из вех в развитии древнегреческой математики, – доказательство того, что $\sqrt{2}$ – не рациональное число, т.е. не представимо в виде дроби p/q, где p и q – целые числа.

41. В процессе развития алгебры был изобретен способ символической записи, позволявший представлять в сокращенном виде все более сложные соотношения между величинами.
42. На протяжении 18 в. находилось все больше подтверждений того, что все следствия, полученные из основных аксиом, в особенности в астрономии и механике, согласуются с данными экспериментов.
43. Точное определение понятия структуры довольно сложно.
44. С понятием структуры тесно связаны многие абстрактные понятия; назовем лишь одно из наиболее важных – понятие изоморфизма.
45. Еще древние греки отчетливо понимали, что математическая теория должна быть свободна от противоречий.
46. Несмотря на заявления о независимости математики, никто не станет отрицать, что математика и физический мир связаны друг с другом.
47. Функция — термин, используемый в математике для обозначения такой зависимости между двумя величинами, при которой если одна величина задана, то другая может быть найдена.
48. Обычно функция (с 17 в.) задается формулой, выражающей зависимую переменную через одну или несколько независимых переменных.
49. Многие конкретные функции имеют свои названия; обычно такие функции задаются формулами.
50. Многие задачи в математике приводят к формулам, содержащим бесконечные суммы; такие суммы называются бесконечными рядами, а их слагаемые – членами ряда.
51. Наряду с числовыми рядами мы можем рассматривать т.н. функциональные ряды, слагаемыми которых являются функции.
52. Многие функции можно представить с помощью функциональных рядов.
53. Изучение числовых и функциональных рядов является важной частью математического анализа.
54. Поскольку сложение бесконечного числа членов ряда физически невозможно, необходимо определить, что именно следует понимать под суммой бесконечного ряда.
55. Сумму функционального ряда можно понимать по-разному.

56. Важная задача дифференциального исчисления – создание методов, позволяющих быстро и удобно находить производные.
57. При изучении площадей криволинейных плоских фигур открываются новые аспекты математического анализа.
58. Геометрические фигуры, переходящие одна в другую при топологических преобразованиях, называются гомеоморфными.
59. Топологическим свойством (или топологическим инвариантом) геометрических фигур называется свойство, которым вместе с данной фигурой обладает также любая фигура, в которую она переходит при топологическом преобразовании.
60. Среди топологических инвариантов поверхности можно также отметить число сторон и число краев.
61. Изучение топологических пространств позволило открыть множество красивейших теорем.
62. Наряду с евклидовой геометрией, возникшей в качестве модели внешнего мира, можно рассматривать и абстрактную, не имеющую прямого к нему отношения геометрию пространства n измерений, где n – любое целое положительное число.
63. Нужно подчеркнуть, что проективная геометрия не есть что-то абстрактное, практически не связанное с внешним миром.
64. Если потребовать, чтобы для каждого действительного числа нашлась соответствующая ему точка на прямой, то мы получим т.н. «непрерывную» геометрию.
65. В последние десятилетия наши представления о пространстве сильно изменились под воздействием повсеместного принятия в физике концепции «пространства-времени».
66. Наше представление о пространстве – это наиболее изученная модель, позволяющая лучше всего понять те абстракции, которые и составляют суть математики в целом.
67. Арифметика — это искусство вычислений, производимых с положительными действительными числами.
68. По-видимому, наибольшую трудность у древних вычислителей вызывала работа с дробями.
69. Индийская система счисления и первые арифметические алгоритмы были заимствованы арабами.
70. Числа в математических таблицах, тригонометрических или таблицах логарифмах, – приближенные, записанные с определенным числом знаков.

2. Translate the following sentences from the field of physics and mechanics into Russian:

1. Новым в системе Ньютона стало понятие силы не просто как некоего действия, а как величины.
2. Наклонная плоскость применяется для перемещения тяжелых предметов на более высокий уровень без их непосредственного поднятия.
3. Примером прямолинейного равномерного движения может служить движение космического аппарата, летящего по инерции в межзвездном пространстве достаточно далеко от всех небесных тел, там, где гравитационные поля ничтожно малы.
4. Если на наклонную плоскость положить брусок, то в отсутствие трения он соскользнет по ней вниз.
5. Закон противодействия утверждает, что взаимодействующие тела прилагают друг к другу равные по величине, но противоположно направленные силы.
6. Теплота сама по себе не является веществом – это всего лишь энергия движения его атомов или молекул. Именно такого понимания теплоты придерживается современная физика.
7. Вначале взгляды Эйлера не встретили понимания, поскольку противоречили ньютоновской корпускулярной теории света, но получили признание после двух решающих экспериментальных подтверждений.
8. Квантовая механика представляет собой теоретическую основу, на которой строится современная теория атомов, атомных ядер, молекул и физических тел, а также элементарных частиц, из которых все это состоит.
9. Методы квантовой механики были применены к проблемам строения молекул, что привело к революции в химии.
10. Если массы уравновешены, т.е. если их веса равны, то в этом случае равны и сами массы.
11. Преимущество телескопа-рефлектора перед рефрактором состоит в том, что лучи любого цвета отражаются от зеркала одинаково, обеспечивая четкость изображения.
12. Ньютоновская механика по-прежнему применяется в практических расчетах и в тех разделах астрономии, где рассматриваемые объекты – планеты, самолеты, автомобили – достаточно велики и движутся со скоростью, намного меньшей скорости света.

13. Теории относительности образуют существенную часть теоретического базиса современной физики.
14. Одним из важнейших инструментов физики стала теория вероятностей, которая раньше применялась главным образом в теории азартных игр и страховом деле.
15. Греческие астрономы наблюдали небо и записывали свои наблюдения, однако не существует никаких свидетельств того, что они проводили научные эксперименты.
16. Демокрит первым из великих математиков оказал глубокое влияние на развитие физики.
17. Второй великий предтеча современной физики, Архимед, был величайшим математиком древности.
18. Иногда математика дает возможность систематизировать все следствия некой физической гипотезы, выражая их в виде соотношений, истинность или ложность которых поддается экспериментальной проверке.
19. По Аристотелю, Земля находится в центре мироздания потому, что состоит из тяжелых веществ, которых заставило собраться в центре мира их естественное движение.
20. Общая теория относительности Эйнштейна явилась первой серьезной модификацией теории планетных движений Ньютона.
21. Бурное развитие науки знаменовалось изобретением новых оптических инструментов и новой волной интереса к зрительному процессу.
22. Параллельно с усовершенствованием оптических приборов и оптических измерений был выстроен ряд теоретических предположений относительно природы света.
23. Ньютон поставил перед собой более трудную задачу – описать на языке математики процесс распространения звуковой волны в воздухе.
24. Хорошо известно, что движение тел при наличии трения порождает тепло.
25. Главная задача фундаментального изучения материи состоит в том, чтобы как можно больше узнать о всех возможных ее формах.
26. Физика твердого тела – быстро развивающаяся область науки.
27. Плазма – это раскаленный газ, состоящий из проводящих электричество ионов и электронов, но его поведение заметно отличается от поведения газа при обычных условиях.

28. Если учесть, что все звезды и значительная часть межзвездного вещества – плазма, то получается, что во Вселенной в таком состоянии находится более 99% материи.
29. Термодинамика — раздел прикладной физики или теоретической теплотехники, в котором исследуется превращение движения в теплоту и наоборот.
30. Термодинамика находит широкое применение в физической химии и химической физике при анализе физических и химических процессов, в современной физиологии и биологии, в двигателестроении, теплотехнике, авиационной и ракетно-космической технике.
31. Одним из видов энергии является работа, которая совершается, когда тело движется, преодолевая действие некой силы.
32. Полную совершаемую работу можно найти как площадь зависимости силы от соответствующего размера.
33. При быстром сжатии газа некоторая часть работы, совершаемой над ним, может заметно повысить его температуру.
34. Полное преобразование работы в теплоту вполне возможно, но обратный процесс преобразования всей теплоты в эквивалентную ей работу невозможен.
35. Полное преобразование теплоты в работу было бы возможно лишь в том случае, если бы минимальная температура была равна абсолютному нулю, при которой рабочее тело не имело бы никакой тепловой энергии.
36. На существование абсолютного нуля указывает закон расширения газов.
37. Согласно закону сохранения энергии во всех преобразованиях энергия не возникает и не исчезает, она лишь меняет форму.
38. Звуковая волна в газе характеризуется избыточным давлением, избыточной плотностью, смещением частиц и их скоростью.
39. Все типы волн математически описываются так называемым волновым уравнением.
40. При простых гармонических колебаниях движение периодически повторяется.
41. Математически простые гармонические колебания описываются простой функцией.
42. Скорость звука – это характеристика среды, в которой распространяется волна.

43. График зависимости относительной энергии звуковых колебаний от частоты называется частотным спектром звука.
44. Звуковые колебания являются периодическими, если колебательный процесс, каким бы сложным он ни был, повторяется через определенный интервал времени.
45. Интенсивность любой звуковой волны в процессе ее распространения уменьшается вследствие поглощения звука.
46. Дифракцией называется огибание волнами препятствия.
47. Волновое сопротивление газов гораздо меньше, чем жидкостей и твердых тел.
48. Наложение двух или большего числа волн называется интерференцией волн.
49. Интенсивность поглощения зависит от частоты звуковой волны и от других факторов, таких, как давление и температура среды.
50. Если на твердое тело действуют силы, которые нельзя свести к одной, то они заставляют тело вращаться.
51. Если на твердое тело действуют несколько сил, то тело не будет вращаться только при условии, что сумма моментов всех сил равна нулю.
52. Свойственная всем телам способность сопротивляться изменению состояния покоя или движения называется инертностью или инерцией.
53. Движение по направлению к земле вызывается силой гравитационного притяжения, которая при малой высоте падения практически постоянна.
54. Если мяч движется по окружности с постоянной скоростью, то может показаться, что он находится в равновесии относительно центра окружности.
55. Энергетические машины преобразуют один вид энергии в другой.
56. Простейшие механизмы можно найти почти в любых более сложных машинах и механизмах.
57. Силой земного притяжения обусловлено ускорение свободного падения предметов.
58. Сила земного притяжения, или гравитация, удерживает Луну на орбите, а атмосферу – вблизи земной поверхности.
59. Измерения ускорения силы тяжести позволяют получать информацию о внутреннем строении Земли.

60. Еще в 1600 английский физик У.Гильберт, показал, что Земля ведет себя, как огромный магнит.
61. Центробежная сила противодействует гравитационной силе и уменьшает эффективный вес тела на малую, но доступную измерению величину.
62. По сравнению с электрическими силами притяжения и отталкивания между двумя заряженными элементарными частицами тяготение очень слабо.
63. По мере увеличения масс гравитационные эффекты становятся все более заметными и в конце концов начинают доминировать над всеми остальными.
64. При переходе к еще бóльшим масштабам гравитация организует отдельные небесные тела в системы.
65. Тяготение помогает также удерживаться вместе в космическом пространстве газовым и пылевым облакам, а иногда даже сжимает их в компактные и более или менее шарообразные сгустки материи.
66. Во всех космологических теориях принимается, что тяготение – свойство любого вида материи, проявляющееся повсюду во Вселенной, хотя отнюдь не предполагается, что создаваемые тяготением эффекты везде одни и те же.
67. Тяготение всегда явно или неявно переплеталось с космологией, так что оба эти предмета неразделимы.
68. Методами математического анализа Ньютон показал, что сферическое тело, например Луна, Солнце или планета, создает тяготение так же, как и материальная точка, которая находится в центре сферы и имеет эквивалентную ей массу.
69. Мысленно представляя себе всю Землю целиком, мы предпочитаем считать ее неподвижной, полагая, что на тела, находящиеся на поверхности Земли, действуют гравитационные силы, а не силы инерции.
70. Согласно теории относительности, луч света, проходя вблизи большой массы, искривляется.

3. Translate the following extracts into Russian:

а) Теория множеств. Под множеством понимается совокупность каких-либо объектов, называемых элементами множества. Теория множеств занимается изучением свойств как произвольных множеств, так и множеств специального вида независимо от природы образующих их элементов. Терминология и многие результаты этой теории широко используются в математике, например в математическом анализе, геометрии и теории вероятностей.

b) Абстрактная алгебра (общая алгебра) — раздел современной математики, выросший из исследования уравнений и теории чисел. Свою теперешнюю форму абстрактная алгебра начала приобретать лишь в двадцатом веке. Занимается главным образом изучением систем, элементы которых можно сочетать по различным правилам, получая в результате новые элементы, вне зависимости от конкретной природы самих элементов. В последние десятилетия абстрактная алгебра все глубже проникает в различные разделы математики, становясь неоценимым средством исследования в столь различных ее областях, как геометрия, топология, математический анализ и дифференциальные уравнения.

c) Векторы и матрицы находят все более широкое применение и вне математики. Они были изобретены в середине 19 в. в связи с изучением n-мерной геометрии. С тех пор их стали использовать везде, где приходится иметь дело с обработкой больших массивов данных. С использованием матриц решаются многие технические задачи, связанные с расчетом напряжений, деформаций, колебаний. Решение системы линейных уравнений с несколькими переменными по существу является задачей матричного исчисления.

d) Математика. Математику обычно определяют, перечисляя названия некоторых из ее традиционных разделов. Прежде всего, это арифметика, которая занимается изучением чисел, отношений между ними и правил действий над числами. Факты арифметики допускают различные конкретные интерпретации; например, соотношение $2 + 3 = 4 + 1$ соответствует утверждению, что две и три книги составляют столько же книг, сколько четыре и одна. Любое соотношение типа $2 + 3 = 4 + 1$, т.е. отношение между чисто математическими объектами без ссылки на какую бы то ни было интерпретацию из физического мира, называется абстрактным. Абстрактный характер математики позволяет использовать ее при решении самых разных проблем. Например, алгебра, рассматривающая операции над числами, позволяет решать задачи, выходящие за рамки арифметики. Более конкретным разделом математики является геометрия, основная задача которой – изучение размеров и форм объектов. Сочетание алгебраических методов с геометрическими приводит, с одной стороны, к тригонометрии (первоначально посвященной изучению геометрических треугольников, а теперь охватывающей значительно больший круг вопросов), а с другой стороны – к аналитической геометрии, в которой геометрические тела и фигуры исследуются алгебраическими методами. Существуют несколько разделов высшей алгебры и геометрии, обладающих более высокой степенью абстракции и не занимающихся изучением обычных чисел и обычных геометрических фигур; самая абстрактная из геометрических дисциплин называется топологией.

e) Современная математика. Хотя теоретически возможно существование любых аксиом, до настоящего времени было предложено и исследовано лишь небольшое число аксиом. Обычно в ходе развития одной или нескольких теорий замечают, что какие-то схемы доказательства повторяются в более или менее аналогичных условиях. После того как свойства, используемые в общих схемах доказательств, обнаружены, их формулируют в виде аксиом, а следствия из них выстраивают в общую теорию, не имеющую прямого отношения к тем конкретным контекстам, из которых были абстрагированы аксиомы. Получаемые при этом общие теоремы применимы к любой математической ситуации, в которой существуют системы объектов, удовлетворяющие соответствующим аксиомам. Повторяемость одних и тех же схем доказательства в различных математических ситуациях свидетельствует о том, что мы имеем дело с различными конкретизациями одной и той же общей теории. Это означает, что после соответствующей интерпретации аксиомы этой теории в каждой ситуации становятся теоремами. Любое свойство, выводимое из аксиом, будет справедливо во всех этих ситуациях, но необходимость в отдельном доказательстве для каждого случая отпадает. В таких случаях говорят, что математические ситуации обладают одной и той же математической «структурой».

f) Физика. Древние называли физикой любое исследование окружающего мира и явлений природы. Такое понимание термина «физика» сохранилось до конца 17 в. Позднее появился ряд специальных дисциплин: химия, исследующая свойства вещества, обусловленные особенностями его атомной структуры, биология, изучающая живые организмы и т.д. Помимо традиционных предметов исследования, о которых пойдет речь ниже, физика занимается столь разными проблемами, как поведение смазки в машинах, процессы образования химических связей, хранение и передача генетической информации в живых системах и т.д. Объединяющий принцип физики как науки кроется не столько в предметах исследования, сколько в подходе к их изучению, и этим физика отличается от других наук. Опираясь на определенные аксиомы и гипотезы, проводя эксперименты и используя математические методы, она стремится объяснить все многообразие природных явлений исходя из небольшого числа взаимосогласующихся принципов. Физик надеется, что, когда о природных явлениях станет известно достаточно много и когда они будут достаточно хорошо поняты, множество других, на первый взгляд разрозненных и не связанных с ними фактов уложатся в простую, допускающую математическое описание схему.

g) Звук и акустика. Звук – это колебания, т.е. периодическое механическое возмущение в упругих средах – газообразных, жидких и твердых. Такое возмущение, представляющее собой некоторое физическое изменение в среде (например, изменение плотности или давления, смещение частиц), распространяется в ней в виде звуковой волны. Область физики, рассматривающая вопросы возникновения, распространения приема и обработки звуковых волн, называется акустикой. Звук может быть неслышимым, если его частота лежит за пределами чувствительности человеческого уха, или он распространяется в такой среде, как твердое тело, которая не может иметь прямого контакта с ухом, или же его энергия быстро рассеивается в среде. Таким образом, обычный для нас процесс восприятия звука – лишь одна сторона акустики.

Section 12 (Supplementary Texts)[1]

What is ... "How Many?"

Number is the within of all things.
– *Pythagoras*

Number proceeds from unity.
– *Aristotle*

FOUNDATIONS

Read the text and answer the following questions:

1. *Do you agree with the statements about numbers made by Pythagoras and Aristotle?*
2. *How did mathematicians in the late 19th and early 20th centuries treat the problem of "what is a number"?*
3. *What is meant by a "well-defined collection of objects" in the definition of a set?*
4. *What are the possible ways of writing down sets?*
5. *What is an empty set?*
6. *What is the union and intersection of sets?*

It would be natural to suppose that mathematical foundations might begin with arithmetic, or plane geometry, rather than numbers; after all, everyone knows what a number is, right? Not so fast: it can be remarkably difficult to say exactly what a number is. Go ahead, try it. If you are tempted to say, "it's what tells you how much or how many of something," then that's a good effort. Only trouble is, that's not a definition, but a characterization. It's like saying, "a chair is something you sit on." Useful information, no doubt, but we'd be a bit disappointed if we looked up "chair" in a dictionary and found no more than "something you sit on." After all, one can sit on lots of things that aren't chairs.

The problem of "what is a number," is an old one. To tackle it, mathematicians in the late 19th and early 20th centuries (particularly

[1] Supplementary texts come from Prime Mathematics Encyclopaedia: www.mathacademy.com/pr/prime

Cantor, Dedekind, Frege, Peano, Russell, and Whitehead) turned to a new (at the time) branch of mathematics called *set theory*. They didn't solve the problem, but they developed a beautiful theory which can be used to model and extend our primitive (i.e., “given” or “intuitively obvious”) sense of number. A good deal of modern mathematics is now founded on this work – using, at root, nothing more sophisticated[2] than the set operations of union and intersection with which you may already be familiar. We'll be wanting to use these operations shortly, so let's review them now. We begin with defining the notion of a *set*:

A set is any well-defined collection of objects.

In other words, any collection considered as a single thing. By “well-defined,” we mean that we can always tell when something is an element of the set in question, or when it isn't – no ambiguity[3]. By “object” we mean absolutely anything: physical objects, ideas, colours, abstractions, and anything else you care to think of (that can form part of a well-defined collection).

We want to be able to write our sets down, and there is an established way of doing this. We first designate a symbol to stand for the set itself, usually a capital letter like *A* or *S* or some such. Then, we use (curly) braces to enclose some representation of the elements of the set, as follows:

A = { ... elements of A ...}

How do we represent the elements inside the braces? There are two ways. The first and best is simply to list them. For example, if *A* is a set of colours, we could write it down this way:

A = {red, green, blue}

Sometimes, however, listing the elements is not convenient or even possible. In that case we would use a rule method, using a statement like “all shades of green,” or even, “all colours” to represent our set.

When something is an element of a set, we denote this with a special symbol (that looks kind of like a curvaceous “E”):

blue ∊ A

This means, “‘blue’ is an element of the set *A*.”

Much of the power of set theory arises from the fact that we can form sets whose elements are other sets. For example, if *A*, *B*, and *C* are sets of colours, we could form a set of sets of colours.

[2] sophisticated — complicated
[3] ambiguity — something that is not clear because it has more than one possible meaning

$$A = \{\text{red, green, blue}\}$$
$$B = \{\text{purple, blue, orange}\}$$
$$C = \{\text{red, yellow, green}\}$$

sets

$$S = \{A, B, C\}$$

a set of sets

$$= \{\ \{\text{red, green, blue}\},\ \{\text{purple, blue, orange}\},\ \{\text{red, yellow, green}\}\ \}$$

Notice that the sets *A*, *B*, and *C* have some elements in common. For instance, *A* and *B* both contain the element "blue." If it happens that *every* element of a set is also contained in some other set, then we say that the first set is a *subset* of the other set, and we denote this with a big "U-shape" lying on its side. For instance, if *D* is the set containing only "blue," then we could write,

$$D \subset A$$

or, equivalently,

$$\{\text{blue}\} \subset \{\text{red, green, blue}\}$$

We consider every set to be a subset of itself. (Funny thing to do, really, but it makes sense, sort of. At least, it matches the definition of subset.) Also, there is one set that is a subset of *every* set, namely[4] the empty set – the set with no elements. This is often denoted by a circle with a line through it, or a pair of braces with nothing between them.

$$\emptyset = \{\ \}$$
$$\emptyset \subset X, \quad \text{for every set } X$$

Finally, we are ready for the "set operations" of *union* and *intersection*. The union of two sets *A* and *B* is the set containing all the elements that are in either *A* or *B*. Thus, if *A* and *B* are the two sets of colours above, then we have,

$$A \cup B = \{\text{red, green, blue}\} \cup \{\text{purple, blue, orange}\}$$
$$= \{\text{red, green, blue, purple, orange}\}$$

The intersection of two sets is the set containing only elements that are in both. For example, the intersection of *A* and *C* would be denoted as follows:

$$A \cap C = \{\text{red, green, blue}\} \cap \{\text{red, orange, green}\}$$
$$= \{\text{red, green}\}$$

[4] namely — that is to say

Armed with these ideas, we may now turn to our real purpose – nailing down[5] numbers.

NATURAL NUMBERS

Read the text and answer the following questions:

1. *What is the definition of natural numbers and how is the set of natural numbers symbolised?*
2. *What are the properties of the set of natural numbers?*
3. *When is a set said to be well-ordered?*
4. *Why are natural numbers closed under addition and multiplication?*
5. *What do you know about the fundamental theorem of arithmetic?*
6. *Why isn't zero included in the set of natural numbers?*
7. *What do you know about the historical development of zero?*

We will begin where every child begins – with counting. The first mathematical thing we learn to do as children is to say "this many" and hold up our fingers. When we do this, we mean that there is the same "many" as the "many" of fingers we are holding up. Then we learn that the "many's" have names, and are ordered, and slowly we memorize the names and the order they come in (in base ten), starting with "one, two, three, ..." These numbers are called the *natural numbers*, and are denoted as follows:

$$\mathbf{N} = \{1, 2, 3, \ldots\}$$

The *set* of natural numbers is always symbolized by a boldface (or "chalkboard") capital *N*, as above. Notice the set notation. This is critical[6], and provides us with an important characterization:

A number is an element of a set.

Thus, the *counting* numbers – one, two, seventy-three, a million, and so on – are elements of the set of *natural* numbers.

The set of natural numbers has some properties that should be noted. First, of course, is that this set is *ordered*. This means that, given two different natural numbers, one always comes "after" the other, and the other comes "before." (This isn't true, for example, of the set of colours. One colour doesn't "come after" another in any necessary sense.) This may seem so obvious as to be beneath our notice, but we will find as we start to

[5] to nail down — to definitely decide, arrange, or complete something
[6] critical — very important

really *learn* mathematics (as opposed to just memorizing procedures) that such niceties[7] can sometimes take on surprising significance[8].

In fact we can do better with the natural numbers than saying merely that they are ordered. They have a property that we call being *well-ordered:*

A set is said to be well-ordered if every subset of it has a smallest element.

So in other words, given any collection of natural numbers, say for instance { 67, 4, 9, 345, 22 }, then there is always a *smallest* one in the set (4 in this instance). Notice that there is *not* always a *largest* element; the set of all even numbers, the set of all multiples of 5, and the set of all the natural numbers greater than 37 are examples of sets that have no greatest element.

This business of not having a largest element is something every child notices at some point (and then experiences her or his first brush[9] with the idea of infinity). We know that there isn't a largest natural number because, intuitively at least, we know the following principle (which is sometimes called the *Archimedian principle*):

If *n* is a natural number, then *n* + 1 is a natural number.

Another way of saying this is that the natural numbers are *closed* under addition. That is, take any two natural numbers and add them, and you get another natural number[10].

Addition can't take you outside of the set. Notice that the natural numbers are also closed under multiplication (which makes sense, since multiplication is just repeated addition).

The natural numbers are the only numbers we need for one of the most important results in classical mathematics, which comes down to us from antiquity (it is found in Euclid's *Elements*). This result is called the Fundamental Theorem of Arithmetic, which every numerate[11] person should know.

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[7] nicety — a very small difference or detail
[8] significance — importance
[9] brush — a brief encounter, contact with something
[10] GETTING FORMAL
We can establish in a concrete way that the natural numbers have no largest element by reasoning as follows: suppose that there *was* a largest natural number, and let's designate it by *B*. Then by the principle that you can always add one, *B* + 1 is also a natural number. Observe that *B* + 1 is larger than *B*. But we said that *B* was the largest. This is a contradiction, and whenever a chain of reasoning leads to a contradiction we conclude that one or more of our premises was false. The only premise in this case was that there *does* exist a largest natural number, so this must be false, i.e., there does *not* exist a largest natural number.
[11] numerate — able to use numbers, esp. in arithmetical operations

Students often ask why zero isn't included in the set of natural numbers. Many texts describe a set called the *whole numbers*, which is just like the set of natural numbers except that it also includes zero. This set, however, is not used much, and it is as well to separate zero conceptually from the natural numbers because it is really a very different kind of thing. When you count a collection of objects, you don't begin by saying, "zero, one, two, ... ", after all. Its historical development is quite different, too. People were counting for millennia before zero was ever thought of. (In fact its first use is thought to have been in India in the 6th or 7th century, and came to us – like so much of the mathematics that we use in the western world – by way of Arabic culture in about the 11th century. It's notable that native Americans, specifically the Mayan civilization, also developed a concept of zero independently of the old world.) So anyway, we don't include zero in the natural numbers. We will find it, however, in our next set...

INTEGERS

Read the text and answer the following questions:

1. *What other operations with numbers do you know?*
2. *Which numbers does a set of integers consist of and why is it denoted by the letter Z?*
3. *How and when did negative numbers appear?*
4. *Who was the first to recognise the importance of negative numbers and why?*

We said that the set of natural numbers is closed under addition and multiplication, but of course there are other operations with numbers. Subtraction, for instance. If we take away two from three, then there is no problem because the remainder is one, and one is in the set of natural numbers. What happens, however, if we want to go the other way and take three from two? We get a negative number, and negative numbers aren't included in the set of natural numbers. In order to talk about negative numbers, we will need to introduce a new set:

$$\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

This is the set of *integers*, and is always denoted by a bold faced *Z*. (The *Z* is from the German word *zahlen*, which means "to count.") Notice that it includes all the natural numbers, zero, and all the negatives of the natural numbers. This means the natural numbers are a subset of the integers, i.e.,

$$\mathbf{N} \subset \mathbf{Z}$$

Historically, negative numbers didn't come into wide use until the late middle ages, circa[12] the 14th century. Before that time, negative quantities weren't considered "real," and so it was thought that one shouldn't try to calculate with them. After all, when did you last see a negative quantity? No one is "minus five feet tall," for instance. However, negative numbers can be very handy[13] for calculations involving debt, and the Italians (who invented banks) were the first to recognize their importance in finance and to use them for that purpose.

RATIONAL NUMBERS

Read the text and answer the following questions:

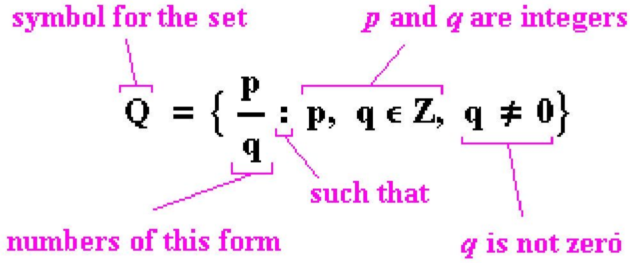
1. *What is a rational number and what is the set of rational numbers?*
2. *How did rational numbers develop historically?*
3. *Why do natural numbers and integers form subsets of rational numbers?*

The integers are closed under addition, subtraction, and multiplication, but what about division? If we divide two into four we're all right, but what about dividing two into three? Then we get catapulted right out of the integers and into the world of *ratios*, or rational numbers.

$$\mathbf{Q} = \left\{ \frac{\mathbf{p}}{\mathbf{q}} : \mathbf{p},\ \mathbf{q} \in \mathbf{Z},\ \mathbf{q} \neq 0 \right\}$$

This looks much more complicated than anything we've done before, but don't be alarmed – we'll take it apart piece by piece. What it says is, the set of rational numbers is the set consisting of all numbers of the form *p* divided by *q*, where *p* and *q* are elements of the set of integers and *q* is not zero.

[12] circa — at the approximate time of
[13] handy — useful



This is an example of using the "rule method" to designate the elements of a set, rather than the "list method" we used before.[14]

We need to use the rule method because there is no way to "list" rational numbers that suggests the complete list in an unambiguous way. Even using an ellipsis (...) doesn't help.

Historically, the rational numbers are nearly as old as the natural numbers. They go all the way back to the ancient Babylonians and Egyptians, and the Greeks were particularly fond of them (though none of these cultures used our notation, which is Arabic). The Pythagoreans of ancient Greece even believed that everything in creation could be understood and analysed in terms of natural numbers and their ratios. (As we'll see below, this idea wasn't to last – its days, so to speak, were numbered!)

Notice that the natural numbers and the integers are both subsets of the rational numbers, since any integer can be expressed as a ratio:

$$3 = \frac{3}{1}$$

The rational numbers are closed under all the arithmetic operations, and if all we ever needed to do was arithmetic we'd never need any other numbers at all. However, sometimes we need more than arithmetic to construct adequate models of the world around us. For this reason, much of the work we do in mathematics will require us to add to the rational numbers a new set – a set of numbers which is the topic of our next section.

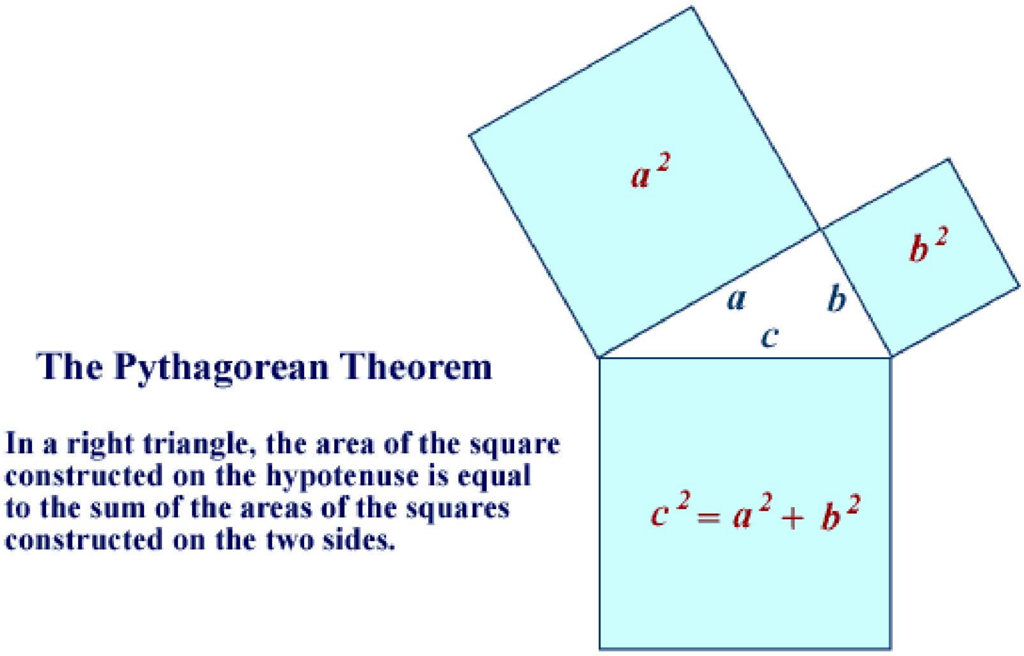
[14] WHY NOT?
What makes giving a "list" of the rational numbers difficult is their *denseness*: Suppose a and b are rational numbers. Then, since the rational numbers are closed under addition and division, the number $(a + b)/2$ (the average of a and b) is also rational, and it lies between a and b. Thus, between *any* two rational numbers you can find another one. This is what "dense" means in this context.

IRRATIONAL NUMBERS

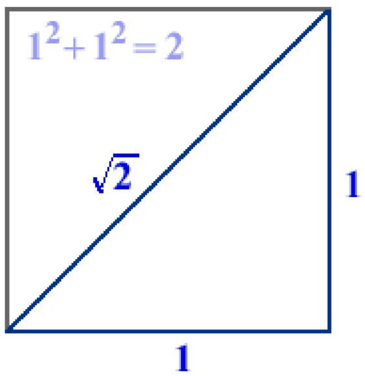
Read the text and answer the following questions:

1. *What does the Pythagorean theorem state?*
2. *Why isn't the square root of two a rational number?*
3. *What is an irrational number?*

In addition to their fascination[15] with numbers and ratios, the Greeks were very keen on[16] geometry. Among the most important results in all of mathematics is the famous Pythagorean Theorem:



This theorem and its many important uses wants a mini-text of its own, but for now let's just apply it to a simple square with sides of length one, and ask the obvious question, "how long is the diagonal?"



[15] fascination — the state of being very interested in something or attracted by something
[16] to be keen on something — to be very interested in something

We see that the length of the diagonal is the square root of two, since one squared plus one squared is two, so the length of the diagonal squared is equal to two.

SO . . . is the square root of two a rational number? That is, can it be written as the ratio of two integers? Remember that the Pythagoreans believed that *everything* was either a whole number or a ratio. In fact, it turns out that the square root of two *cannot* be written as a ratio of integers – it is *irrational*. Note that "irrational" doesn't mean crazy or unreasonable; it means "not expressible as a ratio (of integers)." Legend has it that when the Pythagoreans (who were at sea at the time) first heard about it, they were so overcome[17] by their feelings that they took the poor man who discovered this fact and threw him overboard, drowning him.

The upshot[18] is that we need more than just rational numbers if we wish to work with many kinds of abstract quantities, such as length and proportion in idealized space, for instance. We need *irrational* numbers too. We don't tend to bother much about the set of irrational numbers (usually denoted by a bold-faced *I*) in and of itself, but focus instead on what we get when we mix the rationals and the irrationals together: that most beautiful, strange, and wondrous of sets, the *real* numbers.

REAL NUMBERS

Read the text and answer the following questions:

1. *Which numbers does the set of real numbers contain?*
2. *What is a continuum?*
3. *What does the real number line show?*
4. *In what way is the decimal notation used to represent real numbers?*
5. *Do you know any properties of real numbers?*
6. *Which theory is called the "Queen of Mathematics" and why?*
7. *What do you think of Plato's idea about numbers?*

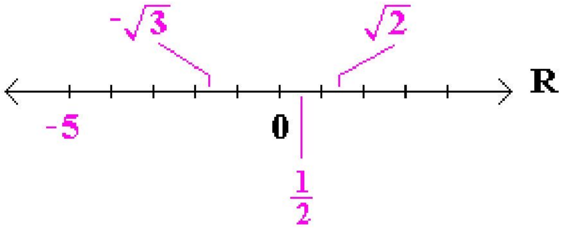
We said above that the set of real numbers is obtained by collecting together the rational numbers and the irrational numbers, i.e.,

$$\mathbf{R} = \mathbf{Q} \cup \mathbf{I}$$

Thus, every rational number is a real number, and every irrational number is a real number. The real numbers, taken all together, form what

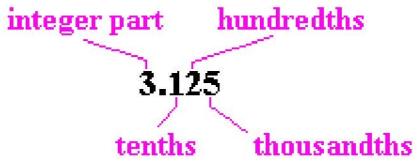
[17] to be overcome by something — to get into a very emotional state
[18] upshot — the result of a process or an event

is called a *continuum*. They are closed under all the arithmetic operations, and they are also closed "geometrically." This last point is important, because it means that we can use geometrical pictures to represent the real numbers and vice versa[19]. The principal and most useful picture we use is called *the real number line*, and will no doubt be familiar:



Here, every point on the real number line corresponds to a unique real number, and every real number corresponds to a unique point on the line. We see that the real numbers are totally ordered, and include every kind of number we might need. A beautiful set.

Notationally, we often represent real numbers using decimal notation, in which we write a real number as its integer and non-integer parts, separated by a dot. Notice that the non-integer part is actually a sum of fractions: so many tenths, plus so many hundredths, plus so many thousandths, and so on.



If the decimal terminates, then it represents a rational number. This is also true if the decimal repeats the same pattern[20] endlessly. In this latter case, we represent the "endlessly repeating pattern" by putting a line over the repeating part.

$$1.23232323\ldots = 1.\overline{23}$$

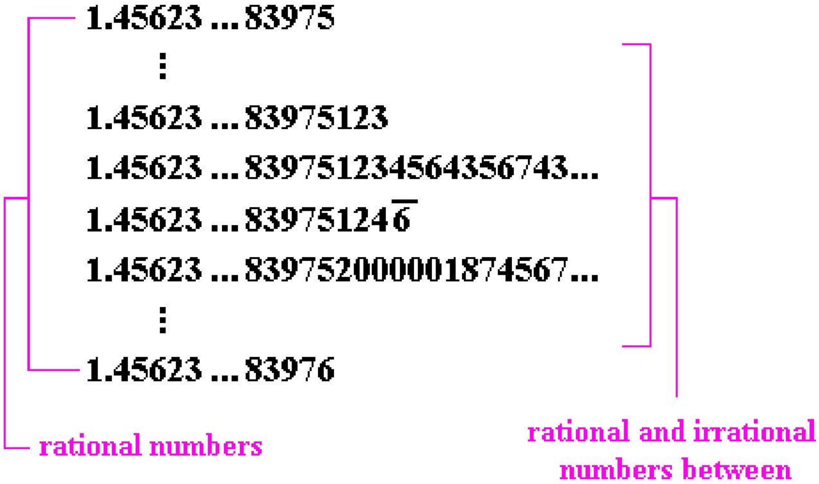
However, if the decimal continues forever without falling into a pattern that repeats endlessly, then it represents an irrational number. Obviously, one cannot write down an endless sequence of digits, so we just use an ellipsis after a few digits to represent that the sequence continues.

$$\sqrt{2} = 1.415\ldots$$

Notice how unimaginably *dense* the set of real numbers is! We already knew that the rational numbers were dense, in that between any two you could find another, but now consider the *ir*rationals – are they

[19] vice versa — the opposite of what has been said
[20] pattern — model, arrangement, style

dense, or what? Just as we can find a rational number between any two rational numbers, so we can find as many irrational numbers as we please[21] between any two rational numbers (or irrational numbers). To see this, take two rational numbers (in decimal representation) that are very close together. Then we can do the following:



Infinities within infinities . . . .

The real numbers have many properties that are both useful and surprising, as you will discover in your continued study. Indeed, an immense[22] body of mathematical science is devoted to these properties, particularly the field of *real analysis*.

There are other numbers sets, as well; the imaginary numbers, the complex numbers, the infinitesimals, and so on. There are even numbers called "hyperreal" and "surreal."

Indeed, this article merely scratches the surface[23] of "number." Even the natural numbers themselves, the ones we learn to count with as toddlers[24], harbour[25] within their depths great riches. It is not for nothing that the great mathematician Karl Gauss called *number theory* (the science of the natural numbers) the "Queen of Mathematics." A good place to start is with the suggested readings below.

Numbers are the highest degree of
knowledge. It is knowledge itself.
– *Plato*

[21] as you please — whatever you like, or in whatever way you prefer

[22] immense — extremely large
[23] scratch the surface — to deal with only the simple or obvious parts of something
[24] toddler — a very young child who is learning how to walk
[25] to harbour — to contain

You can't get there from here!

INTRODUCTION

Read the text and answer the following questions:

1. *How do children become aware of the concept of infinity?*
2. *What is the essence of infinity?*
3. *Is the universe infinite?*
4. *What kinds of infinity do you know?*

Every child becomes aware of infinity when he or she learns to count. We all went through this, and for most of us it snuck up on[26] us gradually. This is because we weren't expected to count very high at first. Getting to “three” was perhaps our first achievement in breaking into the world of “many's.” Then it turned out there was something called “ten,” and it took a little time to work out precisely how to get there from “three.” Once that was mastered, we might have been expected to rest on our laurels[27] – but it was not to be. For there was an “eleven.” And a “twelve.” And *then*, for crying out loud, all those “'teens!” By now, a sneaking suspicion[28] had begun to break across our awareness, and we wanted to ask, “when does this *end?*” And then, dreadfully, “*does* it end?” And at last, the awful truth: it NEVER ends.

Fortunately, after “twenty” it all breaks down to a pretty simple system, and the rest is easy. We turned our attention, gratefully, to other things. Still, though, the fact that it *never* ends remained psychologically vexing[29] for most of us. All children try at some point to see how high they can count, even having contests about it. Perhaps this activity is born, at least in part, of the felt need to challenge this notion of *endlessness* – to see if it really holds up to experiment. “Infinity,” we called it, and used the word cheerfully whenever we needed it.

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[26] to sneak up on smb.— if something sneaks up on you, it happens when you are not expecting it
[27] rest on your laurels — to be satisfied with your achievements and do nothing to achieve more
[28] suspicion — a feeling that something bad is probably true or likely to happen
[29] vexing — making you feel annoyed, confused, or worried

"You're a dumby[30], nyah nyah!"
"Oh yeah? Well you're *twice* as dumb!"
"Well . . . you're a *hundred* times as dumb!"
"Well you're a *million* times . . ."
"Well you're *infinity* times . . ."

... to the last of which a good answer was hard to find. How could you get bigger than infinity? Infinity plus one? And what's that?

Infinity, of course, infected our imaginations, and for some of us it cropped up[31] in our conscious thoughts every now and then in new and interesting ways. I had nightmares for years in which I would think of something doubling in size. And then doubling again. And then doubling again. And then doubling again. And then . . . until my ability to conceive[32] of it was overwhelmed[33], and I woke up in a highly anxious state.

Another form this dream took was "something inside of something," and then all of that inside something else, and all of that inside something else, and . . . and then I was awake, wide-eyed and perspiring. Only when I studied mathematics did I discover that my dream contained the seed of an important idea, an idea that the mathematician John Von Neumann had years before developed quite consciously and deliberately. It is called the Von Neumann hierarchy, and it is a construction in set theory.

There are many ways infinity can catch our imagination. Everyone has wondered if the universe is infinite, for instance. It is an easy mistake to conclude that it must be, reasoning that if it wasn't then it would have to have a boundary, and then what would be on the other side? The answer to this is that the universe may be like a *sphere*. The surface of a sphere doesn't have a boundary, but it is certainly finite in area. (The universe, in other words, could be like a *three*-dimensional surface of a sphere which can be imagined as existing in *four*-dimensional space – most cosmologists think it may well be something like that.)

The trouble here is in thinking that an infinite set must contain everything. However, a little thought shows that this needn't be true.

For instance, if I have an infinite set of natural numbers, must it contain the number 17? Obviously not; there are infinitely many even numbers, yet the set of even numbers doesn't contain the number 17.

[30] dumb(y) — stupid
[31] to crop up — to appear or happen suddenly or unexpectedly
[32] to conceive — to imagine something or think of doing something
[33] to overwhelm — to affect someone's emotions in a very powerful way

(Indeed, there are infinitely many infinite sets of natural numbers that don't contain the number 17!)

Infinities don't have to be large, of course – they can also be small. Instead of something doubling in size, it could be halved[34]. And then halved again. And then halved again. And then halved again. And then… and so on forever. This is the basis of Zeno's Paradox of the Tortoise and Achilles, in which it is proved that motion is impossible. There are other ways of thinking of infinity, too. A circle is infinite, in the sense that one can go round it forever, and many Hindus believe that all of creation is a great circle that repeats itself endlessly. And who hasn't stood between two mirrors? What other kinds of infinity can you think of?

THE HISTORY OF INFINITY

Read the text and answer the following questions:

1. *What were Aristotle's views on infinity and how did he distinguish between potential infinite and actual infinite?*
2. *What is Galileo's Paradox?*
3. *Who was the first to introduce the symbol of infinity we use today?*
4. *In what way did mathematicians of the 17^{th} and 18^{th} centuries treat the problem of infinity?*
5. *How did Cantor's ideas contribute to the development of contemporary mathematics?*

It is a surprising fact that throughout most of the history of mathematics infinity was a taboo subject. Mathematicians just didn't like to talk about it. There were many reasons for this, but a good deal of it goes right back to Aristotle. A student of Plato's and a tutor of Alexander the Great, Aristotle has been considered the most widely learned scholar of all time. He wrote extensively on philosophy, logic, and natural science, and his ideas held great authority long after the end of the classical period. During the middle ages his teachings formed the foundation of almost all formal learning. His views on infinity are summed up in this passage from Book III of his monumental work *Physics*,

[34] to halve — to divide something into two pieces of equal size

Since no sensible[35] magnitude is infinite, it is impossible to exceed[36] every assigned magnitude; for if it were possible there would be something bigger than the heavens.

Aristotle distinguished between the *potential* infinite, and the *actual* infinite. The natural numbers, he would say, are potentially infinite because they have no greatest member. However, he would not allow that they are actually infinite, as he believed it impossible to imagine the entire collection of natural numbers as a completed thing. He taught that only the potential infinite is permissible[37] to thought, since any notion of the actual infinite is not "sensible." So great was Aristotle's influence that more than 2,000 years later we find the great mathematician Karl Friedrich Gauss admonishing[38] a colleague,

As to your proof, I must protest most vehemently[39] against your use of the infinite as something consummate[40], as this is never permitted in mathematics. The infinite is but a figure of speech[41]...

Nonetheless, long before Gauss's time, cracks[42] had begun to appear in the Aristotelian doctrine. Galileo (b. 1564) had given the matter much thought, and noticed the following curious fact: if you take the set of natural numbers and remove exactly half of them, the remainder is as large a set as it was before. This can be seen, for example, by removing all the odd numbers from the set, so that only the even numbers remain. By then pairing every natural number n with the even number $2n$, we see that the set of even numbers is equinumerous with the set of all natural numbers. Galileo had hit upon the very principle by which mathematicians in our day actually *define* the notion of infinite set, but to him it was too outlandish[43] a result to warrant[44] further study. He considered it a paradox, and "Galileo's Paradox" it has been called ever since.

As the modern study of mathematics came into full bloom[45] during

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[35] sensible — perceptible to the mind, material
[36] to exceed — to go beyond the limit or bounds of
[37] permissible — allowed to be done
[38] to admonish — to tell someone that you do not approve of what they have done
[39] vehemently — strongly
[40] consummate — complete or perfect
[41] figure of speech — an expression of language, such as metaphor or personification, by which the usual or literal meaning of a word is not used
[42] crack — a sign that something is weak or beginning to fail
[43] outlandish — extremely strange and unusual
[44] to warrant — to make an action seem reasonable or necessary
[45] in full bloom — if a tree or plant is in bloom, it is covered with flowers

the seventeenth and eighteenth centuries, more and more mathematicians began to sneak[46] the notion of an *actual* infinity into their arguments, occasionally provoking a backlash[47] from more rigorous colleagues (like Gauss).

The English mathematician John Wallis (b. 1616), was the first to introduce the "love knot" or "lazy eight" symbol for infinity that we use today, in his treatise *Arithmetica infinitorum*, published in 1665. Ten years later, Isaac Newton in England and Gottfried Leibnitz in Germany (working independently) began their development of the calculus, which involved techniques that all but demanded the admission of actual infinities. Newton sidestepped[48] the issue by introducing an obscure notion called "fluxions," the precise nature of which was never made clear. Later he changed the terminology to "the ultimate[49] ratio of evanescent[50] increments[51]".

The discovery of the calculus opened the way to the study of *mathematical analysis*, in which the issue of actual infinities becomes very difficult indeed to avoid. All through the nineteenth century, mathematicians struggled to preserve the Aristotelian doctrine, while still finding ways to justify the marvellous discoveries which their investigations forced upon them.

Finally, in the early 1870's, an ambitious young Russian/German mathematician named Georg Cantor upset the applecart[52] completely. He had been studying the nature of something called *trigonometric series*, and had already published two papers on the topic. His results, however, depended heavily on certain assumptions about the nature of real numbers. Cantor pursued[53] these ideas further, publishing, in 1874, a paper titled, *On a Property of the System of all the Real Algebraic Numbers*[54]. With this paper, the field of set theory was born, and mathematics was changed forever. Cantor completely contradicted the Aristotelian doctrine

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46 to sneak — to bring, take, or put secretly

47 backlash — a strong, negative, and often angry reaction to something

48 to sidestep — to avoid something difficult or unpleasant

49 ultimate — elemental, fundamental, basic, or essential

50 evanescent — lasting only for a very short time; ephemeral or transitory

51 increment — a small positive or negative change in a variable or function

52 to upset the applecart — to spoil someone's plan or arrangement

53 to pursue — to follow a course of activity

54 One can without qualification say that the transfinite numbers stand or fall with the infinite irrationals; their inmost essence is the same, for these are definitely laid out instances or modifications of the actual infinite.– *Georg Cantor*

proscribing actual, "completed" infinities, and for his boldness[55] he was rewarded with a lifetime of controversy[56], including condemnation[57] by many of the most influential mathematicians of his time. This reaction stifled[58] his career and may ultimately have destroyed his mental health. It also, however, gained him a prominent and respected place in the history of mathematics, for his ideas were ultimately vindicated[59], and they now form the very foundation of contemporary mathematics.

CANTOR'S SET THEORY

Read the text and answer the following questions:

1. *What idea underlies Cantor's discoveries in set theory?*
2. *What is an equinumerous set?*
3. *When is a set infinite?*
4. *What is cardinality?*
5. *What is the connection between infinite sets and natural numbers according to Cantor?*
6. *What is the idea behind "Cantor's diagonalization argument"?*
7. *Why isn't it possible to make a countable list of real numbers?*

Georg Cantor's discoveries in set theory rest upon a very simple idea, an idea which may be illustrated in the following way. Suppose you couldn't count to five. (Difficult to imagine, I admit – but then for each of us there was a time when this was actually true!)

Now look at your hands. If your were unable to count to five, how would you know there are the same number of fingers on each hand? You couldn't count the fingers on one hand, and then count the fingers on the other hand, to see that there were the same number of fingers on each, because you couldn't count that high. What could you do? The answer is simple: place the thumb of your right hand against the thumb of your left hand. Then place your index fingers together, and then all the other fingers, in a one-to-one match-up. When you are done, each finger of each

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[55] boldness — courage
[56] controversy — disagreement
[57] condemnation — a public statement in which someone criticizes someone or something severely
[58] to stifle — to stop something from developing normally
[59] to vindicate — to prove that someone is right, or that something they said, did, or decided was right, especially when most people believed they were wrong

hand is matched to the corresponding finger of the other hand, with none left over on either side. You still don't know how *many* fingers are on each hand, but you do know that *they are the same number*.

Now, the real trouble with infinity is much the same: we can't count that high! Cantor's insight[60] was that, even though we can't enumerate an infinite set, we can nonetheless apply the same procedure to any well-defined infinite set that we applied above to determining if our hands have the same number of fingers. In other words, we can determine if two infinite sets are the same "size" (equinumerous) by seeking to find a one-to-one match-up between the elements of each set. Now, remember Galileo's Paradox? Galileo noticed that we can do the following:

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 2 & 4 & 6 & 8 & 10 & \dots & 2n & \dots \end{array}$$

Thus, we can assign each number n in the set of natural numbers to the corresponding number $2n$ in the set of even numbers. This rule specifies a one-to-one match-up between the set of all natural numbers and the set of even natural numbers. By definition, then, these sets are equinumerous – the same size. We can play the same game with many subsets of the natural numbers. For example, we can form a one-to-one match-up between the natural numbers and the set of squares, or the set of multiples of five, or the set of prime numbers, or the set of numbers greater than 37.

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 1 & 4 & 9 & 16 & 25 & \dots & n^2 & \dots \end{array}$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 5 & 10 & 15 & 20 & 25 & \dots & 5n & \dots \end{array}$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 2 & 3 & 5 & 7 & 11 & \dots & n^{th}\ \textit{prime} & \dots \end{array}$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & \dots & n & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ 38 & 39 & 40 & 41 & 42 & \dots & n+37 & \dots \end{array}$$

[60] insight — the ability to notice and understand a lot

We may feel some discomfort at the idea that we can remove some of the elements of a set and still have as many as we started with, but this is an artifact[61] of our experience with finite sets (in which removing something means having a smaller set). Infinite sets simply don't behave that way. In fact, this leads us to a definition:

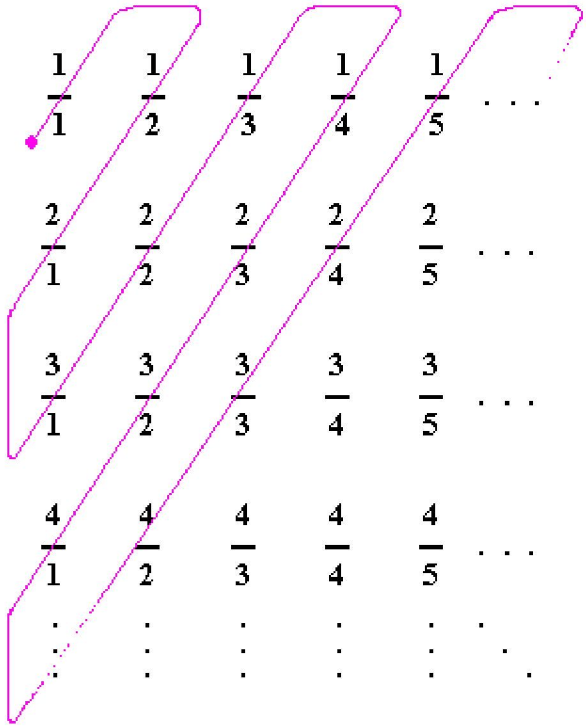
A set is *infinite* if we can remove some of its elements without reducing its size.

We have a special name for the "size" of a set: *cardinality*. We say that the set of natural numbers and the set of even numbers, for instance, have the same cardinality. Also, whenever a set has the same cardinality as the natural numbers, we say that the set in question is *countable*, since it can be put into a one-to-one correspondence with the counting numbers (i.e., the set of natural numbers).

Cantor's next great accomplishment[62] was to ask the question, "do all infinite sets have the same cardinality?" In other words, can all infinite sets be put into a one-to-one match-up with the natural numbers? It is not difficult to find one-to-one match-ups between the natural numbers and the integers (you should try to do this), so the first set to consider as – possibly – cardinally larger is the set of rational numbers. Recall that the rational numbers are *dense*, which means that between *any two* rational numbers on the real number line we can find *infinitely more* rational numbers. This suggests to our intuition that the set of rational numbers may be, in some sense, "bigger" than the set of natural numbers. However, it turns out that the rational numbers are indeed countable, as may be seen by examining the following table:

[61] artifact (artefact) — an object that was made a long time ago and is historically important, for example a tool or weapon

[62] accomplishment — something difficult that you succeed in doing



This table, if completed down and across (an infinite process!) contains *all* the rational numbers. (It contains many duplications, of course. All the fractions on the main diagonal, for instance, are really just the same number – one – but that won't affect our argument.) Now, we can "count" the rational numbers by just following the criss-crossing[63] line. Thus, the rational numbers really are countable – that is, there are just as many natural numbers as there are rational numbers. Given how the set of rational numbers seems to contain infinities within infinities, this is an astounding[64] result.

The next set to ask about, obviously, is the real numbers. After our experience with the rational numbers, it would be understandable to guess that, after all, countable infinities are the only kind of infinities there are in mathematics. But no! Cantor showed that the real numbers are *cardinally larger* than the natural numbers – in other words, there is no way to form a one-to-one match-up between the natural numbers and the real numbers that doesn't leave some of the real numbers out. To show this, Cantor invented a whole new kind of proof, which has come to be called "Cantor's diagonalization argument."

Cantor's proof of the "nondenumerability" of the real numbers (the diagonalization argument) is somewhat more sophisticated than the proofs we have examined hitherto[65]. However, laying aside some purely technical

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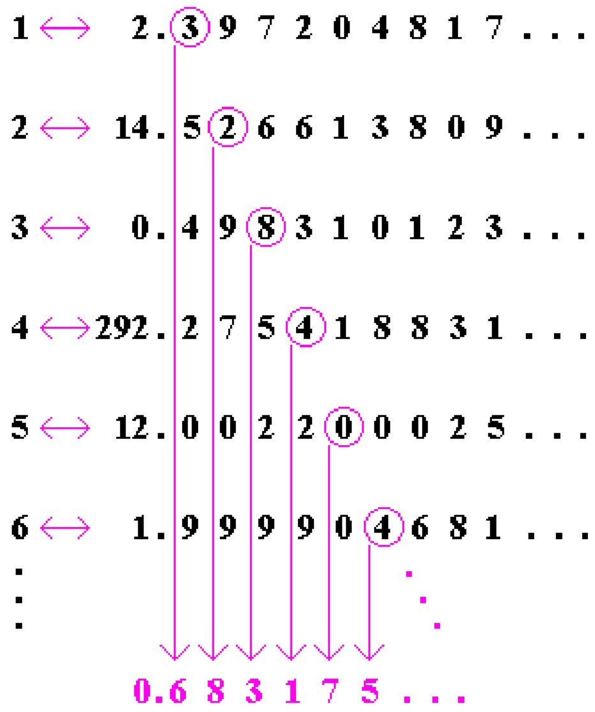
[63] to criss-cross — to form a pattern of straight lines that cross one another
[64] astounding — extremely surprising or shocking
[65] hitherto — until the present time, previously

matters, we can provide a simplified – but still convincing – version of his proof in the following way. Remember that we are attempting to prove a negative statement: that the real numbers are *not* countable. As so often in such cases, we approach this issue through the back door, so to speak, with a *proof by contradiction*. In other words, we begin by assuming, for sake of argument, that the real numbers *are* countable. This would mean that we could form a one-to-one match-up of the natural numbers and the real numbers. Since real numbers may be represented in decimal form (with an integer part and a decimal part), this means that we could provide a *numbered list* of the real numbers which would look something like this:

$$
\begin{array}{ll}
1 \leftrightarrow & 2.\,3\ 9\ 7\ 2\ 0\ 4\ 8\ 1\ 7\ \ldots \\
2 \leftrightarrow & 14.\,5\ 2\ 6\ 6\ 1\ 3\ 8\ 0\ 9\ \ldots \\
3 \leftrightarrow & 0.\,4\ 9\ 8\ 3\ 1\ 0\ 1\ 2\ 3\ \ldots \\
4 \leftrightarrow & 292.\,2\ 7\ 5\ 4\ 1\ 8\ 8\ 3\ 1\ \ldots \\
5 \leftrightarrow & 12.\,0\ 0\ 2\ 2\ 0\ 0\ 0\ 2\ 5\ \ldots \\
6 \leftrightarrow & 1.\,9\ 9\ 9\ 9\ 0\ 4\ 6\ 8\ 1\ \ldots \\
\vdots & \vdots
\end{array}
$$

. . . where the numbers on the left are the natural numbers, and the numbers on the right are a “denumeration” of the real numbers. That is, we are supposing that we eventually have *every* real number running down the right-hand side of this list, with its corresponding natural number next to it. Now, Cantor concluded that there exists at least one real number that can't be on the list, and he reasoned as follows: Create a new real number by first picking any number for the integer part (zero will do), and then let its first decimal place digit be different from the digit in the first decimal place in the first number in our list. Then let our new number's second decimal place digit be different from the digit in the second decimal place in the second real number in our list. Proceed in the same way, so that each decimal place digit in our new number is different from the corresponding digit in the corresponding real number in the list. Thus, we could do something like the following:



Now we ask the question, is our new real number on the list? Well, it can't be the same as the first number on the list, since it is different in the first decimal place, owing to the way we constructed it. Likewise, it can't be the same as the second number on the list, since it is different from that one in the second decimal place. In fact, we see that it can't be the same as *any* of the real numbers in our list, since it differs from *each* number on the list in at least one decimal place.

BUT – we assumed we had a complete list. This is a contradiction. Therefore, our assumption that we could make a countable list of the real numbers is false! The real numbers have a *higher order* of infinity than the natural numbers, i.e., they are cardinally greater. (It is natural to ask, "well, why not just add the new number to the list?" Indeed, we could do so. However, this fails to address the fundamental point of the argument: we *assumed* we had a complete list of real numbers, and then showed that this assumption *cannot be true*. It is the existence of this contradiction which forces the conclusion that the real numbers aren't countable. And of course, even if we added our new one to the list, we could use the same process to create infinitely more. There's just no way to create a completed "match-up" between the sets.)

CARDINALS

Read the text and answer the following questions:

1. *What is meant by cardinal numbers?*
2. *How did Cantor define transfinite cardinals?*
3. *What does Cantor's Theorem state?*
4. *What do you know about continuum hypothesis put forward by Cantor? Did anyone manage to prove it?*

Now that we have two cardinalities – the countable cardinality of the natural numbers and the uncountable cardinality of the real numbers – we have the beginnings of a collection (a set!) of "cardinalities." We'll call them *cardinal numbers* and give them symbols to stand for them. Following tradition, let us denote the countable cardinal by the lower-case Greek letter ω (omega). We'll denote the cardinality of the real numbers by a lower-case *c*, which stands for *continuum*.

Now our set of cardinal numbers contains only two elements, but let's make an adjustment[66] at once. Since cardinal numbers are used to describe the "sizes" of sets, it happens that we really want to call the natural numbers "cardinal numbers" too. After all, they describe the sizes of finite sets. And we may as well have zero, since that's the size of the empty set. Thus, the set of all cardinals will contain both kinds of cardinal; finite cardinals (which are just the natural numbers, really, together with zero) and what Cantor termed *transfinite* cardinals, which include our ω and *c*.

cardinals = { 0, 1, 2, 3, . . . , ω, *c*, . . . ? }

Notice the question mark. We haven't really settled whether there are any *more* transfinite cardinals. Fortunately, Cantor has done that for us, in what is now called Cantor's Theorem. It may be stated as follows.

If *X* is any set, then there exists at least one set, the *power set* of *X*, which is cardinally larger than *X*.

The proof of Cantor's Theorem has a similar flavour to his proof of the nondenumerability of the real numbers, but it is somewhat more abstract. The intrepid[67] will have little difficulty following it, however. It is an interesting fact that the power set of any countable set, i.e., of size ω, has cardinality *c*, the size of the continuum.

[66] adjustment — a change in something that makes it better, more accurate, or more effective
[67] intrepid — fearless, brave

What Cantor's Theorem tells us is that we can construct sets with greater and greater cardinalities. Cantor introduced a special notation for this hierarchy of cardinalities using the Hebrew letter *aleph* (pronounced *AH-leff*), with numeral subscripts showing where they are in the hierarchy. Thus, aleph-null is the first infinite cardinal, and denotes the cardinality of the natural numbers (or any countable infinite set, i.e. of cardinality ω), and aleph-one is the next cardinal number, standing for the next size of infinity, and so on.

$$\text{"cardinals"} = \{0,\ 1,\ 2,\ 3,\ \ldots,\ \aleph_0,\ \aleph_1,\ \ldots ?\}$$

Now we know what to replace the question mark with – since there is no largest cardinality, this list goes on forever, with more and more *aleph's* denoting larger and larger kinds of infinity. And we know where the ω belongs in this list – it is the first infinite cardinal, the aleph-null.

What about *c*, the cardinality of the continuum? Cantor thought that it must be aleph-one, that is, that the size of the continuum was the next highest after the natural numbers. This conjecture is now called the *continuum hypothesis.* Cantor was never able to prove it, however, and this bothered mathematicians for many years. How could we be sure where the "size" of the real numbers fit in the scheme of things?

In the 1930's, Kurt Gödel showed that the continuum hypothesis can't be *disproved* from the axioms of set theory, and in the 1960's another mathematician named Paul Cohen showed that it *cannot be proved*, either. This is a very strange thing, and mathematicians have debated what it means ever since. At the very least, it means that our current understanding of sets is not strong enough to settle the question of the continuum. We have, at present, no way of determining where in the hierarchy of infinite cardinals the cardinality of the continuum belongs.

CONCLUSION

Infinity, as we have seen, is an idea which reaches far beyond that haunting[68] intuition of endlessness which stems from[69] our early childhood experience of number. To a present day mathematician, infinity is both a tool for daily use in his or her work, and a vast[70] and intricate[71] landscape

[68] haunting —remaining in the consciousness; not quickly forgotten
[69] to stem from something — to be caused by something
[70] vast — extremely large
[71] intricate — very complicated and difficult to understand or learn

demanding to be explored. This article has provided only a bare[72] introduction to the topic of infinity, and there remain many beautiful ideas for the interested reader to discover: infinitesimal numbers, surreal and hyperreal numbers, and transfinite ordinals, to name just a few. Indeed, the possibilities are probably . . . well, *you* know!

Even as the finite encloses[73] an infinite series
And in the unlimited limits appear,
So the soul of immensity[74] dwells[75] in minuta
And in the narrowest limits, no limits inhere
What joy to discern[76] the minute in infinity!
The vast to perceive in the small, what Divinity[77]!

– *Jakob Bernoulli*

∞

[72] bare — basic, with nothing extra
[73] to enclose — to contain or hold
[74] immensity — the very large size of something
[75] to dwell — to live somewhere
[76] to discern — see, recognize, or apprehend
[77] divinity — the nature of a deity or the state of being divine

Appendix 1

The most common mathematical symbols[1]

Signs and some sample formulas	Spoken form
1+2=3	1 plus 2 equals 3 *or* 1 and 2 is 3
3-1=2	3 minus 1 equals 2 *or* 1 from 3 equals 2
2x3=6	2 multiplied by 3 equals 6 *or* 2 times 3=6 *or* two threes are 6
6÷2=3	6 divided by 2=3 *or* 6 over 2=3 or 2 into 6 is/goes 3
2+2=4	2+2 equals 4 *or* 2+2 is/are 4 *or* 2+2 make(s) 4
a≠2	*a* is not equal to 2 *or* a does not equal 2
a≈2	*a* is approximately equal to 2
a>2	*a* is greater than 2
a≥2	*a* is greater than or equal to 2
a<2	*a* is less than 2
a≤2	*a* is less than or equal to 2
± 4	plus or minus 4
$2^2=4$	2 squared is 4
$2^3=8$	2 cubed is 8
$2^4=16$	2 to the power 4 is 16
$16^{-4}=2$	16 to the minus 4 is 2
$\sqrt{4}=2$	(the square) root (of) 4 is 2
$\sqrt[3]{8}=2$	the cube root of 8 is 2
25%	25 per cent
90°	90 degrees
½	a half *or* one half
⅓	a third *or* one third
¼	a quarter *or* one quarter
¾	three quarters
⅜	three eighths

[1] The information comes from *Webster's New World Dictionary of American English* (1993).

$\frac{1}{8}$	one eighth
0.1	nought ('o'/zero) point one
3.15	three point one five
8.04	eight point nought ('o'/zero) four
$\lvert 3 \rvert$	absolute value for -3 and +3
$\int$	integral; integral of
Σ	sum; algebraic sum
π	pi [paɪ]
$A \cap B$	logical product; intersection; the intersection of A and B
$A \cup B$	the logical sum; the union of A and B
$B \subset A$	B is contained in A
$A \supset B$	A contains B
$x \in y$	*x* is a member of the set *y*
Ø	empty set
∞	infinitely great
( )	parentheses *or* brackets
[ ]	square brackets
{ }	braces
(a,b)	a pair of (elements) *a, b*
{a,b}	the set of *a,b*
a'	*a* prime
a''	*a* double prime
f(x)	*f* of *x*
f ' *or* df/dx	the derivative of *f*
$\int^b_a f(x)\,dx$	the integral of *f* with respect to *x*, between the limits *a* and *b*
d/dx	derivative with respect to *x*
l_n	*l* sub *n*

Examples of some mathematical formulas read in English

$x(y+z)=xy+xz$

x [eks] times the sum of *y* [wai] and *z* [zed] is equal to *xy* plus *xz*

$ax^2+bx+c=0$

ax squared plus *bx* squared plus *c* is equal to 0

$x^2+2px+p^2=(x+p)^2$
x squared plus two *px* squared is equal to the sum of *x* and *p*

$(2a-a)/a=1$
two *a* minus *a* divided by *a* is equal to one

$a(b+c)=ab+ac$
a times, parenthesis, *b* plus *c*, close parenthesis, is equal to *ab* plus *ac*

$a^2<p<(a+1)^2$
p is greater than *a* squared and less than *a* plus one all squared

$a+(bc) \neq (a+b)\ (a+c)$
a plus the product of *b* and *c* is not equal to the product of *a* plus *b* and *a* plus *c*

$A=(a,b)$
Capital *A* is equal to the pair *a,b*

Appendix 2

A short list of the most common scientifically-oriented Russian words and expressions and their English equivalents

- анализ чего-либо — the analysis of sth.
- благодаря чему-либо — due to sth.
- более или менее — more or less
- быть вызванным чем-либо — to be caused by sth.
- быть исключением из правила — to be the exception to/from the rule
- быть способным на что-то — to be capable of (doing) sth.
- в общем — in general
- в противном случае — otherwise
- в связи с чем-либо — in connection with sth.
- в следующем смысле — in the following sense
- в частности, в особенности — in particular
- верный — true
- включать — to include
- влияние, воздействие чего-либо на что-либо — the influence (the effect) of sth. on sth.
- вместо чего-либо — instead of (doing) sth.
- выполнять вычисления — to perform (to do, to make) calculations
- выраженный в форме (в виде) — to be expressed as sth.
- данный пример — given example
- доказательство от противного — reductio ad absurdum proof
- доказывать / опровергать — to prove / to disprove
- достижение — achievement
- если не — unless
- за исключением чего-либо — except (for) sth., with the exception of sth.
- зависеть от чего-то — to depend on sth.
- задаваться формулой — to be given as a formula
- заданный, установленный, конкретный — specified
- задача чего-либо — the task of sth.
- из-за чего-либо — because of sth.
- изобретать что-либо — to invent, to devise

- изучение чего-то — the study of sth.
- иметь дело с чем-то (заниматься чем-то) — to deal with sth. (to be concerned with sth.)
- иметь много общего с чем-либо, не иметь ничего общего с чем-либо — to have much in common with sth. (to have much to do with sth.), to have nothing in common with sth. (to have nothing to do with sth.)
- иметь силу, быть справедливым для чего-либо — to be valid for sth.
- использование (польза) чего-то — the use of sth.
- исследовать что-либо — to research (to study, to examine) sth.
- как…, так и — both sth. and sth.
- касающийся чего-либо — concerning sth.
- конкретизировать что-либо — to specify sth.
- кроме того — moreover, furthermore
- лежать в основе чего-либо — to underlie sth.
- ложный — false
- считаться (рассматриваться) как что-либо — to be considered (to be regarded) as sth.
- например — for example (e.g.)
- не только…, но и — not only … but
- несмотря на что-либо — in spite of sth. (despite sth.)
- обеспечивать что-либо — to provide sth.
- обзор чего-либо — the review of sth.
- область чего-либо — the area of sth., the field of sth., the sphere of sth.
- означать то же самое — to mean the same thing
- обозначаться чем-либо — to be denoted (designated) by sth.
- обсуждать в деталях — to discuss in detail
- объяснять что-либо — to explain sth., to account for sth.
- ознакомиться с чем-либо — to be familiar with sth.
- оперировать чем-либо — to operate with sth.
- определять (давать определение) — to define sth.
- определять (устанавливать, обусловливать) — to determine sth.
- основы чего-либо — the fundamentals of sth.
- основываться на чем-либо — to be based on sth.
- особенность чего-либо — the peculiarity of sth.

- оставаться неизменным, постоянным — to stay constant
- открывать (закон и т.д.) — to discover
- открытие — discovery, finding
- отличительная черта чего-либо — the distinctive feature of sth.
- относится к чему-то (быть связанным с чем-то) — to be related to sth. (to be connected with sth.)
- отсюда (следовательно) — hence
- очевидно, что — it is obvious (evident) that
- первый из двух, последний из двух — the former, the latter
- передавать(ся) — to transmit
- по сравнению с чем-либо — in comparison with sth., compared with/to sth.
- поддерживать теорию — to support the theory
- подход к чему-либо — an approach to sth.
- подходящий — appropriate, suitable
- подчиняться правилам — to obey the rules
- получать (выводить) из чего-то — to derive sth. from sth.
- помимо чего-либо — apart from sth.
- понимается, что — it is understood that
- поскольку — since, as
- последствие чего-либо — the consequence of sth.
- посредством чего-либо — by means of sth, by the use of sth.
- предложение (суждение, теорема) — proposition
- предмет (объект) — object
- предмет статьи (обсуждения) — the subject of the article (discussion), the topic of sth.
- предполагать что-либо — to assume sth., to suppose sth.
- представленный в виде чего-либо — to be represented as sth.
- (не) представлять интерес (важность, значение) — to be (no) of interest (importance, significance)
- предшествующий — preceding, previous
- преобразовывать что-либо во что-либо — to convert sth. into sth.
- претерпевать изменения — to undergo changes
- при одинаковых условиях — under the same conditions
- при условии, что — provided that
- признавать что-либо — to recognise sth.
- применяться (прилагаться) к чему-либо — to be applied to sth.

- пример чего-либо — an example of sth.
- принимать во внимание (учитывать) что-либо — to take sth. into account (consideration)
- принято называть — it is customary to call sth.
- приписывать чему-то — to assign to sth.
- причина чего-либо — the cause of sth., the reason for sth.
- понятие чего-либо — the notion (the concept) of sth.
- проверять что-либо — to test, to verify, to check
- проводить исследование, эксперимент в какой-то области — to do (carry out, perform, conduct) research into/on sth., experiment on/with sth.
- проводить различие между чем-либо — to show the difference between sth., to distinguish between sth., to differentiate between sth.
- проводить сравнение с чем-либо — to make (draw) a comparison with sth.
- продвижение (прогресс) в чем-либо — an advance in sth.
- противоречить — to contradict sth.
- прояснять что-либо — to clarify sth., to make sth. clear
- развитие чего-либо — the development of sth.
- раздел чего-либо — the branch of sth.
- разрабатывать что-либо — to develop, to work out
- реагировать на что-либо — to react to sth., to respond to sth.
- результат чего-то — the result of sth.
- с одной стороны, с другой стороны — on the one hand, on the other hand
- с точки зрения чего-либо (в терминах чего-либо) — in terms of sth.
- систематизировать что-либо — to systematise sth.
- следующим образом — as follows
- сложная проблема (система) — complicated problem (system)
- сложный аппарат (машины) — complex apparatus (machinery)
- случай, в любом случае, в этом случае — case, in any case, in this case
- совмещать — to combine
- согласно чему-либо — according to sth.
- содержать что-либо — to contain sth.
- составлять (входить в состав) что-либо — to constitute sth.

- состоять из чего-либо — to consist of sth.
- спорные вопрос — controversial question (issue, problem)
- стремиться сделать что-то — to tend to do sth.
- так же, как — as well as
- так называемый — the so called
- таким образом — thus, therefore, that is why
- такой, как — such as
- тем не менее — however, nevertheless
- типичный для чего-либо — typical of sth.
- тогда и только тогда — if and only if
- то есть — i.e. (that is)
- точка зрения — point of view
- требуемый, искомый — required
- трудность чего-либо — the difficulty of sth.
- углубляться в предмет — to go into the subject
- удобный — convenient
- устанавливать правило — to establish the rule
- утверждать что-либо — to state (to assert) sth.
- характеристика чего-либо — the characteristic of sth.
- характерный для чего-либо — characteristic of sth.
- хорошо известно, что — it is well known that
- хотя — (al) though
- цель исследования — the purpose of the study
- четко определенный — well-defined
- что касается чего-либо — as far as sth. is concerned
- явление (явления) — phenomenon (phenomena)

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