Global Behavior of Nonlinear Difference Equations of Higher Order with Applications

by G. Ladas and V. L. Kocic
reviewed by John R. Graef
abridged for academic purposes

The last few years have witnessed a strong resurgence in interest in difference equations as a study in their own right. The focus of this work is the study of nonlinear difference equations of the general form

$$x_{n+1} = F(x_n, x_{n-1}, \ldots, x_{n-k}), n = 0, 1, \ldots (E),$$

where k is a fixed positive integer and F is a continuous non-negative function. The first of the six chapters presents some motivating examples, such as

$$x_{n+1} = \frac{bx_n}{A + x_{n-k}, b > A},$$

which Pielou has offered as a discrete analogue of the well-known delay logistic differential equation, and

$$x_{n+1} = \frac{1 + x_n}{x_{n-1}},$$

which arises in number theory. This chapter also introduces the basic necessities for the discussion of the oscillation, stability, periodicity, and attractivity of solutions.

Chapter 2 discusses the global asymptotic stability and global attractivity of the equilibrium of

$$x_{n+1} = x_n f(x_n, x_{n-k_1}, x_{n-k_2}, \ldots, x_{n-k_r})$$

and some of its special cases. It is in this chapter that the important notion of permanence is introduced and studied. Although it may not be obvious to the casual reader (it is made clear in the notes in the last section of the chapter), much of what is here is a survey of previously known results.

In Chapter 3, the authors consider rational equations; that is, those of the form

$$x_{n+1} = \frac{a + \sum_{i=0}^{k} a_i x_{n-i}}{b + \sum_{i=0}^{k} b_i x_{n-i}},$$

where the coefficients are non-negative real numbers and k is a positive integer. Once again, much of what appears here can already be found in the literature.

Chapter 4, entitled "Applications", is in some sense the most interesting. Various models from biological phenomena are described, and the existence and stability of equilibrium solutions are discussed. The models have mostly appeared elsewhere, but they make an interesting collection.

Chapter 5 deals with periodic cycles of various cases of the equation

$$x_{n+1} = \frac{1 + x_n + x_{n-1} + \ldots + x_{n-k+2}}{x_{n-k+1}},$$

where k is a non-negative integer. The final chapter, "Open Problems and Conjectures", contains a number of problems, partial results, and directions for further research. The book also contains appendices on the Riccati equation, a contraction principle, and global behavior of systems. The first five chapters in the book are interspaced with a variety of "Research Projects". Some of these are obvious suggestions for possible generalizations of known results, whereas others are more subtle and broader in scope.

Although the book is not suitable as a first introduction to difference equations, the material is accessible to anyone who has a firm background in analysis and a passing knowledge of some of the fundamental ideas found in the introductory texts mentioned above.