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English for Students of Mathematics and Mechanics

(Part three book two)

Учебное пособие

под ред. Л. Н. Выгонской

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Unit 5

The first text you are supposed to read and discuss in this unit is concerned with a prize. A prize is awarded generally to the best in a competition. Various competitions have been and are being arranged all over the world in different spheres of human activities. For scientists the most desirable is the Nobel prize. The second text deals with Poincaré's description of how creative thinking operates.

Reading I

Task 1. Below goes the entry from an encyclopedia for the Nobel prize. Read it and extend it contributing the information known to you about this prize. Share it with your classmates.

Nobel prize — annual international prize, first awarded in 1901 under the will of Alfred Nobel, Swedish chemist, who invented dynamite. The interest on the Nobel endowment fund is divided annually among the persons who have made the greatest contributions in the field of physics, chemistry, medicine, literature and world peace.

Task 2. As you have noticed mathematics is not in the list for the Nobel prize, why is it so?

Do you know about any prizes both national and international a brilliant mathematician can be awarded?

Task 3. Look at the list of terms which will appear in the text. *Ellipse, velocity, gravity, focus, a series, convergence, point particle, trajectory, infinity, sum.*

Tell which ones sooner belong to physics while the others belong to mathematics. Give arguments in each case.

Task 4. Read the text quickly.

PRIZE MISTAKE

The n-body problem is solved — too late

In 1885 three famous mathematicians — Karl Weierstrass, Charles Hermite and Gösta Mittag-Lefler — drew up a list of outstanding problems. Any person who solved one would receive a medal and 2,500 gold crowns on the Swedish king's 60th birthday in 1889. Foremost was the classical n-body problem: given the initial positions and velocities of a certain number, n, of objects that attract one another by gravity — say, the sun and its planets — one had to predict their configuration at any later time.

More than 100 years later the problem, as stated by Weierstrass, was finally solved in 1991 by Wang Qiu-Dong, a student at the University of Cincinnati. But no one noticed until last year, when Florin N. Diacu of the University of Victoria in British Columbia described it in the *Mathematical Intelligencer.*

Weierstrass had framed the n-body problem in a specific way. He was sure that for more than two objects, there is no neat, "closed-form" solution. (An example of a closed-form solution is that of the two-body problem — formed by the sun and one planet — which is an ellipse, with the sun at one of the foci.) For n exceeding 2, Weierstrass asked instead for a single series that could yield the answer for all times. So the series had to converge: the successive terms, which serve as refinements to earlier ones, had to get small sufficiently fast. The series $1 - a^2 + a^4 - a^6 + \ldots$, for example, can be summed only if a lies between -1 and 1.

The primary difficulty was collisions. Only a mathematician would worry about point particles — as the bodies are supposed to be approximated — hitting one another. But if they do, their trajectories could cease to exist. Such singular events change the pattern of the series, preventing it from always converging. Wang introduced a measure of time that ran faster as two or more objects approached one another; according to this clock, the collision would occur at infinite time. Having relegated all conflicts to eternity, Wang could then show that there is a converging series.

The solution, unfortunately, is quite useless. As Wang himself states, one has to sum "an incredible number of terms" even for an approximate answer. Nor will he get the prize. It was awarded in 1889 to French mathematician Henri Poincaré, for a paper suggesting that no solution exists. Interestingly, Poincaré's original treatise was so full of mistakes that the publishing journal, *Acta Mathematica*, had to recall and reprint the issue. After correction, however, Poincaré's error-ridden paper laid the foundations of chaos theory. In particular, he elucidated why the motions of the planets are ultimately unpredictable. For this achievement, he surely earned his undeserved prize.

Christoph Pöppe and Madhusree Mukerjee

Vocabulary

Task 5. Below are some words taken from the text. Try to guess their meaning from the context. In each case choose one of the three answers which you think best expresses the meaning.

foremost — (line 5)

(a) the part of a picture which seems nearest to the observer;
(b) first in place or time; in dignity or rank;
(c) self-propulsion of a body through water.

refinement — (line 21)

(a) improvement in accuracy;
(b) a subatomic particle that is not made up of smaller particles, and so can be considered one of fundamental units of matter;
(c) the undeveloped areas of a country, knowledge, etc.

cease — (line 27)
(a) to bring together several different elements to form a whole;
(b) to put into words facts or statements;
(c) to bring something to an end, to a total extinction.

pattern — (line 28)
(a) rapid, independent movements of particles in a small space or cluster;
(b) the design or configuration that something takes in actuality (a perfect representative of a type);
(c) graphic material that accompanies a written text to supplement it or help explain it.

relegate — (line 31)
(a) to manipulate someone by using his own weakness against him as a psychological weapon;
(b) to classify something within a larger system or division;
(c) suggests assignment of a matter to an unimportant place.

treatise — (line 38)
(a) indicates an exclusively scholarly context referring to any presentation from essay-to-book length in which conclusions are drawn from a body of data in a formal and systematic way;
(b) a person who makes decisions in situations in which there is a conflict of views;
(c) a group of nations, states, or other parties that have entered into association for a common purpose.

elucidate — (line 41)
(a) to send away or to place apart a person, group of people or things;
(b) to set something on fire or make it burn;
(c) to indicate any enlightening process that puts an end to confusion.

Task 6. You have to guess what is the only word which can help you in filling in the blanks in the following phrases:
outstanding ... ; classical ... ; n-body ... ; to solve ... ; to state ... ; to frame ... ; to yield the answer to ... ; two-body ... .

Task 7. *An adjective is a word used to modify (limit, identify, or describe) a noun.*

There are two groups of words below. In the first list you'll find combinations of adjective + noun taken from the text. In the second list nouns from the text are given. Take a noun from the second group and try to combine it with all the adjectives from the first group. The result could be: 'Yes', 'No', 'Possible'. To prove 'Yes' and 'Possible' find an appropriate example.

Group I	Group II
famous mathematician	configuration
initial position	example
specific way	difficulty
single series	event
successive terms	pattern
primary difficulty	collision
singular event	foundation
infinite time	theory
useless solution	achievement
incredible number of terms	paper
approximate answer	object
original treatise	mistake

Task 8. What does the following sequence of numbers and symbols have to do with the text?
1889; -1; 1; a; n; 2; 1991; 100; 60; $2,500$; 3; 1885.

Grammar tasks

Task 9. Tell what is the grammatical issue underlying the principle of organizing the following sentences into one group. Translate them into Russian.

Group I

1. One had to predict the configuration of a certain number, n, of objects at any later time.

2. For n exceeding 2, Weierstrass asked instead for a single series that could yield the answer for all times.

3. The series had to converge.

4. The successive terms had to get small sufficiently fast.

5. The series can be summed only if a lies between -1 and 1.

6. If the bodies hit one another their trajectories could cease.

7. One has to sum "an incredible number of terms" even for an approximate answer.

8. The publishers had to recall and reprint the issue.

GROUP II

1. Only a mathematician would worry about point particles — as the bodies are supposed to be approximated — hitting one another.

2. The n-body problem as Weierstrass had framed it seemed for a long time to have been solved by Poincaré.

3. Poincaré's error-ridden paper was likely to lay the foundation of chaos theory.

4. More than 100 years later the problem, as stated by Weierstrass was declared to be solved by a student.

5. The introduction of a measure of time that ran faster as two or more objects approached one another appeared to relegate all conflicts to eternity.

6. The solution, unfortunately, proves to be quite useless as one has to sum "an incredible number of terms".

GROUP III

1. Any person who solved any problem from the list would receive a medal and $2,500$ gold crowns.

2. The prize was supposed to be given to a person who would predict the configuration of a certain number, n, of objects attracted to one another at any later time.

3. Only a mathematician would worry about point particles hitting one another.

4. Wang introduced a measure of time; according to this clock, the collision would occur at infinite time.

5. If the undeserved prize had not been given to Poincaré the student would get it.

Group IV

1. The list of outstanding problems drawn up by three famous mathematicians involved the foremost classical n-body problem.

2. Given the initial positions and velocities of a certain number, n, of objects attracted to one another by gravity one had to predict their configuration at any later time.

3. The problem, as stated by Weierstrass, was finally solved in 1991.

4. A closed-form solution is that of the two body problem — formed by the sun and one planet — which is an ellipse, with the sun at one of the foci.

Group V

1. Having relegated all conflicts to eternity Wang could show that there is a converging series.

2. The prize was awarded to Henri Poincaré for a paper suggesting that no solution exists.

3. For n exceeding 2, Weierstrass asked for a single series that could yield the answer for all times.

4. Such singular events change the pattern of the series, preventing it from always converging.

5. The publishing journal had to recall and reprint the issue.

Discussion of facts, ideas and concepts

Task 10. Read the text again and answer the following questions.

1) Have you heard anything about three mathematicians who drew up a list of outstanding problems in 1885? If the answer is positive, tell a few words about each familiar to you.

2) What was the prize for solution of at least one problem?

3) How was the foremost classical n-body problem formulated?

4) Did Weierstrass expect to have a neat "closed-form" solution for the n-body problem with more than two objects?

5) What is an example of a closed-form solution? Can one yield an answer to the two-body problem in a form of a distinct formula?

6) Why do you think, for n exceeding 2, Weierstrass asked for a single series as a solution?

7) What does it mean: "the series had to converge"?

8) Why were collisions a serious concern for a mathematician? How can they affect the pattern of the series?

9) What was the advantage of introducing "a measure of time"?

10) Why does Wang's solution seem to be useless?

11) Who was awarded the prize, when and what for?

12) Why was Poincaré's achievement considered to earn his undeserved prize?

Task 11.

Poincaré, Jules Henri (1854–1912) — French mathematician who developed the theory of differential equations and was a pioneer in relativity theory. He suggested that Isaac Newton's laws for the behaviour of the universe could be the exception rather than the rule. However, the calculation was so complex and time-consuming that he never managed to realize its full implication.

He also published the first paper devoted entirely to topology (the branch of geometry that deals with the unchanged properties of figures).

Three branches are mentioned in the reference given above where Poincaré made his contribution to:

1) He was a *pioneer in relativity theory.*

2) He *published* the first *paper* entirely *on topology.*

3) He *developed the theory of differential equations.*

Read the two articles from *the British Multimedia Encyclopedia* and try to write a text on the theory of differential equations.

Topology — branch of geometry that deals with those properties of a figure that remain unchanged even when the figure is transformed (bent, stretched) — for example, when a square painted on a rubber sheet is deformed by distorting the sheet. Topology has scientific applications, as in the study of turbulence in flowing fluids.

The topological theory, proposed in 1880, that only four colours are required in order to produce a map in which no two adjoining countries have the same colour, inspired extensive research, and was proved in 1972 by Kenneth Appel and Wolfgang Haken.

The map of the London Underground system is an example of the topological representation of a network; connectivity (the way the lines join together) is preserved, but shape and size are not.

Relativity — in physics, the theory of the relative rather than absolute character of motion and mass, and the interdependence of matter, time, and space, as developed by German physicist Albert Einstein in two phases: special theory (1905) starting with the premises that (1) the laws of nature are the same for all observers in unaccelerated motion, and (2) the speed of light is independent of the motion of its source, Einstein postulated that the time interval between two events was longer for an observer in whose frame of reference the events occur in different places than for the observer for whom they occur at the same place. **General theory of relativity (1915).** The geometrical properties of space-time were to be conceived as modified locally by the presence of a body with mass. A planet's orbit around the Sun (as observed in three-dimensional space) arises from its natural trajectory in modified space-time; there is no need to invoke, as Isaac Newton did, a force of gravity coming from the Sun and acting on the planet.

Einstein's theory predicted slight differences in the orbits of the planets from Newton's theory, which were observable in the case of Mercury. The new theory also said light rays should bend when they pass by a massive object, owing to the object's effect on local space-time. The predicted bending of starlight was observed during the eclipse of the Sun (1919), when light from distant stars passing close to the Sun was not masked by sunlight. Einstein showed that for consistency with premises (1) and (2), the principles of dynamics as established by Newton needed modification; the most celebrated new result was the equation $E = mc^2$, which expresses an equivalence between mass (m) and energy (E), c being the speed of light in a vacuum. Although since modified in detail, general relativity remains central to modern astrophysics and cosmology; it predicts, for example, the possibility of black holes. General relativity theory was inspired by the simple idea that it is impossible in a small region to distinguish between acceleration and gravitation effects (as in a lift one feels heavier when the lift accelerates upwards), but the mathematical development of the idea is formidable. Such is not the case for the special theory, which a nonexpert can follow up to $E = mc^2$ and beyond.

Reading II

In 1908 Poincaré published a popular essay *Science and Method* where he gave his description of how creative thinking operates. From it one can understand that he was not only the creator of new knowledge in mathematics but also an analyst of general creative process, i. e. how an individual comes to perceiving the hidden laws of nature. In this work Poincaré made some assumptions about the ways the mind operates. In his particular case he seems to have come to the discovery of the universal unconscious mechanism of human creativity — a mechanism that could generate "mental beauty". The method he used to understand the mechanism is known as the investigator's introspective observation of his own mental phenomena, so he can be called the "anatomist of his own mind".

In this part you are challenged to be an anatomist of your own mind comparing your experience in the ways of seeking and finding solutions to intellectual problems with that of Poincaré.

Task 12. Here is a list of words you will find useful and helpful in presenting your ideas. The left column is a collection of words employed by Poincaré while the right one is their Russian version. Make equivalents as in the example:

creative thinking = творческое мышление.

1) accidental discovery;	(a) понять частично, как наше мышление работает на подсознательном уровне;
2) universal process of discovery;	(b) случайное открытие;
3) conscious work;	(c) безрезультатная работа;
4) unfruitful work;	(d) наука рождается не только как следствие и результат чисто логически сделанных умозаключений;
5) fruitful ideas;	(e) полезное сочетание идей;
6) worthy combination of ideas;	(f) внезапное озарение;
7) to perceive partly unconscious mind at work;	(g) работа на уровне сознания;

8) subliminal ego;	(h) повторение;
9) irrational yet delicate intuition;	(i) обнаружить взаимосвязь между разрозненно существующими фактами;
10) to discriminate order within random facts;	(j) идеи, соприкасающиеся друг с другом;
11) tautology;	(k) подсознательное 'я';
12) not from pure logic alone can any science issue;	(l) не контролируемая сознанием, но точная интуиция;
13) sudden illumination;	(m) универсальный процесс открытия;
14) ideas jostling one another.	(n) плодотворные идеи.

Task 13. Now read an extract from the article titled *Coffee Mates* and subtitled *An artist and a mathematician share a "ready-made" brew* written by RHONDA ROLAND SHEARER and published in the journal *The Sciences.* Try to guess the meaning of the words you don't know. Then consult your dictionary to check their meaning and pronunciation. Translate it into Russian.

Poincaré gave a description of how creative thinking operates, in the popular essay *Science and Method* (1908) — where Poincaré describes his accidental discovery of the so-called Fuchsian functions, as well as the universal process of discovery itself. Following days of "unfruitful" conscious work spent trying to prove the functions do not exist, Poincaré changed his habit one evening and drank black coffee late at night. The next morning, and continuing over the following several days, "fruitful" ideas came into his conscious mind. All he then had to do was "select" among the "worthy combinations" of ideas. Unexpectedly, he suddenly saw a way to prove the existence of the very mathematical functions whose existence he had previously doubted.

Poincaré believed that the "black coffee" had opened to his "over-excited consciousness" a window through which he "partly" perceived his "unconscious" mind at work. Most of the hard work, according to Poincaré, is done by the "automatic machine motion" of the unconscious "subliminal ego." The next step in the mechanical discovery is a required rest, during which *"fruitful" ideas can be selected "ready-made" (Poincaré's term) — not by anyone,* *or even through calculations or reason, but only by a person with great "irrational" yet "delicate intuition."* "Discovery," he wrote, "is discernment, selection." In other words, the intuitive ability to discriminate order within random facts is needed to enable hidden laws to emerge unconsciously from otherwise unfruitful patterns.

Poincaré states, *"Pure logic could never lead us to anything but tautologies; it can create nothing new; not from it alone can any science issue."*

Poincaré describes his "sudden illumination": "a host of ideas kept surging in my head; I could almost feel them jostling one another, until two of them coalesced ... to form a stable combination." The "unconscious work" was like "gaseous molecules ... set in motion," whose collisions "made them produce new combinations." Those "fruitful" combinations of discovery "appear" to be the result of "preliminary sifting," in which the "irrational" and "unruly" intuition "plays the part of the delicate sieve."

Task 14. Structure out the creative process as it was understood by Poincaré. Present it in a form of successive steps.

Task 15. Explain the lines in the text in italics in your own words.

Unit 6

You are going to read an introduction to the article *René Descartes' Curve-Drawing Devices: Experiments in the Relations Between Mechanical Motion and Symbolic Language* by DAVID DENNIS, where he asks the readers to live the history backwards and to consider how the bridge between geometry and algebra was being constructed in the investigations of great scientists to demonstrate the consistency between them. The author does not advocate the revival of geometric methods. On his account progress in mathematics has been made only to the extent to which geometry has been eliminated but he is sure that such progress was possible only after mathematics had achieved basic faith in the ability of algebraic language to represent and model geometry accurately.

Reading

Task 1. What do you know about the intellectual activities of the seventeenth century Europe in science and mathematics? Answer this question, in case you feel you are lacking some factual material read the following text excerpted from *Compton's Interactive Encyclopedia*, it will help you to recollect the developments in mathematics made at the time. Discuss it in the class.

17TH CENTURY

Mathematics received considerable stimulus in the 17th century from astronomical problems. The astronomer Johannes Kepler, for example, who discovered the elliptical shape of the planetary orbits, was especially interested in the problem of determining areas bounded by curved figures. Kepler and other mathematicians used infinitesimal methods of one sort or another to find a general solution for the problem of areas. In connection with such questions, the French mathematician Pierre de Fermat investigated properties of maxima and minima. He also discovered a method of determining tangents to curves, a problem closely related to the almost simultaneous discovery of the differential and integral calculus by Isaac Newton and Gottfried Wilhelm Leibniz later in the century.

Of equal importance to the invention of the calculus was the independent discovery of analytic geometry by Fermat and René Descartes. Of two, Descartes used a better notation and devised superior techniques. Above all, he showed how the solution of simultaneous equations was facilitated through the application of analytic geometry. Many geometric problems could be translated directly into equivalent algebraic terms for solution.

Developed in the 17th century, projective geometry involves, in part, the analysis of conic sections in terms of their projections. Its value was not fully appreciated until the 19th century. The study of probability as related to games of chance had also begun.

The greatest achievement of the century was the discovery of methods that applied mathematics to the study of motion. An example is Galileo's analysis of the parabolic path of projectiles, published in 1638. At the same time, the Dutch mathematician Christiaan Huygens was publishing works on the analysis of conic sections and special curves. He also presented theorems related to the paths of quickest descent of falling objects.

The unsurpassed master of the application of mathematics to problems of physics was Isaac Newton, who used analytic geometry, infinite series, and calculus to make numerous mathematical discoveries. Newton also developed his method of fluxions and fluents — the differential and integral calculus. He showed that the two methods — derivatives and integrals — were inversely related to one another. Newton and Leibniz were studying similar problems of physics and mathematics at the same time. Having made his own discovery of the calculus in 1674, Leibniz published a rather obscure version of his methods in 1684, some years before Newton published a full version of his own methods. The sequence of mathematical developments that flows out of the discovery of the calculus is called analysis.

Although the new calculus was an immediate success, its methods were sharply criticized because infinitesimals were sometimes treated as if they were finite and, at other times, as if they were zero. Doubts about the foundations of the calculus were unresolved until the 19th century.

Task 2. Here are the names of mathematicians you will come across while reading the article: Euclid, Apollonius, Descartes, Leibniz, Euler.
What do you know about them, about their contribution to mathematics?

In case you have some problems you may consult the texts given below, they all come from *Compton's Interactive Encyclopedia*. Read them and comment on the statements dealing with particular discoveries of each of them in geometry and algebra. Discuss this in the class.

EUCLID

It has been said that, next to the Bible, the 'Elements' of Euclid is the most-translated, -published, and -studied book in the Western world. Of the author himself almost nothing is known. It is recorded that he founded and taught at a school of mathematics in Alexandria, Egypt, during the reign of Ptolemy I Soter, who ruled from 323 to about 283 BC. It is assumed from his books that he was not a first-class mathematician, but he was a first-rate teacher of geometry and arithmetic. The 'Elements' remained unchallenged for more than 2,000 years. Not until the mid-19th century was a non-Euclidean geometry devised.

To compile his 'Elements' Euclid relied on the works of several predecessors, so the book is of uneven quality. Once it was published, the 'Elements' superseded all previous mathematical treatises and became the standard text. During the Middle Ages three Arabic translations were made of the book, and it was through these that it became known in Europe. The English traveler and philosopher Adelard of Bath went to Spain disguised as a Muslim student and obtained an Arabic copy, from which he made a Latin translation in 1120. The first Latin translation of the Greek without an Arabic intermediary was made by Bartolomeo Zamberti and published in Venice in 1505. There have been several more recent translations into other languages. Among Euclid's other works on geometry were 'Data' and 'On Divisions'. He also wrote 'Optics' and 'The Elements of Music'. Some of his writings have been lost, while other works are wrongly credited to him.

APOLLONIUS OF PERGA (262?–190 BC)

Admiring friends called him "The Great Geometer" for his numerous accomplishments in the field of geometry. Specifically, it was his theory of conic sections, elaborated in his major work, 'Conics', that earned Apollonius of Perga the plaudits of his contemporaries.

A solid cone can be cut into sections, producing several unusual forms. Apollonius examined these conic sections, noted their shapes, and introduced the terms ellipse, hyperbola, and parabola to describe them. He was the first to recognize that these three forms, along with the circle, are all part of any cone. His 'Conics', which brought order to a confused and ill-defined area of geometry, is considered one of the greatest scientific works of the ancient world. His theory of conic sections is still useful to engineers and mathematicians.

Born in Perga, an ancient Greek town that lies in present-day Turkey, Apollonius studied in Alexandria, Egypt, and later taught at the university there. He traveled to several libraries and universities to expand his understanding of mathematics. In his numerous books (most of which have been lost), Apollonius acknowledged those who had studied the field before him, summarized their work, and proceeded to make his own contribution. Only in the final volumes of 'Conics' did he break new ground. In addition to geometry, he studied the properties of light and the way curved mirrors reflect it.

DESCARTES, René (1596–1650)

Both modern philosophy and modern mathematics began with the work of René Descartes. His analytic method of thinking focused attention on the problem of how we know, which has occupied philosophers ever since. His invention of coordinate geometry prepared the way for advances in mathematics. Descartes offered one of the first modern theories to account for the origin of the solar system of the Earth.

René Descartes was born on March 31, 1596, at La Haye in the Touraine region of France. At the renowned Jesuit school of La Flèche, René was taught philosophy, the humanities, science, and mathematics. After getting a law degree at the University of Poitiers in 1616, he served as a volunteer in Dutch and Bavarian armies to broaden his experience. He resumed his study of mathematics and science when his duties permitted. Dissatisfied with the haphazard methods of science then in use, he began to doubt all but mathematical knowledge.

In 1619 Descartes arrived at the conclusion that the universe has a mathematically logical structure and that a single method of reasoning could apply to all natural sciences, providing a unified body of knowledge. He believed he had discovered such a method by breaking a problem down into parts, accepting as true only clear, distinct ideas that could not be doubted, and systematically deducing one conclusion from another.

Descartes soon gave up army life. Living on private means, he spent several years traveling and applying his analytical system to mathematics and science. Finding, however, that the sciences rested on disputed philosophical ideas, he determined to discover a first principle, which could not be doubted, on which to build knowledge. Retiring to seclusion in Holland in 1629, he methodically doubted all accepted traditions and evidence about the universe and mankind. He could not doubt the statement "I think, therefore I am," and thus his first principle was established.

Descartes's major writings on methodology and philosophy were his 'Discourse on Method' (published in 1637) and 'Meditations' (1641). His application of algebra to geometry appeared in his 'Geometry' (1637). He also published works on his studies in natural science.

Descartes's work brought him both fame and controversy. In 1649 he was invited to teach philosophy to the queen of Sweden. Unused to the climate, he became ill and died in Stockholm on Feb. 11, 1650.

LEIBNIZ, Gottfried Wilhelm (1646–1716)

Although he was not an artist, Leibniz was in many other ways comparable to Leonardo da Vinci. He was recognized as the universal genius of his time, a philosopher and scientist who worked in the fields of mathematics, geology, theology, mechanics, history, jurisprudence, and linguistics.

Gottfried Wilhelm Leibniz was born in Leipzig, Germany on July 1, 1646. He was educated at the University of Leipzig and received a doctorate in law at the University of Nuremberg. Because he was forced to earn a living, he spent his entire adult life in the service of nobility and royalty, particularly for the House of Brunswick-Lüneberg in Germany. His last employer was Duke George Louis of Hanover, who became King George I of England in 1714. This employment enabled Leibniz to travel a great deal throughout Europe and meet the leading scholars of his day. His many duties did not interfere with his extensive intellectual pursuits.

During his lifetime Leibniz perfected the calculating machine invented by Blaise Pascal; laid the ground for integral and differential calculus; founded dynamics, an area of mechanics; worked on mechanical devices such as clocks, hydraulic presses, lamps, submarines, and windmills; perfected the binary system of numeration used today in computer operations; devised the theory that all reasoning can be reduced to an ordered combination of elements such as numbers, words, sounds, or colors (the theoretical basis of modern computers); laid the foundation for general topology, a branch of mathematics; strove to formulate a basis for the unification of the churches; and pursued the goal of writing a universal history. He also continued to perfect his metaphysical system through research into the notion of a universal cause of all being.

Leibniz published his philosophy in several works. 'Reflections on Knowledge, Truth, and Ideas' defined his theory of knowledge. In 'On the Ultimate Origin of Things' he tried to prove that only God could be the source of all things. 'Theodicy', his only major work published in his lifetime, explained his ideas on divine justice. 'Monadology', written two years before his death, spelled out his theory of monads, which he conceived of as simple, unextended, spiritual substances that formed the basis for all composite forms of reality. His theory of monads — a term derived from the Greek words meaning "that which is one" or "unity" — is elaborated in 'Monadology' and 'Principles of Nature and Grace Founded in Reason'. The theory attempts to describe a harmonious universe made up of an infinite number of monads, or units, arranged in a hierarchy and originating in the Supreme Monad, which is God. Monadology had its roots in the philosophy of ancient Greece and was carried on by such eminent thinkers as Immanuel Kant, Edmund Husserl, and Alfred North Whitehead. The hierarchy of monadology was, according to Leibniz, the "best of all possible worlds." The philosopher died in Hanover on Nov. 14, 1716.

EULER, Leonhard (1707–83)

The Swiss mathematician and physicist Leonhard Euler not only made important contributions to the subjects of geometry, calculus, mechanics, and number theory but also developed methods for solving problems in observational astronomy. A founder of pure mathematics, he also demonstrated useful applications of mathematics in technology and public affairs.

Euler was born in Basel, Switzerland, on April 15, 1707. In 1727 he became, upon the invitation of Catherine I of Russia, an associate of the Academy of Sciences at St. Petersburg. In 1730 he became professor of physics. The author of innumerable papers, Euler overtaxed himself and in 1735 lost the sight of one eye. Invited by Frederick the Great, Euler in 1741 became a member of the Academy of Sciences at Berlin, where for 25 years he poured forth a steady stream of publications. When Frederick became less cordial, Euler in 1766 returned to Russia. Soon after his arrival in St. Petersburg, a cataract formed in the other eye, and Euler spent the last years of his life in total blindness. Despite this and other misfortunes, his productivity continued undiminished, sustained by an uncommon memory and a remarkable ability to compute mentally.

The mathematician J. L. Lagrange, rather than Euler, is often regarded as the greatest mathematician of the 18th century. But Euler never has been excelled either in productivity or in the skillful and imaginative use of computational devices for solving problems.

Task 3. Make sure you understand the terms listed below. Try to explain their meaning. To demonstrate you know the words use them in the sentences of your own:
number theory; analytic geometry; to graph an equation; curve; coordinate; taxonomy of curves; conic sections; calculus; axiomatic proof; locus problem; resultant loci; dynamic geometry software.

Task 4. In the text you will find the words designating devices used for geometric constructions. They are: straight-edge; compass; linkage (a device made of hinged rigid rods, that can add, subtract, multiply, divide and generate integer powers).

Those who are well schooled in mathematics are well aware of the fact that classical geometric construction problems were expected to be solved exclusively with a straight-edge and compass. Descartes analysed the curves that could be drawn with a “linkage”.

Do you have any idea of what kind of a device a linkage is? How does it add, subtract, multiply, divide and generate integer powers?

Discuss this in the class.

Task 5. Now read the text below. What are some of the main ideas?

RENÉ DESCARTES' CURVE-DRAWING DEVICES: Experiments in the Relations Between Mechanical Motion and Symbolic Language

By the beginning of the seventeenth century it had become possible to represent a wide variety of arithmetic concepts and relationships in the newly develoved language of symbolic algebra. Geometry, however, held a preeminent position as an older and far more trusted form of mathematics. Throughout the scientific revolution geometry continued to be thought of as the primary and most reliable form of mathematics, but a continuing series of investigations took place that examined the extent to which algebra and geometry might be compatible. These experiments in compatibility were quite opposite from most of the ancient classics. Euclid, for example, describes in Book 8–10 of the *Elements* a number of important theorems of number theory cloaked awkwardly in a geometrical representation. The experiments of the seventeenth century, conversely, probed the possibilities of representing geometrical concepts and constructions in the language of symbolic algebra. To what extent could it be done? Would contradictions emerge if one moved freely back and forth between geometric and algebraic representations?

The profound impact of Descartes' mathematics was rooted in the bold and fluid ways in which he shifted between geometrical and algebraic forms of representation, demonstrating the compatibility of these seemingly separate forms of expression. Descartes is touted to students today as the originator of analytic geometry, but nowhere in the *Geometry* did he ever graph an equation. Curves were constructed from geometrical actions, many of which were pictured as mechanical apparatuses. After curves had been drawn Descartes introduced coordinates and then analyzed the curve-drawing actions in order to arrive at an equation that represented the curve. Equations did not create curves; curves gave rise to equations. Descartes used equations to create a taxonomy of curves.

It can be difficult for a person well schooled in modern mathematics to enter into and appreciate the philosophical and linguistic issues involved in seventeenth century mathematics and science. We have all been thoroughly trained in algebra and calculus and have come to rely on this language and grammar as a dominant form of mathematical representation. We inherently trust that these symbolic manipulations will give results that are compatible with geometry; a trust that did no fully emerge in mathematics until the early works of Euler more than a century after Descartes. Such trust became possible because of an extensive set of representational experiments conducted throughout the seventeenth century which tested the ability of symbolic algebraic language to represent geometry faithfully. Descartes' *Geometry* is one of the earliest and most notable of these linguistic experiments. Because of our cultural trust in the reliability of symbolic languages applied to geometry, many of those schooled in mathematics today have learned comparatively little about geometry in its own right.

Descartes wrote for an audience with opposite predispositions. He assumed that his readers were thoroughly acquainted with geometry, in particular the works of Apollonius (ca. 200 BC) on conic sections. In order to appreciate the accomplishments of Descates one must be able to check back and forth between representations and see that the result of symbolic algebraic manipulations are consistent with independently established geometrical results. The seventeenth century witnessed an increasingly subtle and persuasive series of such linguistic experiments in the works of Roberval, Cavalieri, Pascal, Wallis, and Newton. These led eventually to Leibniz's creation of a general symbolic language capable of fully representing all known geometry of his day, that being his "calculus".

Because many of the most simple and beautiful results of Apollonius are scarcely known to modern mathematicians, it can be difficult to recreate one essential element of the linguistic achievements of Descartes — checking algebraic manipulations against independently established geometrical results.

I was recently discussing my work on curve-drawing devices and their possible educational implications with a friend. His initial reaction was surprise: "Surely you don't advocate the revival of geometrical methods; progress in mathematics has been made only to the extent to which geometry has been eliminated." This claim has historical validity, especially since the eighteenth century, but my response was that such progress was possible only after mathematicians had achieved a basic faith in the ability of algebraic language to represent and model geometry accurately. I argued that one cannot appreciate the profundity of calculus unless one is aware of the issue of coordination of independent representations. Many students seem to learn and even master the manipulations of calculus without ever having questioned or tested the language's ability to model geometry precisely. Even Leibniz, no lover of geometry, would feel that such a student had missed the main point of his symbolic achievement.

Descartes' curve-drawing devices poignantly raise the issue of technology and its relation to mathematical investigation. During the seventeenth century there was a distinct turning away from the classical Greek orientation that had been popular during the Renaissance in favor of pragmatic and stoic Roman philosophy. During much of the seventeenth century a class in "Geometry" would concern itself mainly with the design of fortifications, siege engines, canals, water systems, and hoisting devices — what we could call civil and mechanical engineering. Descartes' *Geometry* was not about static constructions and axiomatic proofs, but concerned itself instead with mechanical motions and their possible representations by algebraic equations. Classical problems were addressed, but they were all transformed into locus problems, through the use of a wide variety of motions and devices that went far beyond the classical restriction to straight-edge and compass. Descartes sought to build a geometry that included all curves whose construction he considered "clear and distinct." An examination of his work shows that what he meant by this was any curve that could be drawn with a "linkage," i.e., a device made of hinged rigid rods. Descartes' work indicates that he was well aware that this class of curves is exactly the class of all algebraic curves, although he gave no formal proof of this. This theorem is scarcely known among modern mathematicians, although it can be proved staightforwardly by looking at linkages that add, subtract, multiply, divide, and generate integer powers. Descartes' linkage for generating any integer powers was used repeatedly in the *Geometry* and has many interesting possibilities.

This transformation of geometry from classical constructions to problems involving motions and their resultant loci has once again raised itself in light of modern computer technology, specifically the advent of dynamic geometry software such as *Cabri* and *Geometer's Sketchpad*. Many new educational and research possibilities have emerged recently in response to these technological developments. It seems, indeed, that seventeenth century mechanical geometry may yet rise from the ashes of history and regain a new electronic life in our mathematics classrooms.

David Dennis

Vocabulary

Task 6. Below are some words taken from the text. Try to guess their meaning from the context in which they are found. In each case choose one of three answers which you think best expresses the meaning.

preeminent — (line 4)

(a) a position of great power and authority;
(b) points to a small degree of value, usefulness or importance;
(c) refers to what is new.

compatible — (line 9)

(a) means extreme in degree, power, fury or the like;
(b) capacity for exising or coming together without disagreement, discord, disharmony, absence of conflict between two things;

(c) quick in the mental processes especially in comprehending a situation.

awkward(ly) — (line 12)

(a) refers to the things extending without end in space, time, or number;
(b) having defects that stem from misproportion or lack of grace;
(c) impressively large or heavy.

converse(ly) — (line 14)

(a) being familiar with something;
(b) that which is opposite of another thing;
(c) applied to acts being free from excitement.

profound — (line 19)

(a) familiar through frequent or regular repetition;
(b) applies to actions, thoughts which are unwilled and unconscious;
(c) exceedingly great depth.

poignant(ly) — (line 83)

(a) applies to that which is motionless or at rest;
(b) implies freedom from relations or dependence on anything else;
(c) emphasizes the presence of any sort of keen, sharp feelings.

Task 7. Read and translate the sentences given below. Each of them has at least one word, or its derivative, or word combination from the text you have just read. Find the sentences with the italicized words in the text. Tell if their meanings in both sentences coincide or differ.

1) *Science*, itself, while comprised of enumerable specialities, also grows increasingly interdependent, and the trends within science are likely to be *linked* to the social context within which it works.

2) New *developments* in science *create* new problems and are also affected by, and have an *impact* on man's social environment.

3) Our *culture*, economy, and society have already been modified in dramatic ways by these *impacts*, and the *coming* years will see vastly greater impacts.

4) Not only is microfilm much more compact than paper as a means for storing *graphical* information, it is also much easier to *move* fast and to locate accurately for scanning.

5) In some cases the clouds appear to have been ejected from old *evolved* stars; in other cases the clouds seem to *represent* protostars, cool masses of dust and gas in the earliest stages of stella *evolution*.

6) The process of calculation is believed to *involve* programming, input of information, storage of information, *arithmetic* operations, and output.

7) The activities *involved* in various space programmes are believed to have created a *revolution* in scientific *research* and *technological developments*.

8) In the last 50 years Chapman and others of his generation have promoted world-wide surveys of the field, have given mathematical *representations* of the magnetic data, and have *developed* theories accounting for *validations* in geomagnetic field.

9) *Extension* of the present approach to certain types of model is also possible.

10) Only a few of the major contributors who introduced new *concepts* have been mentioned here.

11) This gave rise to the *concept* of free rotation.

12) The criterion that the law should be *tested* by application *far beyond* the *original* data is therefore satisfied.

13) These data are *compatible* with the predictions.

14) The discovery of the satellites of Jupiter was an extremely important scientific event but its *cultural impact* was even greater.

15) Weyl's geometry would merely interpret this as evidence that the two *rods* had travelled by different routes.

16) What was striking about the hypothesis was its *implication.*

17) If the hypothesis proved to be *in opposition* to a single firmly established law then of course it must be abandoned.

18) The alternative hypothesis had been shown to be *incompatible* with the observed phenomenon.

19) Some aspects of these ideas were *originated* by Hawkins.

20) Thus the idea has *arisen* that expression and content are *coordinate* equal entities in every respect.

21) Dumas *extended* the ideas *inherent* in the substitution of one element or radical by another into a theory of types.

22) One *concern* is with the idea of progress as applied to the historical process.

23) The 2000-year-old art of storing and retrieving information in archives is still done by specialists with college *training.*

24) A new method of greater *power* and flexibility, based on a computer, was tried.

25) Answering these questions *raises issues* that lie at the root of much of present day astrophysics.

26) Study of the speech process is interesting *in its own right.*

27) The *assumption* that electrons are vibrating around positions of stable equilibrium in the atom offered a simple *picture* of the *origin* of spectral lines.

28) It was not until after 1890 that discriminating *taxonomic* work began.

29) In the last 20 years general relativity theory has *developed* dramatically in two *opposite* directions.

Task 8. Come back to the text and find the sentences fragments of which are given below. Change these sentences filling them with the ideas of your own, leaving the fragments untouched.

1) By the beginning of the … it had become possible to … .
2) … held a preeminent position as … .
3) Throughout … continued to be thought of as … .
4) A continuing series of investigation took place that … .
5) … were quite opposite from … .
6) … probed the possibilities of … .
7) The profound impact of … was rooted in … .
8) … demonstrated the … of these seemingly separate … .
9) … analyzed … in order to arrive at … .
10) It can be difficult for a person … to enter into and appreciate … .
11) … have come to rely on … .
12) We inherently trust that … will give results that are … .
13) … is one of the earliest and most notable of … .
14) Because … are scarcely known to … it can be difficult to … .
15) … advocate the revival of … .

16) One cannot appreciate the profundity of ... unless one is aware of the issue of ... .

17) ... poignantly raise the issue of ... .

18) During ... there was a distinct turning away from ... in favor of ... .

19) ... theory is scarcely known among ... .

20) ... has raised itself in light of modern ... .

21) Many new ... possibilities have emerged recently in response to ... .

Grammar notes and grammar tasks

Past Participle forms are rather frequently mistaken for *Simple Past* forms of verbs in texts owing to the identity in spelling (work — worked — worked; hold — held — held). The only possibility to diverge them is to consider their function in sentences, Past Participle being attribute while Simple Past being predicate. The correct function identification helps better understanding of texts.

Task 9. Look at the list given below. Rearrange the words taken form the text into two distinct groups: the first — embracing all the Predicates and the second — collecting all the Attributes. The number of the line in the text is indicated.
Held (4), trusted (5), continued (6), examined (8), cloaked (12), probed (14), moved (17), shifted (20), introduced (27), analyzed (27), represented (29), used (30), schooled (32), involved (34), conducted (42), tested (43), applied (46), established (55), led (58), argued (75), concerned (93), sought (97), included (98), considered (99), meant (100), made (101).

Participle and Participle Construction

Participles used in speech can give more information about nouns (people, things) they are connected with.

E.g. Most of the people *involved in the investigation* are my colleagues from the department of geometry.
People *investigating* this problem are my colleagues from the department of geometry.

Participles can also be used to say more about the action of the verb or about the idea expressed by a sentence as a whole.

E.g. *Involved* in the investigation I had a clear idea that most of my colleagues are from the department of geometry.
Conducting the investigation with my colleagues I had a clear idea that most of them are from the department of geometry.

Normally the subject of a participle phrase is the same as the subject of the main clause in a sentence. In other cases a participle construction can be given its own subject. The participle construction with its own subject is called *Absolute Participle Construction*.

E.g. A group of geometers was involved in this investigation, *most of them being* from the department of geometry.

The text in this unit gives you an example of Absolute Participle Construction, read it and translate into Russian.

These led eventually to Leibniz's creation of a general symbolic language capable of fully representing all known geometry of his day, that being his "calculus".

(More information about Participles you can find in your grammar book, p. 22.)

Task 10. Translate the following sentences with Absolute Participle Construction into Russian.

1) It is certainly obvious that a system of equations possesses a number of solutions, no solution at all being one of them.

2) Perhaps the formula is too categorical, certain exceptions being allowed.

3) An initial agenda of the conference posed some of key questions but they were not covered with equal attention; some topics having received relatively detailed consideration, some being only touched upon and others not discussed at all.

4) Exact science in its generally accepted sense may be referred to as a family of specialized natural sciences, each of them providing evidence and information about the different aspects by somewhat different working methods.

5) Social scientists and physical scientists, each group representing a diversity of specialized disciplines, were brought together to review some implications of interaction between science and society.

6) One of the main arguments against using computer programmes in planning is, that, the accuracy and validity of forecast data being questionable, there is no sense in trying to refine planning procedure.

7) The trouble with the trend curve being that it may tell you quite accurately what to expect, but does not tell you how it is going to happen.

8) The functional validity of a working hypothesis is not a priori certain, it being based on intuition.

9) There being no alternative, we have to take this implicit concept for granted.

10) One famous question was already raised: that of the "mathematical dream", it having been suggested that the solution of problems may appear in dreams.

11) Our previous errors having been corrected, we now proceed with the evaluation of the results obtained.

12) There are only two methods of communication for scholars, writing and speaking, communication being not merely the desire and the responsibility of the scholar, but his discipline.

Task 11. Look at the following sentence and answer a grammar question.
Descartes is touted to students today as the originator of analytic geometry, but nowhere in the Geometry did he ever graph an equation.

Why do you think the author introduced "did" in the structure of the sentence which is neither interrogative nor negative?

Task 12. Consider the following sentences each of which has *"would"* as its component.

1) *Would* contradictions emerge if one moved freely back and forth between geometric and algebraic representations?

2) Even Leibniz, no lover of geometry, *would* feels that such student had missed the main point of his symbolic achievement.
3) During much of the seventeenth century a class in "Geometry" *would* concern itself mainly with the design of fortifications, siege engines, canals, water systems, and hoisting devices — what we *would* call civil and mechanical engineering.

Tell what is the case when *would* + *verb* is used to show:

1) that the action is considered to be impossible (the time frame is the present);
2) that the action refers to a hypothetical possibility (the time frame is neither present nor future);
3) that the action refers to a hypothetical past possibility;
4) that the action was considered to exist in future;
5) formal request;
6) that the action is a past habit which no longer exists.

Discussion of facts, ideas and concepts

Task 13. Answer the following questions.

1) What position did geometry hold at the beginning of the 17th century?
2) What kind of investigations took place within the frames of geometry and algebra and what did they aim at?
3) How did the experiments in mathematics held in the ancient classics differ from those of the 17th century?
4) What was the profound impact of Descartes' mathematics rooted in and what did Descartes try to demonstrate?

5) Did he ever graph an equation?
6) What were the steps Descartes followed in representing curves algebraically?
7) Why is it difficult for a person well schooled in modern mathematics to enter into and appreciate the linguistic issues involved in 17th century mathematics?
8) How did we come to trust symbolic manipulations?
9) What does the author think one must be able to do to reinforce his trust in symbolic manipulations?
10) When was the progress in mathematics made and under what achievements?
11) What student could miss the main point of Leibniz's symbolic achievements?
12) What issue did Descartes' curve-drawing devices rise poignantly?
13) How did the general orientation change during the seventeenth century?
14) A wide variety of motions and devices used transformed classical problems into locus problems, didn't they?
15) What is "linkage" and what curves can be constructed with it?
16) Where has the finding of equations that model motion always been a fundamental concern?

Task 14. In the text you can find either explicit or implicit indication of time when some events, facts or ideas important for tracing the historic developments in science occur. The time indications given in the list below follow the order they appear in the text. Tell what events, facts, ideas or technologies are associated with this time. Arrange them according to the chronological order: *the beginning of the seventeenth century; throughout the scientific revolution; ancient classics; the seventeenth century; today; modern mathematics; seventeenth century mathematics and science; more than a century after Descartes; throughout the seventeenth century; 200 B. C.; (Leibniz) his day; modern mathematicians; recently; since the eighteenth century; during the seventeenth century; during the Renaissance; during much of the seventeenth century; modern computer technology; advent of dynamic geometry software; recently; seventeenth century mechanical geometry.*

Task 15. Read the following sentences paying special attention to the italicized words.

1) Geometry, however, held a preeminent position as an older and far more *trusted* form of mathematics.

2) Throughout the scientific revolution geometry continued to be thought of as the primary and most *reliable* form of mathematics.

3) We have all been thoroughly trained in algebra and calculus and have come to *rely on* this language and grammar as a dominant form of mathematical representation.

4) We inherently *trust* that these symbolic manipulations will give results that are compatible with geometry; *a trust* that did not fully emerge in mathematics until the early works of Euler.

5) Such *trust* became possible because of an extensive set of representational experiments conducted throughout the seventeenth century which tested the ability of symbolic algebraic language to represent geometry *faithfully.*

6) Because of our cultural *trust* in *reliability* of symbolic languages applied to geometry, many have learned comparatively little about geometry in its own right.

7) Such progress was possible only after mathematicians had achieved a basic *faith* in the ability of algebraic language to represent and model geometry accurately.

As you have noticed all the italicized words have a likeness in denotation and can be identified in the context of this article as synonyms that is having the same or very nearly the same *essential* meaning. They can be assembled in one group with common denotation which in case of verbs “rely' and “trust” means “to have or place full confidence”; and in case of nouns “trust”, “faith”, “reliance” denotes “the feelings that a person or thing will not fail in loyalty, duty or service.” Besides being synonyms the words coming into this group are distinguished from one another by an added implication or application so they may be, and usually are interchangable within limits. Now read the discriminating articles for the words earlier grouped as synonyms.

Verbs: rely, trust

One **relies** on or upon someone or something that one believes he will never fail in giving or doing what one wishes or expects. **Rely** usually connotes a judgement based on previous experience and, in case of persons, actual association; as, *he **relies** on his father to help him out of any trouble he gets into; he never **relies** on the opinions of others.*

One **trusts** (often followed by “in” or by “to”) when one is completely assured or wholly confident that another (often the Supreme Being) will not fail one in need. **Trust** stresses unquestioning faith, though it does not rule out experience as an aid to faith.

Nouns: trust, faith, confidence, reliance

Trust and **faith** suggest the greatest degree of conviction. **Trust** indicates a feeling of certainty that someone or something will not fail in any situaton. The word emphasizes the feeling of certainty whether it is justified or misguided. **Faith** is an intensification of **trust**, suggesting an even deeper conviction of fidelity and integrity, often in spite of no evidence whatever or even in the face of contrary evidence. The word emphasizes such a deep-seated conviction that it is appropriate in a religious context to refer to belief that is based on steadfast loyalty rather than on demonstrable evidence.

Confidence and **reliance** more often suggest **trust** based on the proven reliability of someone or something. One can intuitively **trust** someone at first glance, rightly or wrongly, but **confidence** suggests a conviction born of time-tested familiarity. **Reliance** is even more specific than **confidence**, pointing to an actual dependence on something else, whether out of choice or necessity.

Consider the sentences given at the beginning of this task in the context of the article and try to guess why the author made this or that choice among the words with seemingly equal meaning.

Task 16. In Task 15 we gave the citations from the text where the author presents his viewpoint on the position of geometry and algebra within the frames of mathematics. The terms he used to describe the attitudes toward 17th century geometry and present day algebra seem to have a likeness in denotation. The difference is a time factor.

Answer the questions.

1) Why do you think geometry held an exclusive position in the 17th century?

2) Why did mathematicians begin to trust algebra and calculus a century after Descartes?

3) What is the role of Descartes in the shift of attitudes to symbolic manipulations?

Task 17. As it is clear from the text the 17th century was a crossroad where geometry and algebra met and exchanged the achievements and knowledge accumulated by each of them. By knowledge here we understand a body of facts gathered by study, investigation, observation or the like and also a body of ideas acquired by inference from such facts or accepted on good grounds as truths.

Write a brief essay on how the possibility of representing geometrical concepts and constructions in the language of symbolic algebra impacted the further developments in mathematical science and present it in the class. If you prefer to speak on any other topic dealing with either geometry or algebra, you are welcome. The only restriction is to use as many nouns, verbs and word combinations from the text of this unit as possible.

Verbs: *represent, examine, describe, probe, emerge, root, demonstrate, introduce, create, rise, appreciate, test, assume, check, establish, discuss, advocate, achieve, transform, seek, consider, indicate, involve.*

Nouns: *concept, relationship, investigation, experiment, theorem, contradiction, impact, accomplishment, achievement, problem, proof, transformation, advent.*

Combinations: *to hold a position, a form of mathematics, to be opposite from, to give results, a set of experiments, to conduct an experiment, to be acquainted with, to be consistent with, series of experiments, scarcely known results, an essential element, to make progress, to model accurately (precisely), to raise the issue, turning away from (in favor of), to concern itself with, to address a problem, to be well aware of, in light of, in response to, a fundamental concern.*

Task 18. Read, translate and discuss the grammar areas in the italicized sentences.

THOMAS HARRIOT: Father of English Algebra?

Thomas Harriot, a man of remarkable and original genius has good claim to be considered Britain's greatest mathematical scientist before Newton.

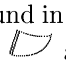
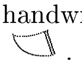
Records concerning Thomas Harriot's early life are exiguous. The Oxford University Register suggests that he was born in Oxford in 1560 and graduated from the University in 1580. He subsequently entered the service of Sir Walter Ralegh as a mathematical tutor, giving lessons in navigation to him and his sea captains. Ralegh sent Harriot out to the newly founded North Carolina colony of "Virginia" as a surveyor with Sir Richard Grenville's expedition of 1585. Harriot returned to England at the end of the following year. He published at London in 1558 *A briefe and true report of the new found land of Virginia, an early description of the colony that included a survey of the merchantable commodities to be found there as well as a warm and sympathetic account of the religion and customs of the native Indians.* Harriot had learnt Indians' language, and moreover, contrived "an Alphabet ... for the American language." *A briefe and true report* was well received. *It turned out to be the only work published by Harriot in his lifetime.*

It was Ralegh who introduced Harriot to Henry Percy, ninth earl of Northumberland, "the Wizard Earl," and when in about 1598 Harriot left Ralegh's patronage Northumberland gave him an annual pension and living quarters in Syon House, Isleworth. Harriot and his mathematical friends, Walter Warner and Thomas Hughes, who became known as Northumberland's "three magi," would often visit him during his imprisonment in the tower after the Gunpowder Plot of 1605. Harriot continued his studies and observations at Syon House until his death, on 2 July 1621, from a cancerous tumour in the nose.

Thomas Harriot possessed a deep and extensive knowledge of the exact sciences of his day. *In astronomy his lunar observations in 1609, with the newly invented telescope, predate those of Galileo;* he, too, observed the moons of Jupiter and calculated their orbits and periods; and he was a diligent observer of sunspots and comets. In optics he formulated the sine law of refraction in about 1601, some 20 years before Willebrord Snell, who is usually credited with the discovery, and used this law to derive a mathematical description of the rainbow. In mechanics he correctly deduced, even before Galileo had begun to study the dynamics of unresisted free fall, that the path of a projectile moving under gravity with a resistance proportional to its speed is a tilted parabola. In navigation he solved original theoretical problems, demonstrating for example that the stereographic projection is angle-preserving, and applied the results to the practical concern of constructing accurate navigational tables. *These are but a few of his achievements, and yet in spite of his technical ability and expertise Harriot was in his own view and that of his contemporaries primarily a mathematician: "As to Harriot, he was so learned, saith Dr. Pell, that had* *he published all he knew in algebra, he would have left little of the chief mysteries of that art unhandled." The tragedy is that Harriot published nothing of his algebra himself, and it is this neglected area of his work that we shall focus on here.*

Histories of mathematics refer to the work associated with his name, the *Artis analyticae praxis* (1631), chiefly in connection with his supposed invention of the inequality signs $<$ and $>$; *in so doing they unwittingly connive in a picture that not only is false, but fails to credit him with his real achievements.* For Harriot, in effect, brought the latest developments in Continental algebra to England, particularly through his familiarity with Vietè's work, which he then developed upon independently and significantly.

We now know that Harriot was not directly responsible for the *Praxis*, which was put together after his death from papers which are no longer extant by Walter Warner (and, perhaps, one or two others), when Nathaniel Torporley had failed to complete the task which had been assigned to him in Harriot's will. *Torporley was a respected mathematician of the day, reputed to have been associated with Viète himself.*

The manuscripts that we do have (in the British library and Petworth) cannot have been the origin of the Praxis, not only on account of their disorder and incoherence, but also because there are significant differences between them and the published work. Notably, the inequality signs associated with his name are never found in his handwriting in the manuscripts but appear throughout as  and . Similarly, equality is denoted in the manuscripts by ıı and not by = (the sign introduced by Robert Recorde), as in the *Praxis*. *The significance of the inequality signs lies in the fact that this is the first time that such signs were used and accorded the same status as the equality sign.*

There are similarities, however, as well as differences between the manuscripts and the book. *Theory of equations and numerical solutions are found in both, and some of the content and layout of the Praxis echoes that of the manuscripts*, although its errors and inconsistencies attest to the mathematical shortcomings of its editor, Warner.

Harriot's notation is extremely workmanlike and, apart from the lack of the exponent, perfectly suited to algebraic manipulation. Although he follows Viète in using vowels for unknowns and consonants for knowns (both lower case), it is free of words and their abbreviations and is fully symbolic. He writes in the manuscripts, using $\lrcorner$ for multiplication:

$$\left.\begin{array}{r} b-a \\ c-a \\ df+aa \end{array}\right| \shortparallel$$

$$\begin{array}{l} bcdf \,-\, bdfa \,-\, dfaa \,-\, baaa \\ \qquad -\, cdfa \,+\, bcaa \,-\, caaa \,+\, aaaa \\ \qquad\qquad\qquad\qquad\qquad\qquad \shortparallel\ 0000 \end{array}$$

Ergo:

$$\begin{array}{ll} bcdf \shortparallel & a \shortparallel b \\ +bdfa - bcaa + baaa & a \shortparallel c \\ +cdfa - dfaa + caaa & aa \shortparallel -df \\ -aaaa & a \shortparallel \sqrt{-df} \end{array}$$

Note the homogeneity, even extending to zero, which is treated as a calculable entity. This last feature is remarkable for his time. He is not afraid of equating all terms to zero and even, elsewhere, to a negative quantity. But most importantly in the above example, note the inclusion of an imaginary root without comment.

Harriot's advances in the theory of equations include a method of "solution" by a sort of reverse factorization, shown briefly in the same example above. The *Praxis* does likewise, but in a more explicatory way.

The well-known inequalities between arithmetic and geometric means (and their generalizations in powers) appear in both the manuscripts and the *Praxis*, and form the basis in the latter of an ingenious attempt at determining the number of (positive) roots in an equation by comparing it with a canonical equation, the number of whose (positive) roots is known, by means of a device closely resembling a discriminant. The method works for positive roots; apart from two examples, *only positive roots are considered in the Praxis, which may well have been intended for students.* The final theoretical part of the *Praxis* is devoted to removing one term in an equation and changing the root; in this, Viète's *De aequationum recognitione et emendatione* is followed pretty closely, albeit with Harriot's highly superior notation.

It is, however, with respect to negative and complex roots that the most significant differences occur between the Praxis and the manuscripts. Although the Praxis states that equations involving negative roots are to be disregarded as they are useless ("tamquam inutiles negliguntur") *and those involving the square-root of a negative quantity are referred to as* "impossible" *because they are "inexplicable" (meaning unsolvable in this context), a different picture emerges in the manuscripts.* Here, negative roots are almost always given, without comment. As for complex roots, called by Harriot "noetic," a folio exists in which a biquadratic is fully worked through and solved in terms of one positive, one negative, and two complex roots, whose values are written

$$5 \shortparallel a, -7 \shortparallel a, a \shortparallel +1 + \sqrt{-32}, a \shortparallel +1 - \sqrt{-32}.$$

Elsewhere in the manuscripts, a number of imaginary solutions are given, although not in all cases in which such solutions exist. *The most likely explanation is that the lists of equations in the manuscripts in which solutions are not derived but are merely stated, and in which complex roots are not given, are simply examples for students to work through, and the complex roots are omitted for pedagogical reasons.* There are, in addition, at least three cubic equations for which only two real roots are given; and it turns out that of the three roots of these equations, two are coincident. However, coincident roots are found elsewhere.

The second part of the Praxis consists of the numerical solution of equations up to the fifth degree, and the method used, as in the examples given in the manuscripts, differs little from that of Viète (De numerosa potestatum recognitione). Harriot's notation is particularly useful in this field. *For, instead of Viète's use of C, Q, and N (standing for the cube of the unknown, its square, and the unknown itself), together with verbal explanations in Latin which appear to us to be very convoluted, Harriot's presentation is crystal* *clear.* It is designed for easy manipulation and very largely speaks for itself. *The impact of this must have been considerable, supplying the basis for further advances.*

Harriot's contribution to algebra, therefore, is considerably greater than that with which he is normally credited. He brought Britain into the mainstream of Continental developments through his wide-ranging theoretical and practical concerns and the accessibility of his methodology and notation. He himself was in the mainstream, as his references to Stevin, Bombelli, Stifel, and Viète in the manuscripts testify. *His achievements later prompted John Wallis to comment that Harriot had laid the foundation "without which the whole superstructure of Descartes had never been" — a remark he later toned down.*

Apart from algebra, other branches of pure mathematics are to be found in the enormous volume of Harriot manuscripts. Examples are worked in binary notation; there is work on Pythagorean triples; meridional parts are calculated with a treatment closely resembling calculus; and there is the quadrature of the equiangular spiral.

Thomas Harriot died on 2 July 1621 in the house of his friend Thomas Buckner in Threadneedle Street. He was buried in the nearby churchyard of St. Christopher's parish church, and a memorial plaque was placed in the chancel of the church. Although St. Christopher's was completely destroyed in the Great Fire in 1666, the wording of Harriot's memorial was, quite fortuitously, preserved in the 1633 edition of John Stowe's *The survey of London*. The Bank of England was founded in 1694 but did not occupy its present site near the former St. Christopher's church in Threadneedle Street until 1734. Three hundred and fifty years after Harriot's death a bronze plaque bearing the original epitaph was installed in a niche in the entrance hall of the bank.

A yet more poignant epitaph might be furnished by Harriot's friend William Lower's urgings to him to publish his discoveries. On 6 February 1610 he wrote to Harriot:

> Doe you not here startle, to see every day some of your inventions taken from you; for I remember longe

> since you told me as much [as Kepler has just published] that the motions of the planets were not perfect circles. So you taught me the curious way to observe weight in Water, and within a while after Ghetaldi comes out with it, in print. A little before Vieta prevented you of the Gharland for the greate invention of Algebra al these were your deues and manie others that I could mention; and yet too great reservednesse hath rob'd you of these glories.

Nevertheless, more than 250 years later, on 5 September 1883, J. J. Sylvester was to write to Arthur Cayley a letter in which he acknowledged Harriot to be "the man who first introduced the Algebraical Zero into Analysis, the father of current Algebra."

MURIEL SELTMAN AND EDDIE MIZZI

Unit 7

In this unit you are supposed to read three fragments from the article *MATHEMATICS FORTY YEARS AFTER SPUTNIK* written by S. W. Golomb in the journal *American Scholar*. The author presents his view on changes in mathematicians' attitudes and emphasis on "pure" mathematics, the historical records of mathematicians' interest in applications for their ideas. He also shows that many current fields depend on applying sophisticated mathematics.

Reading

Task 1. Discuss in the class the following.

1. Why do you think people become mathematicians?
2. What are the motivations for educated, bright, talented young people to become involved in mathematics?
3. What factors are important in deciding what particular discipline within the frames of your future profession to prefer?
4. Say what matters to you and what matters to most people in your opinion.

Task 2. Consider a list of key words taken from the text and try to guess what ideas and concepts are stated in it.

Mathematics department, practical application, good mathematics, pure mathematics, possible applications of pure mathematics, "real" mathematics, "real" mathematicians, "useless" mathematics, trivial mathematics, useful mathematics, real creative work, expository works in and about mathematics, relativity and quantum mechanics, theory of numbers, stellar astronomy, atomic physics, practical importance, to have an effect on war, warlike purpose, pacifism, harmlessness of mathematics, chemistry, poison gases, "useful" science, engineering, designing war planes, service of war, twenty three outstanding unsolved problems, conflict between pure and applied mathematics, unapplicability of pure mathematics, aura of non-respectability.

We hope the following names mentioned in the article will give you a prompt as to what ideas and concepts are stated in the text. Read them.

G. H. Hardy, Fermat, Euler, Gauss, Riemann, J. E. Littlewood, S. Ramanujan, Maxwell, Einstein, Eddington, Dirac, Hitler, Stanley Baldwin, Neville Chamberlain, David Hilbert, Felix Klein, E. T. Bell, Archimedes, Newton, Lagrange, Laplace, Fourier, Herman Weyl, Norbert Wiener, John von Neumann.

Are these names familiar to you? What do you know about them? Share everything you know about these persons with your classmates.

You are unlikely to know much about Stanley Baldwin and Neville Chamberlain. Below you will find two entries for them taken from *The Hutchinson Pocket Encyclopedia.*

Baldwin Stanley, 1st Earl Baldwin of Bewdley (1867–1947). British Conservative politician, prime minister (1923–24, 1924–29, 1935–37); he weathered the general strike (1926), secured complete adult suffrage (1928), and handled the abdication of Edward VIII (1936), but failed to prepare Britain for World War II.

Chamberlain (Arthur) Neville (1869–1940). British Conservative politician, son of Joseph Chamberlain. He was prime minister (1937–40). Trying to close the old Anglo-Irish feud, he agreed to return to Eire those ports that had been occupied by the navy. He also attempted to appease the demands of the European dictators, particularly Mussolini. In 1938 he went to Munich and negotiated with Hitler the settlement of the Czechoslovak question. He was ecstatically received on his return and claimed that the Munich Agreement brought "peace in our time." Within a year, however, Britain was at war with Germany. He resigned in 1940 following the defeat of British forces in Norway.

Task 3. Read the text quickly.

MATHEMATICS FORTY YEARS AFTER SPUTNIK (extract 1)

When I was a graduate student at Harvard in the early 1950s, the question of whether anything that was taught or studied in the mathematics department had any practical applications could not even be asked, let alone discussed. This was not unique to Harvard. Good mathematics had to be pure mathematics, and by definition it was not permissible to talk about possible applications of pure mathematics.

This view was not invented by G. H. Hardy, the great British number theorist, but he was certainly one of its most eloquent and influential exponents. In *A Mathematician's Apology*, published in 1940, Hardy wrote, "Very little of mathematics is useful practically, and ... that little is comparatively dull"; and "The 'real' mathematics of the real mathematicians, the mathematics of Fermat and Euler and Gauss and Riemann, is almost wholly 'useless'"; and "We have concluded that the trivial mathematics is, on the whole, useful, and that the real mathematics, on the whole, is not."

To other number theorists, Hardy is best known for his collaborative work with J. E. Littlewood and with the Indian mathematical prodigy S. Ramanujan. He was well known for claiming that mathematicians did their real creative work by age thirty-five, and he dedicated his own later years to expository works in and about mathematics. The *Apology* was published when Hardy was sixty-three, seven years before his death.

In order to force external reality into the *Apology*'s rhetorical model, Hardy decided to include leading theoretical physicists in his canon of "real" mathematicians, but to justify this by saying that their work had no real utility anyway. Thus, he wrote, "I count Maxwell and Einstein, Eddington and Dirac, among 'real' mathematicians. The great modern achievements of applied mathematics have been in relativity and quantum mechanics, and these subjects are, at present at any rate, almost as 'useless' as the theory of numbers." He also asserted: "Only stellar astronomy and atomic physics deal with 'large' numbers, and they have very little more practical importance, as yet, than the most abstract pure mathematics." And, in a statement that must have had particular resonance in 1940: "There is one comforting conclusion which is easy for a real mathematician. Real mathematics has no effects on war. No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years." Today, fifty years after Hardy's death, it seems incredible that a book so at odds with reality was so influential for so long.

It is ironic that Hardy's *Apology* was in fact not directed toward mathematicians at all. After the dreadful carnage at World War I, pacifism was widespread in England and effectively the established religion at Oxbridge between the wars. The extreme attempts to avoid antagonizing Hitler made by Stanley Baldwin and Neville Chamberlain, who between them occupied 10 Downing Street from 1935 to 1940, can only be understood in this context. It was primarily to the non-scientists at Oxford and Cambridge that Hardy wanted to proclaim the harmlessness of mathematics. Hardy indicated that the *Apology* was an elaboration of his inaugural lecture at Oxford, which he had delivered in 1920, when antiwar sentiment would have been particularly vehement, and that he was reasserting his position that "mathematics [is] harmless, in the sense in which, for example, chemistry plainly is not."

Chemistry, responsible for the poison gases and disfiguring explosives of the Great War, was Hardy's chief example of a "useful" science, closely followed by engineering, which is used for helpful things like building bridges, but also for destructive things like designing war planes and other munitions. Hardy was anxious to persuade his readership that "real" mathematics (especially the kind he did himself) is a noble aesthetic endeavor, akin to poetry, painting, and music, and has nothing in common with merely "useful" subjects like chemistry and engineering, which can be employed in the service of war. Barely two years later, after the blitz bombing of London, Hardy's pacifist audience in England would have almost completely disappeared; but as a mathematician's manifesto, his *Apology* carried a good deal of weight in mathematical circles for decades.

David Hilbert of Germany, who is regarded by many as the leading mathematician of the first four decades of the twentieth century, shared and advocated the view advanced in Hardy's *Apology.* Hilbert largely defined the agenda for twentieth-century mathematics with his famous list of twenty-three outstanding unsolved problems, presented at the International Congress of Mathematicians in Paris in 1900. The most clearly "applied" problem on Hilbert's list was the one that asked for a proper, rigorous mathematical formulation of the laws of physics. Coming just ahead of the discovery of relativity and quantum mechanics, this problem led to interesting mathematical work in directions Hilbert could not have anticipated, but in which he actually participated.

Another famous professor at Gottingen during the Hilbert epoch was Felix Klein, who had a much broader appreciation of applications. A reporter once asked Klein if it was true that there was a conflict between pure and applied mathematics. Klein replied that it was wrong to think of it as a conflict, that it was really a complementarity. Each contributed to the other. The reporter then visited Hilbert and told him, "Klein says there's no conflict between pure and applied mathematics."

"Yes," said Hilbert, "of course he's right. How could there possibly be a conflict? The two have absolutely nothing in common."

Through the ages, the very greatest mathematicians have always been interested in applications. That was certainly true of E. T. Bell's "three greatest mathematicians of all times": Archimedes, Newton, and Gauss. It was equally true of Euler, Lagrange, Laplace, and Fourier. Even in the first half of the twentieth century, it was true of Hermann Weyl, Norbert Wiener and John von Neumann. As we come to the end of the twentieth century, the earlier insistence on the desired inapplicability of pure mathematics seems almost quaint, though one lingering legacy is that in certain circles the label "applied mathematics" retains a pejorative taint and an aura of non-respectability.

Solomon W. Golomb

Vocabulary

Task 4. Below are some words taken from the text. Try to guess their meaning from the context in which they are found. In each case choose one of the three answers which you think best expresses the meaning.

eloquent — (line 9)

(a) able to express oneself clearly or easily, also suggests the stimulus of powerful emotion;
(b) affected by a desire to sleep;
(c) applies to persons or things that do not arrive or take place at the expected and usual time.

exponent — (line 10)

(a) index number or quality, written to the right of or above another to show how often the latter is to be multiplied by itself;
(b) one, who expounds, demonstrates, or explains;
(c) a symbol.

prodigy — (line 20)

(a) a teacher of the highest rank in a university;
(b) a very gifted person;
(c) the son of a king or emperor.

vehement — (line 55)

(a) having or manifesting the power to reach a conclusion without reasoning;
(b) extreme in degree, pitch, power, fury, or the like;

(c) presenting a fine, fresh or elegant appearance.

endeavor — (line 64)

(a) to try to do something requiring unusual and earnest effort;
(b) denotes either liquid food or food having a liquid base;
(c) the mind's power to call up images, to picture or conceive things that are not actually before the eye or within the experience.

akin — (line 64)

(a) not sufficiently intelligible or clearly understood;
(b) closely resembling each other, when in spite of marked differences things reveal essential rather than superficial likeness;
(c) above the average in height.

pejorative — (line 103)

(a) describes a person who is easily engaged in conversation;
(b) indicates a lack of depth;
(c) *(of a word, phrase, etc.)* expressing disapproval or suggesting that someone or something is of little value or importance.

Task 5. Look at the list of nouns from the article all of which end in *-tion*. Write beside each the verb from which it is derived. Use dictionary if necessary.

question —	munition —
application —	formulation —
definition —	direction —
elaboration —	appreciation —
conclusion —	education —

Here are 10 derivation patterns:

a) *-ute, -ate, -ete* verbs: Drop final '*e*' add *-ion*

b) *-ct, -pt* verbs: Add *-ion*

c) *-duce* verbs: Drop final '*e*' add *-tion*

d) *-ise* verbs: Drop final '*e*' add *-ation*

e) *-de* verbs: Drop final '*de*' add *-sion*

f) *-fy* verbs: Drop final '*y*' add *-ication*

g) *-mit* verbs: Drop final '*t*' add *-ssion*

h) *-ss* verbs: Add *-ion*

i) *-scribe* verbs: Drop final '*be*' add *-ption*

j) *-pose* verbs: Drop final '*e*' add *-ition*

Task 6. Review the nouns in Task 5. Tell what pattern they belong to.

Task 7. Most of the nouns below belong to the above patterns, but some are exceptions. Consider them, noting exceptions and grouping the other nouns. Use your dictionary if necessary.
Destruction, decision, imagination, simplification, omission, minimization, repression, qualification, reduction, information, prescription, contribution, exaggeration, contradiction, specification, exception, opposition, construction, proposition, realization, examination, prediction, supposition, intensification, completion.

Task 8. For each of the following sentences, write another with the same meaning, using the word in brackets and the right noun or verb in the correct form.
E.g. Payment will be made on completion of the work. *(when)*
You will be paid when the work is completed.

Sentences using an abstract noun are *more formal* than their equivalents using a verb-phrase. Try to make your converted sentences *more formal* or *less formal* accordingly, changing other words where necessary.

1. His description of the situation is not adequate. *(he, wrongly)*
2. His education suffers the lack of serious background. *(poorly)*
3. This group of scientists contributed significantly to the research programme. *(made)*
4. Prediction of the future is always difficult. *(it)*

5. This is my proposition. *(what)*
6. Nobody expected him to realize his plan so soon. *(unexpected)*
7. It made him very satisfied to see that all his efforts had been rewarded with success. *(great)*
8. My colleague's admission that he had been wrong amazed everybody. *(when)*

Task 9. Make your own sentences with verbs and abstract nouns derived from them. The sentences from the previous task can serve as examples. Ask your classmates to convert the sentences.
To graduate — graduation; to discuss — discussion; to invent — invention; to publish — publication; to collaborate — collaboration; to dedicate — dedication; to decide — decision; to justify — justification; to occupy — occupation; to indicate — indication; to define — definition; to anticipate — anticipation.

Task 10. Imagine yourself speaking to a person who has read the text as you have, but he or she still has vague understanding of some words and phrases. Try to act like an expert and give sound explanation employing both your erudition and information from the text.
E.g. "a *graduate student* — a student who is completing a course of study working on his diploma or degree."

... had any *practical applications* ...	(line 3);
... was not *unique* to ...	(line 4);
... and *by definition* it was ...	(line 5);
... view was not *invented* by ...	(line 8);
... one of its most *eloquent and influential exponents.*	(line 9);
Very little of mathematics *is useful practically* ...	(line 11);
... the *trivial* mathematics is ...	(line 15);
... for his *collaborative* work with ...	(line 18);
... mathematicians did their *creative work* by age 35 ...	(line 21);

... *expository works* in and about mathematics	(line 22);
... to *force* external reality into ...	(line 25);
... a *rhetorical* model ...	(line 25);
... his *canon* of "real" mathematicians ...	(line 27);
... had no *real* utility ...	(line 28);
... only stellar astronomy and atomic physics *deal with 'large' numbers* ...	(line 33);
... a statement that must have *had particular resonance* ...	(line 36);
... *comforting* conclusion ...	(line 37);
... any *warlike* purpose ...	(line 39);
... a book so *at odds* with reality ...	(line 42);
... pacifism was ... effectively the *established religion* at *Oxbridge* ...	(line 46);
... to avoid *antagonizing* Hitler ...	(line 48);
... occupied *10 Downing Street* ...	(line 49);
... was an *elaboration* of his *inaugural* lecture ...	(line 53);
... he was *reasserting his position* ...	(line 55);
... *helpful things* like building bridges ...	(line 60);
... to persuade his *readership* ...	(line 62);
... a *noble aesthetic endeavor* ...	(line 64);
... mathematics ... *has nothing in common with* merely "useful" subjects ...	(line 65);
... which *can be employed* in the service of war	(line 66);
... *carried a good deal of weight* in mathematical circles ...	(line 70);
... *shared and advocated* the view ...	(line 74);
... asked for *a proper, rigorous mathematical formulation* ...	(line 79);
... it was really a *complementarity* ...	(line 88);
... have absolutely *nothing in common*	(line 93);
... the *desired* inapplicability of pure mathematics ...	(line 101);
... one *lingering legacy* is ...	(line 102);
... retains a *pejorative taint* and *an aura of non-respectability* ...	(line 103).

Task 11. We use passive voice when we don't know, don't care, or want to hide who did something. It is used to add formality. Any passive voice construction is more formal than its active voice

equivalent. (Consult your grammar book, p. 27.) Consider the following sentences from the text and tell why the passive voice constructions are used in them.

1. When I was a graduate student at Harvard in the early 1950s, the question of whether anything that *was taught* or *studied* in the mathematics department had any practical applications *could not even be asked, let alone discussed.*

2. This view *was not invented* by G. H. Hardy.

3. Hardy *is* best *known* for his collaborative work with J. E. Littlewood and S. Ramanujan.

4. He *was* well *known* for claiming that mathematicians did their real creative work by age thirty-five.

5. The Apology *was published* when Hardy was sixty-three.

6. No one has yet discovered any warlike purpose *to be served* by the theory of numbers or relativity.

7. It is ironic that Hardy's Apology *was in fact not directed* toward mathematicians at all.

8. The extreme attempts to avoid antagonizing Hitler *can only be understood* in this context.

9. Chemistry was Hardy's chief example of a "useful" science, closely followed by engineering, which *is used* for helpful things, but also for destructive things.

10. Hardy was anxious to persuade his readership that "real" mathematics had nothing in common with merely "useful" subjects, which *can be employed* in the service of war.

11. David Hilbert, who *is regarded* by many as the leading mathematician of the first four decades of the 20th century shared and advocated the view.

12. Through the ages, the very greatest mathematicians *have* always *been interested* in applications.

Task 12. For each sentence given below there are two passive voice variations. Read them.

1. Many think that Steven Spielberg is one of the most talented directors of the world. It is thought by many that Steven Spielberg is one of the most talented directors of the world. Steven Spielberg is thought by many to be one of the most talented directors of the world.

2. Nowadays, people agree that Galileo was a genius. Nowadays, it is agreed that Galileo was a genius. Nowadays, Galileo is agreed to have been a genius.

3. Brazilians claim that Pele was the greatest footballer of all the time. It is claimed that Pele was the greatest footballer of all the time. Pele is claimed by Brazilians to have been the greatest footballer of all the time.

These constructions are extremely common in reporting what people say, believe, allege, etc. They are often found in news reporting and *discussion of an academic kind.*

The following verbs can be used with either construction: *know, think, say, believe, allege, announce, claim, report, feel, find, suspect, assume, fear, consider, recognize, understand.*

Note: In the infinitive construction, the indefinite infinitive is used between the main verb and the rest of the sentence. The perfect infinitive is used when there is a difference in time.

Task 13. In Task 2 there is a list of all the names appeared in the text. Choose 10 persons you know better and make 10 statements about them using the verbs from the previous task. For each statement there could be two passive voice variations. Write the alternative sentences.

Task 14. In the papers of an academic character authors very often present other scientists' ideas to support some viewpoints or to oppose them. Other people's opinions are introduced in the texts either in the form of quoted or reported speech. (Consult your grammar book, p. 32.)

Below are the sentences from the text. Convert the sentences with quoted speech into reported and vice versa.

1. The question of whether anything that was taught or studied in the mathematics department had any practical applications could not even be asked.

2. Hardy wrote, "Very little of mathematics is useful practically, and ... that little is comparatively dull"; and "The 'real' mathematics of the real mathematicians, the mathematics of Fermat and Euler and Gauss and Riemann, is almost wholly 'useless'"; and "We have concluded that the trivial mathematics is, on the whole, useful, and that the real mathematics, on the whole, is not."

3. He was well known for claiming that mathematicians did their real creative work by age thirty-five.

4. Thus he wrote, "I count Maxwell and Einstein, Eddington and Dirac, among 'real' mathematicians. The great modern achievements of applied mathematics have been in relativity and quantum mechanics, and these subjects are, at present at any rate, almost as 'useless' as the theory of numbers."

5. He also asserted: "Only stellar astronomy and atomic physics deal with 'large' numbers, and they have very little more practical importance, as yet, than the most abstract pure mathematics."

6. And, in 1940 (he also asserted): "There is one comforting conclusion which is easy for a real mathematician. Real mathematics has no effects on war. No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems very unlikely that anyone will do so for many years."

7. Hardy indicated that the Apology was an elaboration of his inaugural lecture of Oxford, which he had delivered in 1920, when antiwar sentiments would have been particularly vehement, and that he was reasserting his position that "mathematics (is) harmless, in the sense in which, for example, chemistry plainly is not."

8. A reporter once asked Klein if it was true that there was a conflict between pure and applied mathematics.

9. Klein replied that it was wrong to think of it as a conflict, that it was really a complementarity.

10. The reporter then visited Hilbert and told him, "Klein says there's no conflict between pure and applied mathematics."

11. "Yes," said Hilbert, "of course he's right. How could there possibly be a conflict? The two have absolutely nothing in common."

Discussion of facts, ideas, and concepts

Task 15. Read the text again and answer the following questions:

1. Why was not it permissible to discuss possible applications of pure mathematics at Harvard in the early 1950s?

2. When was Hardy's *A Mathematician Apology* published?

3. What was Hardy's viewpoint on 'real' mathematics?

4. Why did he dedicate his later years to expository works in and about mathematics?

5. Why did Hardy decide to include leading theoretical physicists in his canon of "real" mathematicians?

6. What is the "one comforting conclusion" for a real mathematician?

7. What seems incredible to the author?
8. What examples of "useful" sciences did Hardy give in his *Apology* and why?
9. How soon and why did Hardy's pacifist audience disappear?
10. Who shared and advocated the view advanced in Hardy's *Apology*?
11. What was the most clearly "applied" problem on Hilbert's list?
12. How did Klein and Hilbert understand a conflict between pure and applied mathematics?
13. What seems almost quaint as we come to the end of the twentieth century?

Task 16. In the first paragraph the writer of the article makes the following statement, "Good mathematics had to be pure mathematics, and by definition it was not permissible to talk about possible applications of pure mathematics"; and "This was not unique to Harvard." The time is the early 1950s. So it comes from the text that there were other reputable learning centres where the situation similar to that of Harvard existed.

What do you know about the attitude towards pure and applied mathematics at your faculty in the middle of the 20th century?

Has the situation changed since then?

Do your lecturers speak in any way on the priorities in research work conducted at your faculty?

What is the proportion of pure and applied mathematics at present at your department?

Task 17. In the third paragraph the author writes, "... mathematicians did their creative work by age thirty-five."

What do you think about limits in age for creative work in mathematics?

Do you agree with Hardy?

In case of either positive or negative answer, try to support your viewpoint speaking about mathematicians whose creative activity demonstrates you are right.

How do you think things are in branches other than mathmematics and why so?

Task 18. In the fifth paragraph the author writes, "Hardy's Apology was in fact not directed toward mathematicians at all."

It is but natural to suggest that the greatest motivation for him to write this paper was to present incontrovertible evidence to public that "real mathematics has no effects on war."

Read one more time the end of the fourth, fifth and sixth paragraphs and on their basis try to make an oral presentation on the thesis: "Motives for writing apology."

The following words and phrases are supposed to help you:
To have effects on war; warlike purpose; dreadful carnage; World War I; pacifism; antagonizing Hitler; antiwar sentiments; poison gases; disfiguring explosives; destructive things; designing war planes; munitions; to employ sth in the service of war.

In developing a discourse designed to present our understanding of the problem any writer should select and organize the information for the audience in a clear and convincing way. The following outline can help you develop both your argument and your context.

1. Introduction
   a) Direct the reader's attention to the subject or problem.
   b) Explain your experience in it.
2. Background
   a) Explain the nature of the problem — its history and causes.
   b) Explain why the problem is important.
3. Thesis and explanation
   a) State the thesis.

   b) Develop it. (Include any information that is necessary for making it clear and acceptable. Cite authoritative statements, facts, statistics, personal experience and experience of others, and so on.)

4. Conclusion

   a) Explain the implications of the information.

   b) Summarize your discussion: the problem (2a), your thesis (3a) and your explanation (3b).

You can also use this draft with certain modifications depending on a number of variables: the nature of the message, its length and complexity, the extent of reader's relevant knowledge and attitudes while producing either oral or written presentation of any kind. By using the draft you increase both the efficiency of the writing process and the effectiveness of the discourse.

In expository discourse (scientific writing) the emphasis is on:

1) clarity of focus (a precisely stated thesis);

2) clarity of structure (a well-defined, well-signaled organizational pattern);

3) clarity and appropriateness of explanation (definition of unshared terms; use of shared knowledge, experience, and values).

The writer's chief task is to connect new information with old, stating and ordering his generalizations and explanations as clearly as possible.

Task 19. In the last paragraph the writer states, " ... in certain circles the label 'applied mathematics' retains a pejorative taint and an aura of non-respectability." Why do you think it is so?

Unit 8

The extract from the article *Mathematics Forty Years After Sputnik* you are going to read in this unit traces the university discipline "mathematics" back to its origin in Hellenic times, and forth to the present day division of the subject into modern branches at mathematics departments: it also considers the place of mathematics in the university curriculum and emphasizes the dependence of many current fields on applying sophisticated mathematics.

Reading

Task 1. You are assumed to find useful and cognitive information on universities taken from *Encyclopaedia Britannica.* It is certain to help you in understanding the main text better. Read and discuss the article in the class.

University — institution of higher education, usually comprising a liberal arts and sciences college and graduate and professional schools and having the authority to confer degrees in various fields of study. A university differs from a college in that it is usually larger, has a broader curriculum, and offers graduate and professional degrees in addition to undergraduate degrees.

The modern university evolved from the medieval schools known as studia generalia (singular, studium generale); they were generally recognized places of study open to students from all parts of

Europe. The earliest studia arose out of efforts to educate clerks and monks beyond the level of the cathedral and monastic schools. The inclusion of scholars from foreign countries constituted the primary difference between the studia and the schools from which they grew.

The earliest Western institution that can be called a university was a famous medical school that arose at Salerno, Italy, in the 9th century and drew students from all over Europe. It remained merely a medical school, however. The first true university was founded at Bologna late in the 11th century. It became a widely respected school of canon and civil law. The first university to arise in northern Europe was the University of Paris, founded between 1150 and 1170. It became noted for its teaching of theology, and it served as a model for other universities in northern Europe such as the University of Oxford in England, which was well established by the end of the 12th century. The universities of Paris and Oxford were composed of colleges, which were actually endowed residence halls for scholars.

These early universities were corporations of students and masters, and they received their charters from popes, emperors, and kings. These universities were free to govern themselves, provided they taught neither atheism nor heresy. Students and masters together elected their own rectors (presidents). At the price of independence, however, universities had to finance themselves. So teachers charged fees, and, to assure themselves of a livelihood, they had to please their students. These early universities had no permanent buildings and little corporate property, and they were subject to the loss of dissatisfied students who could migrate to another city and establish a place of study there. The history of the University of Cambridge began in 1209 when a number of disaffected students moved there from Oxford; and 20 years later Oxford profited by a migration of students from the University of Paris.

From the 13th century on universities were established in many of the principal cities of Europe. Universities were founded at Montpellier (1220) and Aix-en-Provence (1409) in France, at Padua (1222), Rome (1303), and Florence (1321) in Italy, at Salamanca (1218) in Spain, at Prague (1348) and Vienna (1365) in central

Europe, at Heidelberg (1386), Leipzig (1409), Freiburg (1457), and Tübingen (1477) in what is now Germany, at Louvain (1425) in present-day Belgium, and at Saint Andrews (1411) and Glasgow (1451) in Scotland.

Until the end of the 18th century, most universities offered a core curriculum based on the seven liberal arts: grammar, logic, rhetoric, geometry, arithmetic, astronomy, and music. Students then proceeded to study under one of the professional faculties of medicine, law, and theology. Final examinations were grueling, and many students failed.

The Protestant Reformation of the 16th century and the ensuing Counter-Reformation affected the universities of Europe in different ways. In the German states, new Protestant universities were founded and older schools were taken over by Protestants, while many Roman Catholic universities became staunch defenders of the traditional learning associated with the Catholic church. By the 17th century, both Protestant and Catholic universities had become overly devoted to defending correct religious doctrines and hence remained resistant to the new interest in science that had begun to sweep through Europe. The new learning was discouraged, and thus many universities underwent a period of relative decline. New schools continued to be founded during this time, however, including ones at Edinburgh (1583), Leiden (1575), and Strasbourg (university status, 1621).

The first modern university was that of Halle, founded by Lutherans in 1694. This progressive-minded school was one of the first to renounce religious orthodoxy of any kind in favour of rational and objective intellectual inquiry, and it was the first where teachers lectured in German (i.e., a vernacular language) rather than in Latin. Halle's innovations were adopted by the University of Göttingen (founded 1737) a generation later and subsequently by most German and many American universities.

In the later 18th and 19th centuries religion was gradually displaced as the dominant force as European universities became institutions of modern learning and research and were secularized in their curriculum and administration. These trends were typified by the University of Berlin (1809), in which laboratory experimentation replaced conjecture; theological, philosophical, and other traditional doctrines were examined with a new rigour and objectivity; and modern standards of academic freedom were pioneered. The German model of the university as a complex of graduate schools performing advanced research and experimentation proved to have a worldwide influence.

The first universities in the Western Hemisphere were established by the Spaniards: the University of Santo Domingo (1538) in what is now the Dominican Republic and the University of Michoacan (1540) in Mexico. The earliest American institutions of higher learning were the four-year colleges of Harvard (1636), William and Mary (1693), Yale (1701), Princeton (1746), and King's College (1754; now Columbia). Most early American colleges were established by religious denominations, and most eventually evolved into full-fledged universities. One of the oldest universities in Canada is that at Toronto, chartered as King's College in 1827.

As the frontier of the United States moved westward, hundreds of new colleges were founded. American colleges and universities tended to imitate German models, seeking to combine the Prussian ideal of academic freedom with the native tradition of educational opportunity for the many. The growth of such schools in the United States was greatly spurred by the Morrill Act of 1862, which granted each state tracts of land with which to finance new agricultural and mechanical schools. Many "land-grant colleges" arose from this act, and there developed among these the Massachusetts Institute of Technology, Cornell University, and the state universities of Illinois, Wisconsin, and Minnesota.

Several European nations in the 19th century reorganized and secularized their universities, notably Italy (1870), Spain (1876), and France (1896). Universities in these and other European countries became mostly state-financed. Women began to be admitted to universities in the second half of the 19th century. Meanwhile, universities' curricula also continued to evolve. The study of modern languages and literatures was added to, and in many cases supplanted, the traditional study of Latin, Greek, and theology. Such sciences as physics, chemistry, biology, and engineering achieved a recognized place in curricula, and by the early 20th century the newer disciplines of economics, psychology, and sociology were also taught.

In the late 19th and 20th centuries Great Britain and France established universities in many of their colonies in South and Southeast Asia and Africa. The independent nations that emerged from these colonies in the mid-20th century expanded their university systems along the lines of their European (or American) models and achieved varying degrees of success. Universities in Japan, China, and Russia underwent a parallel evolution. The state universities of Moscow (1755) and St. Petersburg (1819) are long-established institutions that have retained their preeminence in Russia. Tokyo (1877) and Kyoto (1897) universities are the most prestigious ones in Japan, as is Peking University (1898) in China.

Modern universities may be financed by a national government or by state or provincial ones, or they may depend largely on tuition fees paid by their students. The typical modern university may enroll 10,000 or more students and educate both undergraduates and graduate students in the entire range of the arts and humanities, mathematics, the social sciences, the physical, biological, and earth sciences, and various fields of technology. Universities are the main source of graduate-level training in such fields as medicine, law, business administration, and veterinary medicine.

Task 2. The article you've just read gives you an idea of how the institutions of higher education sprang into being and developed. But there are only a few lines in the text about the universities in Russia.

1. What do you know about the university you study at?
2. Who founded it?
3. What are the mathematics and mechanics disciplines available to be studied?
4. What is the subject you are going to specialize in at your faculty?
5. What is considered to be central to a good education in mathematics and mechanics at your faculty?

6. What does the traditional curriculum for students of your faculty consist of?
7. What are the students seeking a university education supposed to concern themselves with?
8. What science degree are you planning to get?
9. What attracted you to mathematics or mechanics?
10. What was your interest in your future speciality motivated by?

Task 3. In the text you are going to read you'll find a number of terms designating either a discipline or a subject dealing with some certain parts of scientific knowledge. Make sure you know them and if not consult any source of information available. They are: *algebra, geometry, topology, number theory, function theory, non-Euclidean differential geometry, Einstein's general theory of relativity, group theory, quantum mechanics, particle physics, finite fields, error-correcting codes, computer data storage systems, deep-space communications, formal mathematical logic, electronic switching theory, prime number theory, public key cryptography, RSA (Rivest, Shamir and Adleman) algorithm, knot theory, superstring theory, molecular biology, topology of surfaces, graph theory, tiling problems, quasicrystals.*

Task 4. Read the text.

MATHEMATICS FORTY YEARS AFTER SPUTNIK (extract 2)

In the United States, the beginning of the modern research university dates back only to 1876, with the founding of Johns Hopkins, which was based on a German model no more than a few decades older. Before this period, the modern division of knowledge into departments and disciplines was much less rigid. In Newton's day, the term natural philosophy covered all the natural sciences. The chair that Gauss held at Gottingen was in astronomy. Only when there were separate, clearly defined departments of mathematics was it necessary to invent a rationale to defend their independence from either established or newly emerging fields that sought to apply mathematics. On the other hand, it was not necessary to justify the notion that every university needed a department of mathematics.

From the time of Plato's Academy, all through the Middle Ages, and into the rise of post-medieval universities, mathematics had always been central to a good education. The traditional Scholastic curriculum consisted of two parts: the more elementary trivium, with its three language-related subjects — logic, grammar, and rhetoric; and the more advanced quadrivium, with its four mathematics-related subjects — geometry, astronomy, arithmetic (that is, number theory), and music (that is, harmonic relationships). At a time when Latin and Greek were indispensable parts of a university education, no one would have remotely considered eliminating mathematics because it was "impractical." Those students seeking a liberal university education, whether at Oxbridge or the Ivy League, were not supposed to concern themselves with learning a trade and earning a living. That came much later. And high-budget research, with the concomitant requirement to set funding priorities, was not yet a part of the university scene.

In the late nineteenth century, university mathematics departments thus had a firm franchise to exist and considerable latitude to define themselves. It was a time when much was happening in mathematics. The abstract approach was being applied, especially to algebra; the algebraic approach was being applied, especially to geometry and topology; analytic function theory was in full bloom; and a new standard of rigor had emerged. In many areas, mathematics was running so far ahead of applications that it was assumed that most of these fields would never have any. This was also true of certain classical areas, like number theory, which was developing rapidly as the beneficiary of new techniques from function theory and modern algebra and had no foreseeable applications. Rather than apologize for these fields' lack of applications, leading mathematicians and mathematics departments decided to turn a possible defect into a virtue. (In this, they anticipated a basic tenet of Madison Avenue: "If you can't fix it, feature it.")

In fact, the best mathematics consistently found very important applications, but often not until many decades later. When Riemann invented non-Euclidean differential geometry in the 1850s, it was assumed to be inapplicable, since no one then doubted that the physical universe we live in is Euclidean; but it became the mathematical basis for Einstein's general theory of relativity some sixty years later. Purely abstract concepts in group theory from around 1900 became central to the quantum mechanics of the 1930s and 1940s and to the particle physics of the 1950s onward. Finite fields, invented by Evariste Galois, who was killed in a duel in 1832 at the age of twenty, were once considered the purest of pure mathematics, but since 1950 they have become the basis for the design of error-correcting codes, which are now used indispensably in everything from computer data storage systems to deep-space communications to preserving the fidelity of music recorded on compact disks. George Boole's nineteenth-century invention of formal mathematical logic became the basis for electronic switching theory from 1940 onward and, in turn, for digital computer design.

Hardy's most precious area of inapplicable pure mathematics was prime number theory. Edmund Landau, in his 1927 work *Vorlesungen uber Zahlentheorie ("Lectures on Number Theory")*, quoted one of his teachers as frequently remarking, "Number theory is useful because you can get a Ph. D. with it." Today, the most widely used technique for "public key cryptography" is the so-called RSA (Rivest, Shamir and Adleman) algorithm, which depends on several theorems in prime number theory, and on the observation that factoring a very large number into primes (especially if it is a product of only two big ones) is much harder than determining if an individual large number is prime.

In 1940, topology would have been high on most people's list of inapplicable mathematics. Within topology, knot theory would have seemed particularly "useless." Yet today knot theory has extremely important applications in physics (for both quantum mechanics and superstring theory) and in molecular biology (for the knotted structures of both nucleic acids and proteins). The topology of surfaces is also much involved in superstring theory, including the structures that superstrings may take in multidimensional spaces. Even graph theory has blossomed into a major discipline in which the boundary between "pure" and "applied" is virtually invisible. Until recently, tiling problems were largely relegated to the domain of "recreational mathematics" (an obvious oxymoron to most non-mathematicians, but a pleonasm to true believers). Then, a decade or so ago, Roger Penrose's work on small sets of tiling shapes that can be used to tile the entire plane, but only if there is no precisely repeating finite pattern, was found to underline the entire vast field of "quasicrystals."

I could give many, many more examples of how topics and results from the "purest" areas of mathematics have found important applications, but I believe I have made my point. It may still be necessary for some mathematics departments to defend themselves against being turned into short-term providers of assistance to other disciplines that are consumers rather than producers of mathematics. But most mathematicians now recognize the basic principle that good "pure" mathematics is almost certain to have major applications eventually. For most mathematicians today, the distinction that matters is between good mathematics and bad mathematics, not between pure mathematics and applied mathematics. To be fair to Hardy, this was the distinction he was trying to make in *A Mathematician's Apology*, when he differentiated real mathematics from trivial mathematics. Where he went off the deep end was in insisting that real mathematics is useless, and that useful mathematics is trivial.

Like many of my generation, I was attracted to mathematics not by Hardy's *Apology*, but by E. T. Bell's *Men of Mathematics*, and my early interest in number theory was partly motivated by the accessibility of the subject. In most branches of "higher mathematics" the basic concepts are opaque to the outsider, but once they are understood, the questions that are easily asked are also easily answered. In contrast, the basic concepts in number theory (whole numbers, prime numbers, whether one number exactly divides another) are clear to almost everyone, but it is easy to ask "simple" questions that no expert can yet answer.

Solomon W. Golomb

Note

Johns Hopkins University — privately controlled institution of higher learning in Baltimore, Md., U.S. Based on the German university model, which emphasized specialized training and research, it opened primarily as a graduate school for men in 1876 with an endowment from Johns Hopkins, a Baltimore merchant. It also provided undergraduate instruction for men. The university, now coeducational, consists of eight academic divisions and the Applied Physics Laboratory, the latter located in Laurel, Md. The Zanvyl Krieger School of Arts and Sciences, the G.W.C. Whiting School of Engineering, and the School of Continuing Studies (for part-time students) are located at the Homewood campus in northern Baltimore.

ˌIvy ˈLeague *adj AmE* — belonging to or typical of a group of old and respected universities of the eastern U.S. These are Brown, Columbia, Cornell, Harvard, Princeton, and Yale Universities, Dartmouth College, and the University of Pennsylvania. The phrase Ivy League is also used more broadly to include other very highly-respected universities and colleges: an Ivy League college / Ivy League clothes / manners / football.

ˌMadison ˈAvenue — a street in New York City in the U.S., famous as the centre of the advertising industry.
Cultural Note: Madison Avenue can be used to mean advertising. People often accuse Madison Avenue of attitudes and practices such as treating people, things, and ideas in a very light way, when they mean that advertising does this.

oxymoron — a combination of words which seem to contradict (=disagree with) each other, such as '*cruel kindness*'.

pleonasm — use of more words than are needed to express an idea: The phrase '*an apple is divided into two halves*' is a pleonasm.

Vocabulary

Task 5. Below are some words taken from the text. Try to guess their meaning from the context in which they are found. In each case choose one of the three answers which you think best expresses the meaning.

rationale — (line 9)

(a) restriction in amount;
(b) logical basis; exposition of principles;

(c) a written or otherwise formulated communication.

concomitant — (line 28)

(a) one of two things which are opposite, contradictory, etc;
(b) exhibiting a sense of one's superiority;
(c) customary or necessary associating.

franchise — (line 31)

(a) a formulated plan listing things to be done or to take place especially in their time order;
(b) the right, privilege, or power of expressing one's choice in the determination of policy;
(c) a number of things similar in character.

beneficiary — (line 40)

(a) a definite pattern to be followed;
(b) a battle between opposing forces for supremacy;
(c) one who gains or derives advantage from something.

virtue — (line 44)

(a) qualities that are highly desirable and give a person or thing its value or worth;
(b) various forms of brief expressions of what are supposed to be accepted truths;
(c) the first example of a type from which other examples are developed.

tenet — (line 44)

(a) a principle accepted or belief held as authoritative by members of a church, a school of philosophers, a branch of science or a body of adherents;
(b) more or less regular rise and fall (in intensity) of sounds;
(c) anything that may be used in injuring, destroying, or defeating an enemy or opponent.

fidelity — (line 60)

(a) a change in the appearance of a thing without any change in the observer's point of view;
(b) faithfulness to the original as in representation, portrayal, quotation, etc;

(c) a way of dressing that is generally accepted at a given time by those who wish to follow or to be regarded as up-to-date.

Task 6. The text you've just read mostly tells about past situations, actions and events. Tenses relate actions and events to one another and show relationship in time between them. Time expressions are also important and they indicate:

a) *The point in time* when something happened / was happening.

b) *How long* something took to happen.

c) *The order* in which things happened.

Tell what situations, actions and events are associated with the following time indications in the text:
a few decades, before this period, in Newton's day, only when, from the time, all through, at a time when, much later, in the late 19th century, it was a time when, not until many decades later, when ... in the 1850s, some sixty years later, from around 1900, of the 1930s and 1940s, of the 1950s onward, in 1832, scince 1950, from 1940 onward, today, in 1940, yet today, until recently, then, a decade or so ago.

E.g. back only to 1876
The beginning of the modern research university with the founding of Johns Hopkins dates back only to 1876 in the United States.

Task 7. Read the following sentences. In each you will find one or more words from the text you have just read. Identify them. Translate the sentences into Russian.

1. Gilman introduced the sciences into the curriculum, promoted advanced research, and created professional schools.

2. There is no poet in any tongue who stands so firmly as Dante as a model for all poets.

3. The child initiates new processes of thought and establishes new mental habits much more easily than the adult.

4. That part of northern Ohio where the Bentley farms lay had begun to emerge from pioneer life.

5. Acquaintance with the scientific method of inquiry has become an indispensable element in culture.

6. In most poets there is a conflict between the poetic self and the rest of the man; and it is by reconciling of the two, not by eliminating the one, that they can reach their full stature.

7. My wardrobe had to provide for a wide range in temperature, and social, business, and sport requirements.

8. Nobody can foresee how the necessary restriction on the population will be affected.

9. Most of the great European thinkers of the eighteenth and early nineteenth centuries were in some measure inspired, influenced, or anticipated by Shaftesbury (English politician, organizer of Whig Party 1621–1683).

10. The boundary between France and Germany shifted several times in the course of 150 years.

11. Just to sit in the sun, to bask like an animal in its heat — this is one of my country recreations.

12. He was so intoxicated with dreams of fortune that he had lost all sense of the distinction between reality and illusion.

13. The desire for conquest motivated the explorations of the sixteenth century.

14. The problem in triangulation was extremely difficult, and an expert in geodesy was brought from the United States.

Task 8. Put one preposition in each gap. If you have a problem consult the text.

The beginning ___ modern research; was based ___ a German model; the modern division of knowledge ___ departments; ___ Newton's day; the chair that Gauss held ___ Göttingen; the chair was ___ astronomy; ___ the other hand; all ___ the Middle Ages; mathematics had always been central ___ a good education; curriculum consisted ___ two parts; ___ a time when Latin and Greek were indispensable parts of a university education; were not supposed to concern themselves ___ learning a trade; the abstract approach was being applied ___ algebra; function theory was ___ full bloom; this was true ___ certain areas; to turn a possible defect ___ virtue; basis ___ Einstein's general theory of relativity; abstract concept ___ group theory; became central ___ the quantum mechanics; was killed ___ a duel; ___ the age of twenty; the fidelity of music recorded ___ compact disks; you can get a Ph. D. ___ it; factoring a large numbers ___ primes; high ___ most people's lists; the topology of surfaces is involved ___ superstring theory; blossomed ___ a major discipline; tiling problems were relegated ___ the domain of; Penrose's work ___ small sets; to be fair ___ Hardy; I was attracted ___ mathematics; the basic concepts are opaque ___ the outsider.

Grammar tasks

Task 9. Below you'll find a number of sentences which include specific examples covering some important aspects and problems in the English grammar. They may cause great difficulty for the Russian learners of English. You'll have to identify there frequently used grammar constructions and find the sentences with similar grammar aspects in the main text of the unit. Make sentences to demonstrate you are able to use them in your own speech. Use the help given.

1. *Only* after a year *did I* begin to see the results of my work. (inversion after "*Only* … ")

2. From the time of his election as a head of our department he *had initiated* a number of new researches. (Past Perfect)

3. It was the kind of fort (New York) the British had the time and money and man power and machinery to build. If the forts in the West had been like this one, Brother George couldn't have done what he did. Not even George, audacious as he is, *would have dared* besiege a place like that. (hypothetical past possibility)

4. *The translator* is supposed *to carry* in his brain a very large part of the vocabularies and grammars of both languages, and a great deal of information about the subject matter involved. (Complex Subject)

5. The first thing *to realize* is that we do not have a crisis we just have a problem. (infinitive as attribute)

6. She decided she *would go to* Philadelphia and check him out. She *would not even speak to* him, she *would speak to* his neighbors and *show* them his picture. (Future in the Past)

7. His books were popularization of other people's work; he *had not written* an original paper in fifteen or twenty years. (Past Perfect)

8. I fear he *may* take some drastic actions against you on the spur of the moment. (modal, possibility mixed with doubt)

9. Even her greatest enemies *could* not deny that she had something more than beauty. (modal, ability, a line of action that is possible)

10. We consider *Newton's scientific research to have been* a response to some basic philisophical problems of his age in relation to time, space, matter, movement and eternity. (Complex Object)

11. He (Henry VIII) wanted *everything to be perfect*, and if it were not, those who denied it to him must pay their lives. (Complex Object)

12. I explained to him that I wished *my sister to be released* from the Tower but that a strong guard must be kept on her. (Complex Object)

13. *She* did not seem *to realize* that they were guarding her not only for my safety but for her own. (Complex Subject)

14. She wanted *one to know* that the terrible sin she had committed in allowing *herself to be forced* to pose as Queen was no fault of hers. (Complex Object)

15. He lacked the good sense of his brother Edward. His *were* the handsome *looks*, the dashing *personality,* the *ability* to attract people to him; but without good sense such attributes can be dangerous. (inversion)

16. Katharine gave birth to a little girl. Seymour *would have preferred* a boy. Do not all men? But Katharine, I knew, *would* be content with the child, whatever its sex. (hypothetical past possibility; future in the past)

Task 10. The following text is a fragment from KEN FOLLETT's international bestseller *THE THIRD TWIN.*

Jeannie Ferrami, a brilliant young research scientist studying the genetic components of agression is lecturing to her students at Jones Falls University in Baltimore, an Ivy League college.

Read the extract, translate it into Russian, being particularly careful while translating the italicized phrases.

"Observed variations in the intelligence of human beings can be explained by three factors," Jeannie said. "One: different genes. Two: a different environment. Three: measurement error." She paused. They all wrote in their notebooks.

She had noticed this effect. *Any time she offered a numbered list, they would all write it down. If she had simply said, "Different genes, different environments, and experimental error," most of them would have written nothing. Since she had first observed this* *syndrome, she included as many numbered lists as possible in her lectures.*

She was a good teacher — somewhat to her surprise. In general, she felt her *people skills* were poor. *She was impatient, and she could be abrasive, as she had been this morning.* But she was a good communicator, clear and precise, and *she enjoyed explaining things.* There was nothing better than *the kick of seeing enlightenment dawn in a student's face.*

"We can express this as an equation," she said, and she turned around and wrote on the board with a stick of chalk:

$$\mathsf{Vt{=}Vg{+}Ve{+}Vm}$$

"Vt *being the total variance,* Vg the genetic component, Ve the environmental, and Vm the measurement error." They all wrote down the equation. "*The same may be applied to any measurable difference between human beings, from their height and weight to their tendency to believe in God.* Can anyone here find fault with this?" No one spoke, so she gave them a clue. "*The sum may be greater than the parts. But why?*"

One of the young men spoke up. It was usually the men; the women were irritatingly shy. "Because genes and the evironment act upon one another to multiply effects?"

"Exactly. Your genes steer you toward certain environmental experiences and away from others. Babies with different temperaments elicit different treatment from their parents. We *must add* to the right-hand side of the equation the term Cge, meaning gene-environment covariation." She wrote it on the board then looked at the Swiss Army watch on her wrist. It was five to four. "Any questions?"

For a change it was a woman who spoke up. She was Donna-Marie Dickson, a nurse *who had gone back to school in her thirties,* bright but shy. She said: "What about the Osmonds?"

The class laughed, and the woman blushed, Jeannie said gently: "Explain what you mean, Donna-Marie. *Some of the class may be too young to remember the Osmonds.*"

"They were a pop group in the seventies, all brothers and sisters. The Osmond family are all musical. But they don't have the same genes, they're not twins. *It seems to have been the family environment that made them all musicians.* Same with the Jackson Five." The others, who were mostly younger, laughed again, and the woman smiled bashfully and added: *"I'm giving away my age here."*

"Ms. Dickson makes an important point, and I'm surprised no one else thought of it," Jeannie said. She was not surprised at all, *but Donna-Marie needed to have her confidence boosted.* "*Charismatic and dedicated parents may make all their children conform to a certain ideal,* regardless of their genes, just as *abusive parents may turn out a whole family of schizophrenics.* But these are extreme cases. A malnourished child will be short in stature, even if its parents and grandparents are all tall. An overfed child will be fat even if it has thin ancestors. *Nevertheless, every new study tends to show, more conclusively than the last, that it is predominantly the genetic inheritance, rather than the environment or style of upbringing, that determines the nature of the child.*" She paused. "If there are no more questions, please read Bouchard et al. in *Science*, 12 October 1990, before next Monday." Jeannie picked up her papers.

They began packing up their books. *She hung around for a few moments, to create an opportunity for students too timid to ask questions in open class to approach her privately.* Introverts often became great scientists.

Ken Follett

Discussion

Task 11. Answer the following questions. Consult the main text any time you have a problem.

1. What university was founded in 1876 and what was it based on?

2. What did the term natural philosophy cover in Newton's day?

3. When was it necessary to invent a rationale to defend the independence of departments of mathematics and what did they want to be independent of?
4. What subject was considered to be central to good education?
5. What did the traditional Scholastic curriculum consist of?
6. What was the period of time when students at Oxbridge or the Ivy League were not supposed to concern themselves with learning a trade and earning a living?
7. When "was much happening in mathematics," and what was happening in it particularly?
8. How were certain classical areas, like number theory, developing in the late nineteenth century?
9. How soon did the best mathematics find very important applications?
10. What was the mathematical basis for Einstein's general theory of relativity, the quantum mechanics, the particle physics the design of error-correcting codes, electronic switching theory, digital computer design?
11. Where are error-correcting codes now used indispensably?
12. What technique is the most widely used for "public key cryptography" and what does it depend on?
13. What would have seemed particularly "useless" in 1940?
14. Where does knot theory have extremely important applications?
15. What "inapplicable" fields of mathematics have blossomed into disciplines in which the boundary between "pure" and "applied" is virtually invisible?
16. The author says, " ... but I believe I have made my point." What does he mean?

17. What is the basic principle most mathematicians now recognize?

18. Where did Hardy go off the deep end?

19. What was the author motivated by and attracted to mathematics?

20. How does the author explain the accessibility of number theory?

Task 12. In the second paragraph the author writes, "From the time of Plato's Academy, all through the Middle Ages, and into the rise of post-medieval universities ... students seeking a liberal university education whether at Oxbridge or the Ivy League were not supposed to concern themselves with learning a trade and earning a living."

Naturally the question arises: What was the objective of higher education? The answer can be found in the book *Higher Education in Transition* by J. S. Brubacher and W. Rudy. Read and discuss the fragment from this book in the class.

Universities were definitely a place to learn for the sake of learning and not for the sake of earning. The two great points to be gained in intellectual culture were discipline and furniture of the mind, expanding its power and storing it with knowledge. So students were supposed to develop mental power which could be transferred at will from one study to another and from studies in general to the occupations of life. The old-time college was concerned primarily with training a special elite for community leadership being notably intellectual and theoretical compared to practical and vocational education of the masses. The college might not offer training peculiar to specific occupations, it would provide the broad theoretical foundation logically prior for them all to discipline the mind. The classical curriculum served this objective. The old-time college was primarily involved in conserving of existing knowledge rather than the search for new knowledge. To the end of the 19th century the American graduate school was being modeled after the German university. Two important characteristics of that university were incorporated into the American university — autonomy and objectivity in research. There was a notable tendency at the turn of the century in the United States that a university should give people the practical instruction that they want, specific learning for specific purpose that is to put research results at the service of public. So universities underwent alterations to meet the changing demand of time and place.

The topics supposed to be discussed in the class are: *What is the objective of higher education at present time in your country?* and *What is the place of university education in your country?*

Task 13. In Task 10 you have read one fragment from Follett's book. It describes a lecture. Reread it and use its vocabulary and grammar to tell the class about one of your lectures.

Task 14. One of the forms of university education is a seminar. Generally seminars are called upon to discuss some particular scientific problems, to affirm or to deny a thesis or hypothesis suggested by researchers. You are sure to have experience in participating in some seminars.

Below is an extract from the article published in the journal *Helicopter World*. The author visited the European Rotocraft Forum and is summarizing his impressions and what was duscussed there. Read the introduction. Translate it into Russian and then try to describe what kind of researchers come to the seminar you are attending at the moment, what problems are discussed and what your impression is.

If you could channel the electricity from the collection of brains that typically turns up at the peripatetic European Rotocraft Forum (this year's event was in Rome), the multi-megawatt output would probably make several nuclear power stations redundant. ERF is where helicopter engineers meet — people who juggle Euler equations for fun, are soul-mates with their computers, scamper around neural networks in their sleep and who apply their genius to making better helicopters.

ERF is an opportunity for them to strut their stuff and for the rest of us to get an idea of the directions in which their creative juices are flowing. The official language for the sessions is English, but subjects such as acoustics, aerodynamics, dynamics and flight mechanics have arcane languages of their own that might as well be dialects of Martian to the uninitiated. But they undoubtedly do make better helicopters, the new generation of machines entering service now is proof of that, and there are nuggets among the ERF papers that reveal how they will continue to do so.

PETER DONALDSON

Task 15.

1. You are going to read the final part of the article *MATHEMATICS FORTY YEARS AFTER SPUTNIK* by S. W. GOLOMB. At ten points, fragments have been removed. What do you think was in each gap? Discuss this in small groups. Cover the list of fragments following the article below.

2. Study the list which contains the missing fragments and decide where they go in the extract. Check with your teacher, then fill in the gaps.

MATHEMATICS FORTY YEARS AFTER SPUTNIK (extract 3)

As you will remember, Hardy had contended in *A Mathematician's Apology* that real mathematics is much more similar to poetry and painting than it is to chemistry or engineering. **1.**___ The rationale for including mathematics in the largesse required a commitment to the principle that basic research in mathematics, like basic research in chemistry or engineering, will ultimately have significant consequences. Suddenly, there was a reason for trying to show that one's mathematics had practical uses and implications. This was not the only reason for the change in attitude about whether good mathematics could be useful, but it certainly played a part.

Another major influence has been development of digital technology, which has placed new emphasis on areas of discrete mathematics that were previously considered inapplicable — for example, as I've mentioned, finite fields. **2.**___ Another development was Shannon's mathematical theory of communication, which asks questions that are motivated by applications but are more abstract mathematically than anything in physics. Shannon's "bit of information" is purely mathematical concept. **3.**___

It is also true that science and engineering have changed dramatically in the fifty years since Hardy's death. Semiconductors and lasers make very nontrivial use of quantum mechanics, and the people who study them are not restricted to using what Hardy, derisively referred to as "school mathematics." Today, many other fields, from molecular biology to control theory, also apply sophisticated mathematics.

In fairness to G. H. Hardy, there are many things in *A Mathematician's Apology*, with which most mathematicians will agree or identify. Hardy asserted that mathematicians are attracted to the subject by its inner beauty rather than by any overwhelming desire to benefit humanity. Most mathematicians I know would agree with that. Even more important, Hardy identified himself as a realist (as the term is used in philosophy) about mathematics. **4.**___ This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards." The great majority of mathematicians share this view about mathematics. Where Plato went overboard was in trying to extend mathematical reality physical reality.

Immanuel Kant explicitly distinguished between the transcendental reality of mathematics and the (ordinary) reality of the physical universe. **5.**___ The Platonic (that is, realist) view of mathematics has dissenters. Some who, in my opinion, are overly influenced by quantum mechanics would argue that $2^p - 1$, where p is some very large prime number, is neither prime nor composite, but rather in some intermediate "quantum state," until it is actually tested. Of course, the realist view is that it is already one or the other, and we find out which when we test it. Euclid gave a lovely proof — included in Hardy's *Apology* as an example of mathematical beauty that an educated non-mathematician could appreciate — that there is no largest prime number. **6.**___

Even less palatable to most mathematicians is the postmodern criticism of all science: that it is just another human cultural activity, and that its results are no more absolute or inevitable than works of poetry, music or literature. The extreme form of this viewpoint would assert that "$4+7=11$" is merely a cultural prejudice. **7.**___ Also, culture can play an important role in determining which mathematical questions are asked and which mathematical topics are studied. (Our widespread use of the decimal system is undoubtedly related to the fact that we have ten fingers.) **8.**___

It was Isaac Newton who wrote, "**9.**___ " It is hard to give a better formulation of the realist view of mathematical truth to which most mathematicians adhere.

Newton is firmly entrenched in the pantheons of both mathematics and physics. He certainly erected no artificial barriers between theory and applications. **10.**___ In the forty years that have elapsed since those satellites first circled the earth, most mathematicians — myself included — have moved further from Hardy's outlook and closer to Newton's.

Solomon W. Golomb

a) My own version of this distinction is that if the Big Bang had gone slightly differently, or if we were able to spy on an entirely different universe, the laws of physics could be different from the ones we know, but 17 would still be a prime number.

b) That is something that many of us, as mathematicians, might still like to believe; but the new government funding didn't extend to poetry and painting.

c) What I will not concede is that if the same mathematical questions are asked, the answers would come out inconsistently in another culture, on another planet, in another galaxy, or even in a different universe.

d) That is, the sequence $2, 3, 5, 7, 11, \dots$ of the prime numbers is unending, or infinite. But Euclid's proof does not identify any specific large numbers as primes. It is particularly easy to test whether numbers of the form $2^p - 1$ are prime, and the largest specific number known to be prime at any time is likely to be one of these "Mersenne primes."

e) Then there is computer science itself, which studies questions in pure mathematics, such as determining the computational complexity of various procedures, the results of which turn out to be extremely practical.

f) I will readily concede the obvious: it requires a reasoning device like the human brain (or a digital computer) to perform the sequences of steps that we call mathematics.

g) I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great Ocean of Truth lay all undiscovered before me.

h) It has no mass, no spin, no charge, no momentum — and yet the issues involved in measuring information in bits, in storing information, and in moving information from one place to another are so important that we are told daily that we live in the Age of Information.

i) In the three centuries since he published the *Philosophiae Naturalis Principia Mathematica* (the *Principia*, for short), we have, with electromagnetism and relativity and quantum mechanics, waded deeper into that Ocean of Truth; but Newton's laws of motion and gravitation were actually sufficient for the launching of Sputnik and Explorer.

j) "I will state my own position dogmatically," he wrote. " … I believe that mathematical reality, lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations … "

Appendix I

Task 1. Look through the summaries following the article below. Decide which are satisfactory. Give your reasons.

ON SOLID ANGLES AND THE VOLUMES OF REGULAR POLYHEDRA

In the interesting article [2], Paul Shutler presents three different approaches to the problem of computing the volumes of the regular (Platonic) polyhedra. These five polyhedra (the cube and the regular tetrahedron, octahedron, dodecahedron, and icosahedron) display perfect symmetry, i.e., all faces are congruent regular polygons and all polyhedral angles are also congruent to each other. The most mysterious of his methods is based on the Biot-Savart Law of magnetostatics and the concept of solid angle. In this note we show how the physics used in this approach can be replaced by a bit of vector calculus. The result is a method that is geometric and conceptually quite simple.

We first discuss the geometric concept of solid angle, a natural generalization of the idea of planar angle to dimension three. Let K denote a surface in $\mathbb{R}^3$ that intersects each ray from the origin in at most one other point (see Figure 1).

The solid cone of lines that meet K intersects the unit sphere centered at the origin in a surface K^*. The *solid angle* Ω subtended by K at the origin is defined to be the area of K^*. Notice that the two-dimensional version of this idea results in the ordinary angle

Figure 1 (from [1], p. 521)

subtended by a curve in the plane at the origin. It is possible to express Ω as an integral provided that K is sufficiently (piecewise) smooth. Let $\mathbf{r}$ denote the vector from the origin to a general point in K, $r = |\mathbf{r}|$, $\hat{\mathbf{r}} = \mathbf{r}/r$, and $\mathbf{n}$ the outward-pointing unit normal to K at $\mathbf{r}$. Then, since the inverse square field $\mathbf{r}/r^3$ is divergence-free, a direct application of the divergence theorem shows (see [1], p. 521) that Ω is equal to the surface integral

$$\Omega = \int\int_K \frac{\mathbf{r} \cdot \mathbf{n}}{r^3} dA. \tag{1}$$

Consider a regular convex polyhedron P inscribed in a sphere of radius R. Assume that P has N congruent faces, each with n sides. Following [2], we introduce three auxiliary quantities: w, the distance from the center of each face to the center of the sphere; l, the length of each side of a face; and h, the perpendicular distance from the center of a face to a side (classically known as the *apothem*). By decomposing P into a union of V congruent pyramids, each with one vertex at the origin, and then using the usual formula for the volume of a pyramid, we find a simple expression for the volume of P: $V = Nnlhw/6$. The problem now is finding constraints to eliminate the quantities l, h, and w, so that V is given in terms of only N, n, and R. Such a formula would then allow us to determine, for instance, the volume of a dodecahedron inscribed in the unit sphere ($R = 1$, $N = 12$, and $n = 5$).

Figure 2

The Pythagorean theorem in three dimensions shows that $R^2 = w^2+h^2+l^2/4$, and simple trigonometry on one face yields $\tan(\pi/n) = l/(2h)$. To find a third constraint, we consider the solid angle Ω subtended by one face of the polyhedron, where the origin has been placed at the center of the sphere. By the symmetry of P, we have $\Omega = 4\pi/N$, since the area of the unit sphere is 4π. But Ω can also be calculated using (1), and the resulting surface integral can be evaluated (by hand!). We sketch the calculation.

Divide a face into n triangles with a common vertex placed in the center of the face. Let K denote the right triangle obtained by taking half of one of these n isosceles triangles, which we situate in the xy-plane as shown in Figure 2.

Then the center of the sphere lies at distance w directly above the origin, and $\mathbf{r}$ points from the center of the sphere to a general point in the triangle. We have $\mathbf{r} \cdot \mathbf{n} = r\cos\theta$, where θ is the angle that $\mathbf{r}$ makes with the z-axis. Thus $r = \sqrt{x^2+y^2+w^2}$ and $\cos\theta = w/r$; hence Ω is given by the iterated integral

$$\Omega = 2nw \int_0^h \int_0^{\frac{lx}{2h}} \frac{1}{(x^2+y^2+w^2)^{3/2}} dy dx.$$

This integral can be evaluated by using, in the outer integral, the formula

$$\int \frac{x}{(x^2+a^2)\sqrt{a^2+b^2}} dx = -\frac{1}{\sqrt{a^2-b^2}} \arcsin \frac{\sqrt{a^2-b^2}}{\sqrt{a^2+x^2}} + C,$$

valid when $a > b$. The formula can be derived using standard calculus techniques. We write the result in the form $w = \lambda h$, where

$$\lambda = \frac{\sin\left\{\frac{\pi}{n}\left(1-\frac{2}{N}\right)\right\}}{\sqrt{\sin^2\frac{\pi}{n} - \sin^2\left\{\frac{\pi}{n}\left(1-\frac{2}{N}\right)\right\}}}.$$

This is the third of our needed constraints. Using elementary algebra, it is now possible to express V (as well as l, a, h, w, and the surface area A) in terms of R, N, and n. In particular, we obtain the formula for V which was derived in [2] using physics:

$$V = \frac{\frac{1}{3}NnR^3\lambda\tan\left(\frac{\pi}{n}\right)}{\left\{1+\lambda^2+\tan^2\frac{\pi}{n}\right\}^{3/2}}. \qquad (2)$$

It follows from (2) that the volume (to four decimal places) of a dodecahedron inscribed in the unit sphere is 2.7852; that of an icosahedron is 2.5363. Some readers may be surprised that the dodecahedron, which has fewer faces, has the larger volume.

Jeffrey Nunemacher

References

1. Peter Baxandall and Hans Liebeck. *Vector Calculus*, Oxford University Press. New York, NY, 1986.
2. Paul Shutler. Magnetostatics and the volumes of the platonic solids. *Int. J. Math. Educ. Sci. Technol.* 27 (1996). 413–419.

* * *

1. The present paper replaces the physical method of computing the volumes of the regular polyhedra by the method of vector calculus.

   Firstly, the paper introduces the concept of solid angle and gives an integral form of it. The rest part is devoted to the calculation of the volume of a regular polyhedron inscribed in a sphere. The method uses standard division of the polyhedron into N congruent pyramids and calculates the volume of each pyramid. Then three constraints are found to get rid of auxiliary parametres. This is made by means of vector calculus applied to an integral form of a solid angle Ω.

2. For computation of volumes of the regular polyhedra several methods have been invented. Some of them were affected by Paul Shutler and used the physical approach. In the article a modification of one of his methods is presented, that is essentially geometric and quite simple. It is based on the concept of solid angle, a generalization of the notion of planar angle to the 3-dimensional case. The solid angle, subtended by a surface K can be calculated as an integral of special expression over K. There can be found simple relationship between the volume of a polyhedron and the solid angle of one of its faces. Having calculated the integral, it is easy to find the volume.

3. The article gives another approach to the problem of computing the volumes of the regular polyhedra in terms of vector calculus. First, the notion of the solid angle is introduced, then a formula for its calculation is presented ($\Omega = \int\int_K \frac{\mathbf{r}\cdot\mathbf{n}}{r^3} dA$), which is to be transformed in further procedures.

   These are as follows: assuming a regular convex polyhedron to have N congruent faces, each with n sides, we introduce auxiliary quantities and decompose the polyhedron P into N congruent pyramids. In this way the volume of P is easily written. So as to obtain the final result in the form needed constraints for auxiliary quantities elimination are found. The integral having been evaluated in terms of initial values, the formula in question has the following form

$$V = \frac{\frac{1}{3} N n R^3 \lambda \tan\left(\frac{\pi}{n}\right)}{\left\{1 + \lambda^2 + \tan^2 \frac{\pi}{n}\right\}^{3/2}},$$

   where R — radius of a sphere in which P may be inscribed, and

$$\lambda = \frac{\sin\left\{\frac{\pi}{n}\left(1 - \frac{2}{N}\right)\right\}}{\sqrt{\sin^2 \frac{\pi}{n} - \sin^2\left\{\frac{\pi}{n}\left(1 - \frac{2}{N}\right)\right\}}}.$$

   The method presented seems to have an easier derivation of the formula than that based on the Biot-Savart Law of magnetostatics.

4. The article under consideration shows an approach to computing the volumes of the regular polyhedra. It is based on the geometric notion of solid angle. The author considers a regular convex polyhedron P, insribed in a sphere of radius R. Using simple geometric techniques, he finds the formula which expresses the volume of the polyhedron $V = Nnlhw/6$, where N is the number of congruent faces of P, each with n sides, w is the distance from the center of each face to the center of the sphere; l — the length of each side of a face; h is the apothem. Then finding the constraints to eliminate the quantities l, h and w, the author determines the volume in terms of only N, n and R. Surprisingly, it turns out that the volume of a dodecahedron inscribed in the unit sphere is larger than that of an icosahedron, which has more faces.

5. The article investigates the problem of computing the volumes of the regular polyhedra (also known as Platonic). The author describes an approach where sophisticated physics is replaced by vector calculus. He states that such an approach is geometric and thus conceptually transparent.

   The concept of solid three-dimensional angle is presented and three constraints are considered to calculate the volume of the polyhedron. Those three constraints suggested provide with no help of elementary algebra expressions for calculating volumes in terms of only R, N and n, which are respectively the radius of a sphere containing the polyhedron, the number of its congruent faces and the number of its sides. A sketch of calculations is also given.

6. This article discusses a certain approach to the problem of computing the volume of the Platonic polyhedra. For this purpose the concept of solid angle, that is a natural generalization of planar angle, is introduced. It is stated, that with the help of the divergence theorem, one can see, that $\Omega = \int\int_K \frac{\mathbf{r}\cdot\mathbf{n}}{r^3} dA$, where Ω — the solid angle, and K is a sur-

face in $\mathbb{R}^3$, that intersects each ray from the origin in at most one other point. After these notes, a regular polyhedron P is introduced, and w denotes the distance from the center of each face to the center of the sphere, in which P is inscribed, h is the perpendicular distance from the center of a face to a side and l is the length of each side of a face. Also, P is known to have N congruent faces, each with n sides, and the sphere radius is R. If we have all these data, we can easily find the volume of P: $V = Nnlhw/6$. The problem is to eliminate l, h, w from this expression, and so one has to find three constraints for these variables. The Pythagorean theorem, simple trigonometry and the solid angle allow to find that, finally, $V = \frac{\frac{1}{3}NnR^3\lambda\tan\left(\frac{\pi}{n}\right)}{\left\{1+\lambda^2+\tan^2\frac{\pi}{n}\right\}^{3/2}}$.

7. Several approaches to the problem of computing the volumes of the regular polyhedra are known. Among them there is one based on the Biot-Savart Law of magnetostatics, which was introduced by Paul Shutler and then simplified and developed through replacement of physics with raw vector calculus by Jeffrey Nunemacher.

   First, the concept of solid angle is discussed. Its relation to a specific kind of surface integral is established. Second, obvious geometric statements are applied and the volume is expressed in terms of three unknown quantities. To find them constraints are imposed. At this point the first result is obtained. At last, the volume is expressed using the well-known algebraic techniques:

$$V = \frac{\frac{1}{3}NnR^3\lambda\tan\left(\frac{\pi}{n}\right)}{\left\{1+\lambda^2+\tan^2\frac{\pi}{n}\right\}^{3/2}},$$

$$\text{where } \lambda = \frac{\sin\left\{\frac{\pi}{n}\left(1-\frac{2}{N}\right)\right\}}{\sqrt{\sin^2\frac{\pi}{n}-\sin^2\left\{\frac{\pi}{n}\left(1-\frac{2}{N}\right)\right\}}}.$$

   Besides, an interesting numerical result is provided by the author.

Task 2. Consider the summaries following the article below. Decide which are acceptable. Then discuss the characteristics of an effective summary and the steps important in writing it.

FIBONACCI AT RANDOM
Uncovering a new mathematical constant

It all started out with imaginary rabbits. In a book completed in the year 1202, mathematician Leonardo of Pisa (also known as Fibonacci) posed the following problem: How many pairs of rabbits will be produced in a year, beginning with a single pair, if every month each pair bears a new pair that becomes productive from the second month on?

The total number of pairs, month by month, forms the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and so on. Each new term is the sum of the previous two terms. This set of numbers is now called the Fibonacci sequence.

Fibonacci numbers come up surprisingly often in nature, from the number of petals in various flowers to the number of scales along a spiral row in a pine cone. They also arise in computer science, especially in sorting or organizing data.

Amazingly, the ratios of successive terms of the Fibonacci sequence get closer and closer to a specific number, often called the golden ratio. It can be calculated as $(1+\sqrt{5})/2$, or $1.6180339887\ldots$. For instance, the ratio $55/34$ is $1.617647\ldots$, and the next ratio, $89/55$, is $1.6181818\ldots$.

Now, computer scientist Divakar Viswanath of the Mathematical Sciences Research Institute in Berkeley, Calif., has taken a fresh look at Fibonacci numbers and unexpectedly discovered a new mathematical constant: the number $1.13198824\ldots$. He describes his result in a paper to be published in *Mathematics of Computation.*

Viswanath's research represents an intriguing gateway to heavy-duty mathematics, says mathematician Keith Devlin of Saint Mary's College of California in Moraga. It relies on powerful mathematical techniques that also are used, for instance, to elucidate the behavior of disordered materials.

Viswanath wondered what would happen to the Fibonacci sequence if he introduced an element of randomness.

Here's one way to proceed: Start with the numbers 1 and 1, as in the original Fibonacci sequence. To get the next term, flip a coin to decide whether to add the last two terms or subtract the last term from the previous term.

Suppose that heads means add and tails means subtract. Tossing heads would result in adding 1 to 1 to get 2, and tossing tails would lead to subtracting 1 from 1 to get 0. According to this scheme, the successive coin tosses H H T T T H, for example, would generate the sequence 1, 1, 2, 3, −1, 4, −5, −1.

It's easy to write a short computer program to generate these random Fibonacci sequences, notes Lloyd N. Trefethen of the University of Oxford in England. Looking for patterns and trends among such sequences of numbers can be a fascinating pastime, he says.

Indeed, infinitely many sequences follow Viswanath's rule. A few have special characteristics. If the coin always comes up heads, for instance, the result is the original Fibonacci sequence. Other strings of coin tosses can produce a repeating pattern, such as 1, 1, 0, 1, 1, 0, 1, 1, 0, and so on.

Nonetheless, such special cases are sufficiently rare among all possible sequences that mathematicians ignore them.

The standard Fibonacci sequence has an intriguing property. The hundredth Fibonacci number, for example, is roughly equal to the hundredth power of the golden ratio.

By examining typical random Fibonacci sequences based on coin tosses, Viswanath uncovered a similar pattern. He ignored the minus signs, thereby taking the absolute value of the terms. He found that the hundredth term in such a sequence, for example, is approximately equal to the hundredth power of the number 1.13198824.... In fact, the higher the term, the closer its value gets to the appropriate power of 1.13198824....

Despite the element of chance and the resulting large fluctuations in value that characterize a random Fibonacci sequence, the absolute values of the numbers, on average, increase at a well-defined exponential rate.

It's not obvious that this should happen, Viswanath observes. Random Fibonacci sequences might have leveled off to a constant absolute value because of the subtractions, for example, but they actually escalate exponentially.

Providing a rigorous proof of the result was a tricky business. To get the answer he required, Viswanath had to delve into several different areas of mathematics, including the intricacies of geometric forms known as fractals, and finish with a computer calculation.

Viswanath's achievement "showed persistence and imagination of a very high order," Trefethen remarks.

Now, Devlin adds, "mathematics has a new constant." No one has yet identified any link between this particular number and other known constants, such as the golden ratio.

Surprisingly, Viswanath's constant provides one answer to a mathematical puzzle that arose several decades ago from the work of Hillel Furstenberg, now at Hebrew University in Jerusalem, and Harry Kesten of Cornell University.

In a different mathematical context involving so-called random matrix multiplication, Furstenberg and Kesten had proved that in number sequences generated by certain types of processes having an element of randomness, the value of the nth term of the sequence gets closer to the nth power of some fixed number. However, they provided no hint of what that fixed number might be for any particular sequence.

Because random Fibonacci sequences fit into this category of sequences, Viswanath's new constant represents an accessible example of these fixed numbers.

"It is a beautiful result with a variety of interesting aspects," Trefethen says. It's a nice illustration, for example, of how a random process can lead to a deterministic result when the numbers involved get very large.

Moreover, although Viswanath's result by itself has no obvious applications, it serves as an introduction to the sophisticated type of mathematics developed by Furstenberg, Kesten, and others. That mathematical machinery has proved valuable in accounting for properties of disordered materials, particularly how repeated random movements can lead to orderly behavior, Devlin says.

Such mathematics underlies explanations of why glass is transparent and how an electric current can still pass in an orderly fashion through a semiconductor laced with randomly positioned impurities.

Viswanath's original work was done at Cornell University, under Trefethen's supervision. Trefethen and Oxford's Mark Embree have recently studied slightly modified versions of the random Fibonacci sequence. If, for example, one combines the last known term with half the previous term, adding or subtracting according to the toss of a coin, the sequence's numbers decrease at a certain rate, displaying exponential decay.

By using fractions other than one-half, it's possible to find fractions for which one gets exponential decay, exponential growth, or merely equilibrium. "All this quickly gets under your skin when you start trying it on the computer," Trefethen says, adding that it becomes an addictive pastime.

There's still plenty of room for mathematical exploration and experimentation in a problem that began centuries ago as a decidedly unrealistic model of a population of rabbits.

IVARS PETERSON

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1. This article deals mainly with another development of the old, widely known the Fibonacci sequence. The new element, that was introduced by Divakar Viswanath, was a random factor, and so a new term of the sequence can be equal to the sum of two previous terms with a certain probability in one case or to their difference in another. The main property of the Fibonacci sequence, i.e. the n-th number is roughly equal to the n-th power of the golden ratio, holds with some changes for these new random sequences. It is not obvious, but the absolute values of the numbers, on average, increase at a well-defined exponential rate. The proof of this fact was based upon facts from different areas of mathematics, including geometric forms known as fractals, and as a result "mathematics has a new constant." But this is not the only result of

Viswanath's achievement; it is also an important example of a wide class of number sequences, generated by a certain type of random processes. It was proved that a sequence from such a class gets closer and closer to the exponent of some fixed number, but there was no any idea how to find this number.

So, Viswanath's result serves as an introduction to the theory of random matrix multiplication; this theory in its turn has proved valuable in the investigation of how repeated random movements can lead to orderly behavior.

2. The article is devoted to the so-called Fibonacci numbers, which were invented in 1202 by Leonardo of Pisa. He counted the total number of rabbit pairs, considering that each pair gives a new one every month, beginning with the second. So he obtained the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89.

   Later studies, concerning the sequence, showed that the ratio of successive terms strives for $\frac{1+\sqrt{5}}{2}$. This number was called the golden ratio. Only recently another "universal number" was found by Divakar Viswanath of the MSRI (in Berkeley, Calif.), which is 1.13198824. Viswanath introduced an element of randomness into the Fibonacci sequence, subtracting or adding the next term according to the result of a coin tossing. In such a way he generated an enormous number of "random Fibonacci sequences" and discovered that despite the fact that the elements of the sequence were taken "by chance" the absolute values of the terms, on average, increase exponentially, and that the hundredth term in the sequence is close to the hundredth power of 1.13198824.

   Amazingly, this constant provides applications in the theory of fractals, studies of disordered materials, properties, electric current in semiconductors, random matrix multiplication and gives plenty of possibilities for further investigations.

3. A new approach to the well-known Fibonacci sequence has been recently suggested by a computer scientist Viswanath of MSRI in Berkeley. He introduced an element of randomness into the Fibonacci pattern according to which the successive

term of a sequence is obtained from the two previous ones according to the toss of a coin. Thus, say, a head means add, and a tail means subtract the last term from the previous one.

Amazingly, when ignoring the minus signs in the terms of the sequence, they get closer and closer to the appropriate power of a certain number: 1.131.... This number, however, turns out to be a new mathematical constant.

It is also involved in different mathematical and physical contexts, such as random matrix multiplication and exploring properties of disordered materials.

Although these results seem to be a kind of a puzzle, providing a rigorous proof is a tricky business, including sophisticated mathematics.

4. The article under consideration tells us about a new discovery in the field of mathematics. Computer scientist Divakar Viswanath studied the behaviour of the sequence, which he had obtained from the Fibonacci sequence by introducing an element of randomness to it. He found that the n-th term of such a sequence is approximately equal to the n-th power of the fixed number 1.13198824.... This property is similar to the analogous one of the original Fibonacci sequence, and gives confirmation of some results obtained in the field of random matrix multiplication. Moreover, this result can have some valuable applications to the investigation of properties of disordered materials.

5. The article is concerned with new sequences having properties similar to the Fibonacci numbers. The sequences have been obtained with the help of randomness by computer scientist Divakar Viswanath. The number 1.13198824 plays the same role for these new random sequences as the famous golden ratio $\frac{1+\sqrt{5}}{2}$ for the Fibonacci numbers.

   The result of Viswanath provides an illustration for one of the theoretical works. It also underlies explanation of why glass is transparent and how an electric current can pass in an orderly fashion through a semiconductor laced with randomly positioned impurities.

6. The article deals with random sequences which originated from the Fibonacci sequence. As it is known the ratios of successive terms of this sequence tend to the number $(1+\sqrt{5})/2$, called the golden ratio. The article tells about another mathematical constant: the number 1.13198824. It is obtained as follows: the first two numbers are 1, then we flip a coin, if we get a head, we add the last two terms of the sequence, if a tail we subtract the last term from the previous one. The absolute value of the numbers obtained being taken, we can see that the hundredth term is approximately equal to the hundredth power of the number 1.13198824. Whatever the construction of the sequence may be, it is not a simple matter to give a rigorous proof of the fact, which employs fractals and computer calculation. By means of other operations with random numbers one can get sequences with similar properties. Mathematics dealing with and accounting for such phenomena underlies explanation of properties of disordered materials and is of great value to science.

7. A fresh look at the Fibonacci numbers revealed some new mathematical results. Divakar Viswanath introduced random sequences similar to those of Fibonacci. By examining the absolute values of terms in these sequences, he discovered their asymptotic behaviour. Particularly, these sequences have an exponential growth and tend to powers of a specific number — 1.13198824 . . . . A rigorous proof of the result was also found.

   Moreover, the result illustrates some theoretical investigations in the field of random matrix multiplication where similar results having had no appropriate instances were found. Besides, it has some other practical aspects.

8. Let us recall the Fibonacci sequence — the model of rabits' population. It has an interesting property. The ratios of the successive terms in it get closer and closer to the golden ratio

which is equal to $\frac{1+\sqrt{5}}{2}$ or 1.61803398.

Another curious mathematical constant was discovered by computer scientist Divakar Viswanath. He used an element of randomness in the Fibonacci sequence. To get the next term he added the last 2 terms or subtracted the last term from the previous term. It occurred that the 100th power of the number 1.13198824 was approximately equal to the hundredth term in such a sequence.

This finding became an illustration of the problem proved by Furstenberg and Kesten. It asserts that the value of the nth term of the sequence is close to the nth power of some fixed number. This is a very amazing result which can be used for explaining different physical phenomena such as glass transparency and the passing of an electric current through a semiconductor with randomly positioned impurities.

Appendix II

Just for fun

The article you are going to read in this section was published in *The Mathematical Intelligencer* by B. SURY. It is titled *MIXED MOTIVES*. If you like jokes, moreover if you like to know what non-mathematicians sometimes could think of mathematicians you will surely read it. Note down any funny story which happened to you. Tell it in the class.

MIXED MOTIVES

The train jiggled as it ran, and gave periodic shakes like a man caught without his sweater on a cold night. In the second-class compartment, Monsieur Pi Rho was covertly studying the passenger opposite him. The young man had bloodshot eyes; evidently he hadn't slept properly for many days. He was bespectacled and clad in a Khadi Kurta and jeans which hadn't seen water for a considerable period of time; obviously a man of spartan tastes in the matter of attire. His eyes gleamed with a ferocity that spoke of some deep and sinister purpose. Every once in a while, he would jump out of his seat in his restlessness and walk along the aisle, cross the vestibule, and return after a five-minute stroll.

Monsieur Pi Rho (though having retired from his profession two years back) could place him unerringly. Yes, his instincts told him that the young man was up to something, which, whatever it was, would not be too long in coming.

The dinner was long over, most of the lights had been put off, people had given up the pretense of reading, and indeed most of them were snoring gently. The young man took out a packet of cigarettes and moved toward the door. Monsieur Pi Rho got up and, peeping surreptitiously out of the compartment, saw the young man begin his smoke. Tiptoeing back, he pulled from under the seat the dirty-looking brown bag which seemed to be the young man's only piece of baggage. He quickly unzipped it and looked inside. In the dim light, he could see sheets of paper.

Monsieur Pi Rho pulled out one and peered at some handwritten instructions. He began reading.

"We are provided with a scheme and a map which is proper to a point," read Monsieur Pi Rho. With widening eyes, he skimmed through the page to see if any person or place was mentioned. Beyond cryptic words like "the group can act freely but discreetly," (the last word had been misspelt), "go to a cover to kill the classes," and "blow-up if necessary," nothing specific was mentioned. Evidently, some group was planning an ambush, but where and on whom? Who were they? (These people expressed themselves in a strange Pickwickian language — surely in the interest of security.)

Monsieur Pi Rho didn't have too much time before the young man's return. He quickly turned a couple of pages and saw the heading "Motives." Here the language was even of a group whose representatives were deemed to traceless. Also mentioned was a corpse (also misspelt) which was totally disconnected (!), on which some functions still existed but were rapidly decreasing. Whose could it be? And where? Presumably in some local field. Somehow the job of this corpse was threefold:

(i) split some (presumably rival) group;

(ii) decompose certain representatives of the group; and

(iii) infiltrate by powers of ideals.

These people even talked of ideals! Monsieur Pi Rho pondered for a moment on this mysterious group's ways. He turned to the last page and THEN HE KNEW! There it was clearly written: "BULLET IN THE AMERICAN MATHEMATICAL SOCIETY!"

Hastily trusting the manuscript back in its bag, Monsieur Pi Rho returned innocuously to his seat. He must act quickly. What should he do? Would confronting the youth solve the problem? Involving, as it did, such a radical group, solvability depended on their Killing form. He came to a quick decision. No! He would wait until the train reached Hampur and wire for help. The critical point was to use Morse. THAT would certainly put them in their cells.

B. Sury

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