Московский государственный университет
имени М.В. Ломоносова
Механико-математический факультет

О.В. Корецкая

Mastering English in Mathematical Discourse

Учебное пособие

Москва
2013

Корецкая Ольга Владимировна
Совершенствование английского языка в рамках математического дискурса. – М.: МАКС Пресс, 2013. – 96 с.

Учебное пособие «Совершенствование английского языка в рамках математического дискурса» (*Mastering English in Mathematical Discourse)* предназначено для студентов и аспирантов механико-математических факультетов вузов и ставит своей целью помочь учащимся освоить английский язык в рамках «математического дискурса». Прежде всего, это особый терминологический слой лексики, а также определенное грамматическое построение текста, свойственное работам академического характера, знание которых должно входить в компетенцию специалистов по математике и механике.

Пособие состоит из 10 оригинальных статей, взятых из современных британских и американских энциклопедий и журналов. Все тексты, представленные в пособии, объединяются общей тематической направленностью и являются ярким примером «английского языка для специальных целей» (English for Specific Purposes), составляющего основу научного стиля изложения.

К каждому тексту прилагается целый ряд упражнений, направленных на отработку основных аспектов английского языка — фонетики, лексики и грамматики. Предлагаются следующие виды заданий: обсуждение текста и подготовка ответов на связанные с ним вопросы; чтение слов по транскрипции и объяснение их значения на английском языке; упражнения на словообразование, сочетаемость синонимов и перевод с русского на английский язык с использованием лексики из текста; повторение ключевых грамматических конструкций, которые наиболее часто вызывают затруднения у изучающих язык.

Данное пособие может быть использовано как в качестве основного, так и дополнительного учебного материала для учащихся, владеющих языком на среднем и продвинутом уровне.

Koretskaya, Olga Vladimirovna
Mastering English in Mathematical Discourse. – M.: MAKS Press, 2013. – 96 pp.

The book *Mastering English in Mathematical Discourse* is meant for undergraduate and postgraduate students who study at the faculties of mechanics and mathematics. Its goal is to help learners of English master the language of mathematical discourse. In this case, it is mainly particular terminology and a certain grammatical structure of scientific texts whose knowledge is required from specialists in mathematics and mechanics.

The book consists of 10 original articles taken from contemporary British and American encyclopedias and magazines. All texts come from the same field of science and are a perfect example of English for Specific Purposes, which forms the basis for academic writing.

Every text is followed by exercises aimed at practising the main aspects of English — its phonetics, lexis, and grammar. The exercises include: discussing the text and answering the questions; reading out the transcription of words and defining their meaning in English; word formation, choosing the correct synonyms, and translation of sentences from Russian into English using vocabulary from the text; revision of the key grammatical constructions that most often present difficulties to students.

The book can be used as the main or supplementary teaching material for intermediate or upper-intermediate learners of English.

C O N T E N T S

Text № 1: Fractals[1]

Read and translate the text.

A fractal in mathematics is any of a class of complex geometric shapes that commonly have "fractional dimension," a concept first introduced by the mathematician Felix Hausdorff in 1918. Fractals are distinct from the simple figures of classical, or Euclidean, geometry—the square, the circle, the sphere, and so forth. They are capable of describing many irregularly shaped objects or spatially nonuniform phenomena in nature such as coastlines and mountain ranges. The term *fractal*, derived from the Latin word *fractus* ("fragmented," or "broken"), was coined by the Polish-born mathematician Benoit B. Mandelbrot.

Although the key concepts associated with fractals had been studied for years by mathematicians, and many examples, such as the Koch or "snowflake" curve were long known, Mandelbrot was the first to point out that fractals could be an ideal tool in applied mathematics for modelling a variety of phenomena from physical objects to the behaviour of the stock market. Since its introduction in 1975, the concept of the fractal has given rise to a new system of geometry that has had a significant impact on such diverse fields as physical chemistry, physiology, and fluid mechanics.

Many fractals possess the property of self-similarity, at least approximately, if not exactly. A self-similar object is one whose component parts resemble the whole. This reiteration of details or patterns occurs at progressively smaller scales and can, in the case of purely abstract entities, continue indefinitely, so that each part of each part, when magnified, will look basically like a fixed part of the whole object. In effect, a self-similar object remains invariant under changes of scale—i.e., it has scaling symmetry. This fractal phenomenon can often be detected in such objects as snowflakes and tree barks. All natural fractals of this kind, as well as some mathematical self-similar ones, are stochastic, or random; they thus scale in a statistical sense.

Another key characteristic of a fractal is a mathematical parameter called its fractal dimension. Unlike Euclidean dimension, fractal dimension is generally expressed by a noninteger—that is to

[1] From *Encyclopaedia Britannica* 2006 Ultimate Reference Suite DVD

say, by a fraction rather than by a whole number. Fractal dimension can be illustrated by considering a specific example: the snowflake curve defined by Helge von Koch in 1904. It is a purely mathematical figure with a six-fold symmetry, like a natural snowflake. It is self-similar in that it consists of three identical parts, each of which in turn is made of four parts that are exact scaled down versions of the whole. It follows that each of the four parts itself consists of four parts that are scaled down versions of the whole. There would be nothing surprising if the scaling factor were also four, since that would be true of a line segment or a circular arc. However, for the snowflake curve, the scaling factor at each stage is three. The fractal dimension, D, denotes the power to which 3 must be raised to produce 4—i.e., $3^D = 4$. The dimension of the snowflake curve is thus $D = {}^{\log 4}/_{\log 3}$, or roughly 1.26. Fractal dimension is a key property and an indicator of the complexity of a given figure.

Fractal geometry with its concepts of self-similarity and non-integer dimensionality has been applied increasingly in statistical mechanics, notably when dealing with physical systems consisting of seemingly random features. For example, fractal simulations have been used to plot the distribution of galaxy clusters throughout the universe and to study problems related to fluid turbulence. Fractal geometry also has contributed to computer graphics. Fractal algorithms have made it possible to generate lifelike images of complicated, highly irregular natural objects, such as the rugged terrains of mountains and the intricate branch systems of trees.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. What do you know about mathematicians Felix Hausdorff, Benoit B. Mandelbrot, and Helge von Koch?
2. What are the main features of Euclidean geometry?

■ **Phonetics**

Read the words below and define them in English.

[d*a*ɪˈmenʃn]
[əˈsəʊʃieɪt/əˈsəʊsieɪt]
[aɪˈdiːəl]
[vəˈraɪəti]
[bɪˈheɪvjə]
[d*a*ɪˈvɜːs]
[əˈprɒksɪmətli]
[riˌɪtəˈreɪʃn]
[əˈkɜː]
[ˈpjʊəli]
[ˈentəti]
[stəˈkæstɪk]
[ˈsɜːkjʊlə]
[ˈtɜːbjʊləns]
[ˈælgərɪðəm]
[təˈreɪn]

■ **Vocabulary**

I. ***Study the list of the words and expressions from the text.***

to introduce the concept of sth.
to be distinct from sth.
to be capable of (doing) sth.
phenomenon (pl. phenomena)
to be derived from sth.
to be coined by sb.
to be the first to do sth.
to point (sth.) out
to give rise to sth.
to occur
in the case of sth.
under changes of sth.
in turn
it follows that
to denote
the distribution of sth.
to contribute to sth.
to make sth. possible

II. Translate the following groups of words into English using their derivative forms.[2]

- представлять (вводить) — введение — вводный
- отличный (явный) — неявный — отличительный — отличие
- полученный (заимствованный) — получать — получение (словообразование) — производная
- распределение — распределять — распределительный (дистрибутивный) — распределитель
- способствовать (вносить вклад) — вклад — вкладчик (соучастник, соавтор)

[2] *E.g.* наука — научный — научно — ученый
science — scientific — scientifically — scientist

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian. [3]

1. Continuous and discrete transformations create corresponding types of symmetries.
2. Kinetics attempts to explain or predict the motion that will take place in a given situation.
3. The mathematical use of the term 'abscissa' was apparently invented by Leibniz around 1855.
4. Suppose points D, E, F are respectively on lines BC, AC, AB and each point is different from the vertices of AABC.
5. It has already been indicated that mathematics is an essential discipline because of its practical role to the individual and society.
6. Calculus comes from the Latin word for "pebble" and in its most general sense can mean any method or system of calculation.
7. How are we able to acquire knowledge of mathematical truths?

IV. Translate the following sentences into English using the words and expressions from the list.

1. Открытия Галилея, Кеплера и Ньютона способствовали развитию гелиоцентризма.
2. Задачник состоит из трех глав, которые, в свою очередь, разбиты на параграфы.
3. Из этого следует, что выражение xn+yn = zn не может иметь решений в натуральных числах при n>2.
4. В случае нормального распределения вероятности таких двух событий практически не отличаются.
5. Теория бифуркаций изучает изменения качественной картины при изменении параметров, от которых зависит система.
6. В 17 в. Декарт благодаря методу координат сделал возможным изучение свойств геометрических фигур с помощью алгебры.

[3] In all exercises of this type, the change of the grammatical structure is sometimes required.

7. Индийские математики были первыми, кто ввел понятие «ноль».

V. Fill in the gaps with the following synonymous verbs. [4]

to invent — to discover — to devise — to create — to coin

1. Some people believe the universe was ... by a big explosion.
2. Archimedes spent most of his efforts in mathematics, where he ... ways to calculate areas and volumes, defined π and formulated integral calculus.
3. George Boole was a self-taught mathematician who … the power of mathematics early in life and became a leading figure in mathematical circles.
4. Leibniz ... the word "transcendental" in mathematics.
5. Though Einstein did not ... the atomic bomb, the equation $E=mc^2$ laid the theoretical background for it.

■ Grammar

I. Present Perfect / Past Simple / Past Perfect

a) Comment on the use of the underlined tenses in the following sentences from the text.

1. Although the key concepts associated with fractals had been studied for years by mathematicians, and many examples, such as the Koch or "snowflake" curve were long known, Mandelbrot was the first to point out that fractals could be an ideal tool in applied mathematics for modelling a variety of phenomena from physical objects to the behaviour of the stock market.
2. Since its introduction in 1975, the concept of the fractal has given rise to a new system of geometry that has had a significant impact on such diverse fields as physical chemistry, physiology, and fluid mechanics.

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[4] In all exercises of this type, use the dictionary to look up typical collocations with the synonyms given.

3. Fractal geometry with its concepts of self-similarity and noninteger dimensionality has been applied increasingly in statistical mechanics…For example, fractal simulations have been used to plot the distribution of galaxy clusters throughout the universe and to study problems related to fluid turbulence. Fractal geometry also has contributed to computer graphics. Fractal algorithms have made it possible to generate lifelike images of complicated, highly irregular natural objects, such as the rugged terrains of mountains and the intricate branch systems of trees.

b) Open the brackets using Present Perfect (Continuous), Past Simple, or Past Perfect.

1. Archimedes (to found) hydrostatics in about 250 BC when, according to legend, he (to leap) out of his bath and (to run) naked through the streets of Syracuse crying "Eureka!"; it (to undergo) rather little development since.
2. The universe as a whole (to cool) ever since it (to emerge) from the fireball of the Big Bang.
3. Mathematicians (to interest) in the topic of decision-making since Girolamo Cardano (to explore) the ethics of gambling in his *Liber de Ludo Aleae* of 1564.
4. By the 10th century, Muslim mathematicians (to develop) and (to apply) the theory of trigonometric functions as well as spherical trigonometry. They (to use) symbols to describe the binomial theorem, and (to use) decimals to express fractions that (to aid) accurate solution of complex problems.
5. Two researchers from the Institut de Mathématiques de Luminy recently (to make) an important breakthrough regarding a conjecture that (to formulate) in 1968 by the Russian mathematician Alexandre Gelfond concerning the sum of digits of prime numbers.
6. Before Copernicus, astronomers (to try) to account for the observed motions of heavenly bodies by imagining that they (to rotate) on crystal spheres centred on the Earth.
7. The systematic study of mathematics in its own right (to begin) with the Ancient Greeks between 600 and 300 BC. Mathematics since (to extended) greatly, and there (to be) a

fruitful interaction between mathematics and science, to the benefit of both.

8. Scientists (to make) many discoveries about the origins of our 13 billion-year-old universe but many scientific mysteries remain. What exactly (to happen) during the Big Bang, when rapidly evolving physical processes (to set) the stage for gases to form stars, planets and galaxies?

II. Irregular Plural Nouns of Latin and Greek origin

E.g. They [fractals] are capable of describing many irregularly shaped objects or spatially nonuniform *phenomena* in nature such as coastlines and mountain ranges. — This fractal *phenomenon* can often be detected in such objects as snowflakes and tree barks.

Grammar Note:

Singular form	Plural form
- on [ən] *phenomen**on***	- na [nə] *phenomen**a***
- is [sɪs] *bas**is***	- es [siːz] *bas**es***
- um [əm] *dat**um***	- a [ə] *dat**a***
- ex [eks], - ix [ɪks] *ap**ex**, matr**ix***	- ices [ɪsiːz] *ap**ices**, matr**ices***
- a [ə] *antenn**a***	- ae [iː] *antenn**ae***
- us [əs] *nucle**us***	- i [aɪ] *nucle**i***

a) Translate the following phrases into English.

1. правильный многогранник — выпуклые многогранники
2. вершина угла — вершины треугольника
3. координатная ось — оси симметрии
4. интересная гипотеза — выдвигать гипотезы
5. индекс подгруппы — индексы чисел
6. вывести формулу — представить формулы
7. угловой момент — сумма всех моментов
8. цвет спектра — оптические спектры
9. радиус кривизны — равные радиусы
10. фокус оптической системы — фокусы кривой
11. уравнения тора — комплексные торы

12. геометрическое место точек — геометрические места точек

b) Translate the sentences into English.

1. Открытые математические проблемы часто имеют форму гипотез, которые предположительно верны, но нуждаются в доказательстве.
2. Векторная сумма всех моментов импульса относительно любой неподвижной точки для замкнутой системы остается постоянной со временем.
3. Точки пересечения эллипса с осями являются его вершинами.
4. Эллипсом называется ГМТ плоскости, сумма расстояний которых до двух фиксированных точек плоскости, называемых фокусами, есть величина постоянная.
5. Сектор — это часть круга, заключающаяся между двумя радиусами и дугою.
6. В книге представлены основные формулы алгебры, геометрии и тригонометрии.
7. Исторически раньше всех прочих спектров было начато исследование оптического спектра.
8. Правильные многогранники известны с древнейших времён.

Text № 2: Chaos[5]

Read and translate the text.

Chaos in mechanics and mathematics is apparently random or unpredictable behaviour in systems governed by deterministic laws. A more accurate term, "deterministic chaos," suggests a paradox because it connects two notions that are familiar and commonly regarded as incompatible. The first is that of randomness or unpredictability, as in

[5] From *Encyclopaedia Britannica* 2006 Ultimate Reference Suite DVD

the trajectory of a molecule in a gas or in the voting choice of a particular individual from out of a population. In conventional analyses, randomness was considered more apparent than real, arising from ignorance of the many causes at work. In other words, it was commonly believed that the world is unpredictable because it is complicated. The second notion is that of deterministic motion, as that of a pendulum or a planet, which has been accepted since the time of Isaac Newton as exemplifying the success of science in rendering predictable that which is initially complex.

In recent decades, however, a diversity of systems have been studied that behave unpredictably despite their seeming simplicity and the fact that the forces involved are governed by well-understood physical laws. The common element in these systems is a very high degree of sensitivity to initial conditions and to the way in which they are set in motion. For example, the meteorologist Edward Lorenz discovered that a simple model of heat convection possesses intrinsic unpredictability, a circumstance he called the "butterfly effect," suggesting that the mere flapping of a butterfly's wing can change the weather. A more homely example is the pinball machine: the ball's movements are precisely governed by laws of gravitational rolling and elastic collisions — both fully understood — yet the final outcome is unpredictable.

In classical mechanics the behaviour of a dynamical system can be described geometrically as motion on an "attractor." The mathematics of classical mechanics effectively recognized three types of attractor: single points (characterizing steady states), closed loops (periodic cycles), and tori (combinations of several cycles).

Sporadically during the 1930s and '40s, and with increasing frequency in the 1960s, mathematicians and scientists began to notice that simple differential equations can sometimes possess extremely complex solutions. The American mathematician Stephen Smale, continuing to develop Poincaré's insights on qualitative properties of differential equations, proved that in some cases the behaviour of the solutions is effectively random. Even when there is no hint of randomness in the equations, there can be genuine elements of randomness in the solutions.

In the 1960s Stephen Smale discovered a new class of "strange attractors". On strange attractors the dynamics is chaotic. Later it was recognized that strange attractors have detailed structure on all scales of magnification; a direct result of this recognition was the development of the concept of the fractal (a class of complex geometric shapes that commonly exhibit the property of self-similarity), which led in turn to remarkable developments in computer graphics.

By the end of the 20th century, chaos was found not just in the motion of the planets but in weather, disease epidemics, ecology, fluid flow, electrochemistry, acoustics, even quantum mechanics. The most important feature of the new viewpoint on dynamics—popularly known as chaos theory but really just a subdiscipline of dynamical systems theory—is not the realization that many processes are unpredictable. Rather, it is the development of a whole series of novel techniques for extracting useful information from apparently random behaviour. Chaos theory has led to the discovery of new and more efficient ways to send space probes to the Moon or to distant comets, new kinds of solid-state lasers, new ways to forecast weather and estimate the accuracy of such forecasts, and new designs for heart pacemakers. It has even been turned into a quality-control technique for the wire- and spring-making industries.

PRACTICE

■ Discussion

Provide information on the following points.

1. What is a dynamical system in mechanics?
2. How can the terms “steady state”, “closed loop”, and “torus” be defined?
3. What are Edward Lorenz and Stepen Smale famous for?

■ **Phonetics**

<u>Read the words below and define them in English.</u>

[dɪˌtɜːmɪˈnɪstɪk]	[ɪgˈzemplɪfaɪ]	[ˈdʒenjuɪn]
[ˈkeɪɒs]	[ˈsɜːkəmstəns]	[ɪgˈzɪbɪt]
[ˌʌnprɪˈdɪktəbl]	[ˈkærəktəraɪz]	[əˈkuːstɪks]
[ˈmɒlɪkjuːl]	[ˈtɒraɪ]	[tekˈniːk]
[trəˈdʒektəri]	[ˈsaɪkl]	
[ˈpendjʊləm]	[ˈkwɒlɪtətɪv]	

■ **Vocabulary**

I. ***<u>Study the list of the words and expressions from the text.</u>***

to be governed by sth.	despite (doing) sth.
accurate	simplicity
to suggest	to possess
familiar	circumstance
incompatible	to be recognized that
to be at work	to exhibit the property of sth.
in other words	a technique for (doing) sth.
to accept	

<u>II. Translate the following groups of words into English using their derivative forms.</u>

- точный — неточный — точность
- принимать — приемлемый — неприемлемый
- простота — простой — упрощать
- признанный — признавать (узнавать) — признание (узнавание)

<u>III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.</u>

1. The Babylonian triples <u>make you think</u> that an integer square can be expressed as the sum of two integer squares.
2. All theories can always be replaced by more <u>exact</u>, generalized statements if a disagreement of theory with observed data is ever found.

3. Such collisions are controlled by Newton's laws of motion.
4. The only force having an effect here is the gravitational force between the Sun and planets.
5. An axiom is a statement which is taken to be true without proof.
6. Physicists gladly welcomed Schrödinger's alternative wave mechanics since it involved well-known concepts and equations.
7. Although relatively simple, algebra has a powerful problem-solving tool used in fields ranging from engineering to business.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Квантовый объект может проявлять как волновые, так и корпускулярные свойства в зависимости от условий эксперимента.
2. Из соображений простоты Пуанкаре стремился сохранить евклидову геометрию, а Эйнштейн, наоборот, отказаться от нее.
3. Такую задачу решить невозможно никогда и ни при каких обстоятельствах.
4. Другими словами, познание в математике есть часть познания реального мира.
5. Надо признать, что, несмотря на широкие возможности для научно-исследовательской работы по математической биологии, достигнутые к настоящему времени успехи в этой области очень малы.
6. Во время научной революции старая парадигма замещается целиком или частично новой парадигмой, несовместимой со старой.
7. В отличие от классической математики, математическое программирование занимается математическими методами решения задач нахождения наилучших вариантов из всех возможных.

V. Fill in the gaps with the following synonymous adjectives.

accurate — exact — precise

1. To be more ..., we say that the function *f* is continuous at some point *c* if the following three requirements are satisfied.
2. In mathematics, we often stress getting an ... answer.
3. The sundial developed into a more ... instrument with the introduction of the hemispherical sundial around 300 BC.
4. Unlike astronomy, astrology cannot be described as an ... science.
5. Scientists are getting close to seeing the Big Bang at the ... moment it exploded, even before it happened.
6. Until the second millennium AD, estimations of π were ... to fewer than 10 decimal digits.

VI. Translate these collocations with 'well + Past Participle' into English.

E.g. ...the forces involved are governed by *well-understood* physical laws...

1. Наиболее распространенными и хорошо изученными являются математические модели, описывающие зависимости между данными числового типа.
2. Овладеть в полной мере высокими технологиями может только человек, хорошо подготовленный в области математики и физики.
3. Это широко известный учебник по дисциплине «теория вероятностей».
4. Книга содержит много удачно подобранных примеров, а также задач для самостоятельного решения.
5. В математике понятие выпуклой фигуры имеет четко определенный смысл.

■ **Grammar**

I. The verb 'to suggest'

Grammar Note:

1. *to suggest that smb. should do sth.*: If there is a mechanical problem, we suggest that you should contact the manufacturer directly.
2. *to suggest (that) smb. do sth.:* If there is a mechanical problem, we suggest (that) you contact the manufacturer directly.
3. *to suggest doing sth.*: If there is a mechanical problem, we suggest contacting the manufacturer directly.
4. *to suggest sth.:* He suggested an interesting explanation of this phenomenon.

a) Translate the sentences into English using the verb "to suggest".

1. Ферма предложил правила нахождения экстремумов многочленов.
2. Мы предлагаем решить эту задачу следующим образом.
3. Автор предлагает читателю задуматься над этими вопросами.
4. Ученые предлагают создать новый спутник для изучения Солнца.
5. Эйнштейн предложил рассматривать свет как поток частиц, которые он назвал «квантами света».

II. "That of sth./ those of sth." vs "one/ones"

Grammar Note:

that of sth./those of sth. (formal)	***one/ones***
used to refer to a particular person or thing of the general type that has just been mentioned	used to mean someone or something of a type that has already been mentioned or is known about

E.g. A more accurate term, "deterministic chaos," suggests a paradox because it connects two *notions* that are familiar and commonly regarded as incompatible. The first is *that of* (= the notion of) randomness or unpredictability...	***E.g.*** It was a *problem*, but not a major *one* (=problem).

a) <u>Translate the sentences into Russian.</u>

1. An idea central to modern cybernetics and many other fields is *that of* feedback.
2. The influence of Aryabhata on Indian mathematics almost rivals *that of* Euclid in the West.
3. The vertices of a regular icosahedron are *those of* three mutually orthogonal golden rectangles.
4. Is there a world, completely separate from our physical *one*, that is occupied by the mathematical entities?
5. Problems like *the one* listed above certainly seem to be of this kind, but so far no one has managed to prove that any of them really are so hard as they appear.

b) <u>Translate the sentences into English using "that of sth./ those of sth." or "one/ones".</u>

1. Основное понятие дифференциального исчисления — это понятие производной.
2. Рациональные числа — это те, которые можно представить в виде обыкновенной дроби.
3. В математике обычно используется вещественный анализ, тот, который проходят на курсах по матанализу.
4. Открытие Максвелла сравнимо по научной значимости с открытием закона всемирного тяготения Ньютона.
5. Среди предельных теорем теории вероятностей первыми нужно назвать теоремы Муавра-Лапласа о предельном распределении отклонения частоты события от его вероятности.

III. Active and Passive Participles

Active Participle: E.g. In conventional analyses, randomness was considered more apparent than real, *arising* from ignorance of the many causes at work.
Passive Participle: E.g. Chaos in mechanics and mathematics is apparently random or unpredictable behaviour in systems *governed* by deterministic laws.

a) Translate the sentences into English using Active or Passive Participles.

1. В основе аналитической геометрии лежит так называемый метод координат, впервые примененный Декартом.
2. Однородные координаты — координаты, обладающие тем свойством, что определяемый ими объект не меняется при умножении всех координат на одно и то же ненулевое число.
3. Отрезки касательных к окружности, проведенных из одной точки, равны и составляют равные углы с прямой, проходящей через эту точку и центр окружности.
4. Вычисляя длины кривых линий, можно брать любые вписанные в них ломаные.
5. Проективная геометрия дополняет Евклидову, предоставляя красивые и простые решения для многих задач, осложненных присутствием параллельных прямых.

Text №3: Mathematical Techniques in Astronomy[6]

Read and translate the text.

Mathematics is and always has been of central importance to astronomy. As soon as observations became quantified the possibility or calculation and prediction based on observations was open to

[6] http://www.hps.cam.ac.uk/starry/mathematics.html

astronomers. Mathematical developments were both applied to and motivated by astronomical calculations, and many of the most famous astronomers were also mathematicians and vice versa. Although techniques have become increasingly complex, the majority of mathematical astronomical techniques are concerned with positioning and calculation of relative distances of heavenly bodies. The basis of this is spherical trigonometry, which allows calculations on the celestial sphere based on observations taken from an observer on earth. The projection of the celestial sphere onto a flat surface allowed the construction of instruments such as the astrolabe and the mapping of the heavens. Techniques for increasingly accurate calculation were crucial to the development of astronomy as an exact science. It must be borne in mind, however, that not everyone studying or using astronomy was aware of or capable of applying the latest mathematical techniques. For example, there is evidence of a monk in northern France in the 12th century positioning stars relative to architectural landmarks in his monastery, such as the windows along the dormitory wall.

The first developments of mathematical astronomy came during the Mesopotamian and Babylonian civilisations, especially during the Seleucid Kingdom (ca. 320BC to ca. 620AD). Techniques were developed for prediction of eclipses and positions of the heavenly bodies, in terms of degrees of latitude and longitude, and measured relative to the sun's apparent motion. Tables were calculated and written for reference, based on arithmetic methods. These tables were available to the Greeks, who adopted many elements of the approach taken by the Babylonians (the sexagesimal system of calculation remained in use in astronomy right up to the Early Modern period) in areas of maths. Many of the Egyptian methods developed for surveying could also be applied to mathematical problems in astronomy.

Among the techniques developed and improved by the Greeks were geometric solutions of triangulation problems, including their application to three dimensions. Systems based on combinations of uniform circular motions were proposed to explain and predict the motion of the heavenly bodies, Eudoxus being among the first to suggest a model based on the rotations of concentric spheres. This kind of model wasn't very accurate at predicting positions, but generated a new type of curve, the hippopede, which provided a new area of research for geometry. Other popular types of model were based on epicycles (planets orbit along a circular path whose centre is at or near the earth) or eccentrics (the planets rotate around the sun, which in turn rotates around the earth). The development of ever more complex models of the celestial sphere required more complex calculations, and more sophisticated geometry to back them up. Textbooks on the sphere were written, consolidating mathematical techniques for astronomy. These were called spherics.

Mathematicians and astronomers including Hipparchus developed techniques for the measurement of angles, and tables for calculations with these angles. Archimedes and Aristarchus studied the numerical ratios in triangles, and sophisticated theories and treatises on the application of these new theories to astronomy were published. These texts were the precursors of spherical trigonometry, which became vital to astronomy. Ptolemy's 'Almagest' summarised and advanced these techniques and Hipparchus and Menelaus of Alexandria produced tables of what would today be called values of the sine function.

The learning of the Greeks was transmitted to Arabic areas, who in turn added Indian and Chinese mathematical and astronomical texts to the corpus of works available. The Arab scholars improved and combined the methods they read about, predictive astronomy being central to many aspects of Islam. Important advances in mathematical techniques included al-Khwarizmi's predictions of the times of visibility of the new moon, and calculations of the qibla or direction of Mecca, in which to pray, from astronomical observations.

The Arabs worked with the sexagesimal system inherited from the Babylonians via the Greeks, but often converted numbers to the decimal system for complex calculations since it was easier. They called base-60 numbers the arithmetic of the astronomers. They also incorporated elements of the Indian system including zero to the number system.

Thabit and Ibrahim developed geometric methods for sundials, including the solution of conic sections and the application of this to the construction of sundials. In the late 10th century Abu al-Wafa and Abu Nasr Mansur proved theories of plane and spherical trigonometry and derived the laws of sines and tangents. Highly accurate tables and techniques for the calculation of trigonometric problems were produced. Abu Nasr's pupil, al-Biruni (973-1050) applied these techniques with great success to geographic and astronomical problems.

The Greeks had developed the astrolabe, but the Arabs applied their new techniques to its improvement and the development of universal astrolabes that did not require separate plates for each degree of latitude. They perfected techniques for the projection of the celestial sphere onto a flat plane, described by the Greeks, and the marking of scales and lines to enable calculations of positions on the celestial sphere to be carried out on a flat surface.

From the end of the 10th century West European scholars became increasingly interested in the writings of the Greeks and Arabs, and translations were made of important texts. Astronomy was part of the quadrivium (arithmetic, geometry, astronomy and music) of mathematical subjects which were taught to students in church educational institutions. With the founding of the universities came an increased study of Greek and Arab texts, including the mathematics of astronomy. The techniques of spherical trigonometry and other important applied geometry techniques were studied, commented on, and used to calculate astronomical tables for West European latitudes. For the next few centuries the majority of work on mathematical astronomy concentrated on consolidation and improvement of existing techniques.

By the 17th century mathematics was becoming more institutionalised, with increasingly efficient means of communication between mathematicians and their colleagues. This enabled advances in maths to become widely known and applied more quickly. The publication in 1614 of John Napier's work on logarithms was quickly adopted as a way of simplifying mathematical calculations in astronomy, and new logarithmic, trigonometric and astronomical tables followed. These included Kepler's Rudolphine Tables, which made great use of the new techniques and was based on elliptic orbits about the sun. The accuracy of tables and techniques increased quickly, as did the accuracy with which the heavenly bodies could be observed.

The development of calculus in the 17th century allowed calculation of changing quantities with greater accuracy and ease, including quantities like the speed a body is moving at. Developments in the representation of geometric quantities by algebraic expressions facilitated the further refinement of astronomical models. Increased understanding of the forces at work in the universe enabled calculations and predictions to take account of why things behaved as they did more effectively, and build this into the mathematical models used for calculation.

The development of mathematical techniques for astronomy did not stop at the end of the 17th century, although much of the groundwork had been laid. In the following centuries more sophisticated mathematical methods were developed, building on the fundamentals of trigonometry and calculus and were applied to astronomy. The principles of spherical trigonometry underpin the calculations of modern astronomy, although the calculations are now carried out by computers rather than slide rules.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. What does spherical trigonometry deal with?
2. How is triangulation defined?
3. How did Mesopotamians and Babylonians develop mathematics?
4. What is an astrolabe?
5. What is a hippopede?
6. What is John Napier famous for?
7. What did "the Rudolphine Tables" serve for?

■ Phonetics

Read the words below and define them in English.

[ˈkwɒntɪfaɪ]	[əˈpærənt]	[ˈsʌməraɪz]
[ˌvaɪs(i) ˈvɜːsə]	[ˌseksəˈdʒesɪml]	[ˈvaɪə/ˈviːə]
[məˈdʒɒrəti]	[traɪˌæŋgjuˈleɪʃn]	[pəˈfekt]
[səˈlestiəl]	[ˈreɪʃiəʊ]	[ˈskɒlə]
[ˈæstrəleɪb]	[səˈfɪstɪkeɪtɪd]	[ˈkɒliːg]
[ɪˈklɪps]	[ˈtriːtɪs/z]	[fəˈsɪlɪteɪt]

■ **Vocabulary**

I. Study the list of the words and expressions from the text.

to be of (central) importance to sth.
vice versa
the majority of sth./sb.
to be concerned with sth.
to consolidate
to be crucial to sth.
to bear sth. in mind
to be aware of sth.
relative to sth.
in terms of sth.
to be available to sb.
to adopt
to take an approach to sth.
to generate
sophisticated
to back sth.up
treatise
to summarise
to inherit
via
to convert to sth.
to incorporate
to enable sth./sb. to do sth.
refinement
to take account of sth./to take (sth.) into account
to carry out calculations

II. Translate the following groups of words into English using their derivative forms.

- относительный — относительность — относиться
- производить — производитель (генератор)— порождающий (генеративный) — поколение — выродившийся (дегенеративный)
- принимать (усваивать, усыновлять) — принятый (приемный) — принятие (усыновление)
- подытоживать — сумма — сводка (краткое изложение)

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. His claims are supported by recent research.
2. Clearly, the hypothesis does need some small improvement, in the light of these surprising results.
3. New electrical equipment includes all the latest safety features.
4. Although arithmetic computation is extremely important to accountants, they mostly deal with verifying that computations are correct through a system of double checks.

5. The scientific method combines previous knowledge.
6. Bohr knew about the fact that the quantum conditions spoil in some way the consistency of Newtonian mechanics.
7. Most distinguished physicists and astrophysicists returned eagerly to the question of how stars like the sun produce energy.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Нам представляется, что к решению этой проблемы нужно использовать другой подход.
2. С точки зрения современной математики, числа Фибоначчи представляют собой одну из разновидностей последовательности Люка, формулы Бине и возвратной последовательности одновременно.
3. Мы должны не забывать, что это открытие принципиально важно для науки.
4. С учетом упомянутых выше свойств, матрицы образуют кольцо относительно операций сложения и умножения.
5. Сейчас исследователям доступны высокотехнологичные методы решения многих задач.
6. Большое количество встроенных в систему математических функций позволяет ученым решать сложные прикладные и теоретические задачи, а также производить много вычислений.
7. Я хотел бы подытожить результаты своего исследования.
8. Вопрос о роли математики в искусстве волновал еще древних греков, причем свой интерес они унаследовали от предшествующих цивилизаций.
9. Рассмотрены случаи, как преобразовать куб в сферу и наоборот.
10. В трактате «Измерение круга» Архимед предлагает метод определения π, который использовался до конца 17 в.

V. Fill in the gaps with the following synonymous verbs.

to adopt — to accept — to take — to receive

1. We are now compelled to … the fact that there is no such thing as an absolute proof.

2. As a student of Charles Hermite, Poincaré … his doctorate in mathematics from the University of Paris in 1879.
3. In this paper, we … the functional approach to algebra which widens the meaning of algebraic thinking.
4. Mathematical models can … many forms.
5. An axiom is a statement that mathematicians … as being true without demanding proof.
6. In 270 BC, Aristarchus of Samos came up with the first known heliocentric model of the solar system but no one … it seriously in those days.
7. The International System of Units, or SI, was … by the 11th General Conference on Weights and Measures in 1960.

■ Grammar

I. Participle Clause (Absolute Clause)

Grammar note:
Participle Clause (Absolute Clause) — a non-finite or verbless clause containing its own subject, separated from the rest of the sentence by a comma (or commas) and not introduced by a subordinator. Absolute clauses with Present Participle or Past Participle can be transformed into a separate clause logically linked to the other one by a conjunction ("when", "where", "because", "as", "so that", etc.). Participle clauses are used particularly in formal or literary writing.
E.g. The Arab scholars improved and combined the methods they read about, predictive *astronomy being central* to many aspects of Islam. = The Arab scholars improved and combined the methods they read about, *so that* predictive *astronomy was central* to many aspects of Islam.
E.g. Systems based on combinations of uniform circular motions were proposed to explain and predict the motion of the heavenly bodies, *Eudoxus being among the first to suggest* a model based on the rotations of concentric spheres. = Systems based on combinations of uniform circular motions were proposed to explain and predict the motion of the heavenly bodies, *and Eudoxus was among the first to suggest* a model based on the rotations of concentric spheres.

E.g. *With* a substantial number of *mathematicians* these days *accepting* the use of computational and experimental methods, mathematics has indeed grown to resemble much more the natural sciences. = *As* a substantial number of *mathematicians* these days *accept* the use of computational and experimental methods, mathematics has indeed grown to resemble much more the natural sciences.

a) Turn the sentences with Participle Clause into two complex ones joined by a conjunction. Translate the sentences into Russian.

1. For smaller angular measurements, the right angle is divided into 90 equal parts, each part being one degree of arc.
2. The Greek astronomer Eratosthenes used a sexagesimal system to divide a circle into 60 parts in order to devise an early geographic system of latitude, with the horizontal lines running through well-known places on the earth at the time.
3. Seconds were once derived by dividing astronomical events into smaller parts, with the International System of Units (SI) at one time defining the second as a fraction of the mean solar day and later relating it to the tropical year.
4. One may distinguish fractions from rational numbers, the latter being equivalence classes of fractions.
5. This science is usually divided into two parts, theoretical geometry and practical geometry, the former showing the principles of the science, and the latter their application.

II. Auxiliary verbs "to do", "to be", "to have" in comparisons with "as" and "than"

Grammar note:

In formal written language comparisons with "as" and "than" are often accompanied by inversion.

E.g. *The accuracy* of tables and techniques *increased* quickly, *as did the accuracy* with which the heavenly bodies could be observed.

E.g. *Algebra requires* a stronger understanding of the properties of operations *than does arithmetic.*

a) ***Make a complex sentence using "as" or "than" and the word in brackets.***

1. Fractions were soon found to obey the same rules for manipulations (the integers).
2. Matrix multiplication is slightly less intuitive for the beginning student of linear algebra (scalar multiplication).
3. In science, for example, understanding sources of error and their impact on the confidence of conclusions is vital (the use of mathematical models in other disciplines).
4. Riemannian geometries are more useful in some situations (Euclidian geometry).
5. The nature of mathematics, its content and its aims, have changed throughout history (the methods used in order to keep pace with it).
6. These questions are addressed in an appendix, which assumes a much greater level of mathematical knowledge (the main text).

Text №4: Number Game[7]

Read and translate the text.

A number game is any of various puzzles and games that involve aspects of mathematics. Mathematical recreations comprise puzzles and games that vary from naive amusements to sophisticated problems, some of which have never been solved. They may involve arithmetic, algebra, geometry, theory of numbers, graph theory, topology, matrices, group theory, combinatorics (dealing with problems of arrangements or designs), set theory, symbolic logic, or probability theory. Any attempt to classify this colourful assortment of material is at best arbitrary.

[7] From *Encyclopaedia Britannica* 2006 Ultimate Reference Suite DVD

At times it becomes difficult to tell where pastime ends and serious mathematics begins. An innocent puzzle requiring the traverse of a path may lead to technicalities of graph theory; a simple problem of counting parts of a geometric figure may involve combinatorial theory; dissecting a polygon may involve transformation geometry and group theory; logical inference problems may involve matrices. A problem regarded in medieval times—or before electronic computers became commonplace—as very difficult may prove to be quite simple when attacked by the mathematical methods of today.

Mathematical recreations have a universal appeal. The urge to solve a puzzle is manifested alike by young and old, by the unsophisticated as well as the sophisticated. An outstanding English mathematician, G.H. Hardy, observed that professional puzzle makers, aware of this propensity, exploit it diligently, knowing full well that the general public gets an intellectual kick out of such activities.

People have always taken delight in devising "problems" for the purpose of posing a challenge or providing intellectual pleasure. Thus, many mathematical recreations of early origin that have reappeared from time to time in new dress seem to have survived chiefly because they appeal to man's sense of curiosity or mystery. A few survived from the ancient Greeks and Romans: little was known about them during the Dark Ages, but a strong interest in such problems arose during the Middle Ages, stimulated partly by the invention of printing, partly by enthusiastic writers of arithmetic texts, and partly by the rivalry and disputations among early algebraists and scholars. Such activities were most prominent on the Continent, particularly in Italy and Germany. Notable contributors included Rabbi ben Ezra (1140), Fibonacci (Leonardo of Pisa; 1202), Robert Recorde (1542), and Gerolamo Cardano (1545).

Kinds of problems

The problems in general were of two kinds: those involving the manipulation of objects, and those requiring computation. The first required little or no mathematical skill, merely general intelligence and ingenuity, as for example, so-called decanting and difficult crossings problems. A typical example of the former is how to measure out one quart of a liquid if only an eight-, a five-, and a three-quart measure are available. Difficult crossings problems are exemplified by the dilemma of three couples trying to cross a stream in a boat that will hold only two persons, with each husband too jealous to leave his wife in the company of either of the other men. Many variants of both types of problems have appeared over the years.

Some examples

Problems involving computation also took on a variety of forms; some were as follows:

Finding a number

Think of a number, triple it, and take half the product; triple this and take half the result; then divide by 9. The quotient will be one-fourth the original number.

"God-Greet-You" problems

For example, in "God greet you, all you 30 companions," someone says: "If there were as many of us again and half as many more, then there would be 30 of us." How many were there?

The chessboard problem

How many grains of wheat are required in order to place one grain on the first square, 2 on the second, 4 on the third, and so on for the 64 squares?

The lion in the well

This is typical of many problems dealing with the time required to cover a certain distance at a constant rate while at the same time progress is hindered by a constant retrograde motion. There is a lion in a well whose depth is 50 palms. He climbs $^1/_7$ of a palm daily and slips back $^1/_9$ of a palm. In how many days will he get out of the well?

Courier problems

These are typified by the movements of bodies at given rates in which some position of these bodies is given and the time required for them to arrive at some other specified position is demanded.

About the middle third of the 20th century, there was a gradual shift in emphasis on various topics. Up to that time interest had focussed largely on such amusements as numerical curiosities; simple geometric puzzles; arithmetical story problems; paper folding and string figures; geometric dissections; manipulative puzzles; tricks with numbers and with cards; magic squares; those venerable diversions concerning angle trisection, duplication of the cube, squaring the circle, as well as the elusive fourth dimension. By the middle of the century, interest began to swing toward more mathematically sophisticated topics: cryptograms; recreations involving modular arithmetic, numeration bases, and number theory; graphs and networks; lattices, group theory; topological curiosities; packing and covering; flexagons; manipulation of geometric shapes and forms; combinatorial problems; probability theory; inferential problems; logical paradoxes; fallacies of logic; and paradoxes of the infinite.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. Why is G.H. Hardy referred to as an outstanding English mathematician?
2. What do you know about the following scientists mentioned in the text: Rabbi ben Ezra, Fibonacci, Robert Recorde, and Gerolamo Cardano?
3. Which properties does a magic square possess?
4. What is a flexagon?

■ Phonetics

Read the words below and define them in English.

[naɪˈiːv]	[ˌmediˈiːvl]	[ˌɪndʒəˈnjuːəti]
[ˈmeɪtrɪsiːz[	[ɜːdʒ]	[kwɔːt]
[ˈɑːbɪtrəri/ˈɑːbɪtri]	[ˈdɪlɪdʒəntli]	[ˈemfəsɪs]
[ˈtrævɜːs]	[ˌkjʊəriˈɒsəti]	[ˈfæləsi]
[ˌteknɪˈkæləti]	[ˈraɪvlri]	[pɑːm]
[ˈɪnfərəns]	[məˌnɪpjuˈleɪʃn]	[ˈkwəʊʃnt]

■ Vocabulary

I. Study the list of the words and expressions from the text.

to involve	to be manifested by sth.
to comprise	to exploit
at best	for the purpose of sth.
arbitrary	to pose sth.
at times	origin
to have a (universal) appeal	to appeal to sb.

to arise	to be exemplified by sth.
partly	as follows
particularly	a shift in sth./from sth. to sth.
the former/the latter	(to put/place/lay) emphasis on sth.

II. Translate the following groups of words into English using their derivative forms.

- вовлекать — вовлеченный — вовлечение
- происхождение — происходить — изначальный — изначально
- привлекать (призывать) — привлекательность (призыв, апелляция) — привлекательный
- акцент (подчеркивание, упор) — акцентировать (подчеркивать) — выразительный

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. In general relativity gravity is clearly shown by the curvature of space-time.
2. The most sophisticated examples of Mayan thought are illustrated by their calendar and the mathematics which preceded it.
3. The journal also features papers that make best use of recent advances in computing.
4. Another great Muslim mathematician Omar Khayyam asks the question of whether a ratio can be regarded as a number but leaves the question unanswered.
5. The books under discussion will interest scientists taking part in this research.
6. Occasionally a formula is solved for a specific variable when we are given the numerical values for the other variables.
7. With the increasing availability of ever cheaper, faster, and more powerful computers there was a significant, though gradual, change in the way mathematicians viewed their discipline.

<u>IV. Translate the following sentences into English using the words and expressions from the list.</u>

1. Деление <u>произвольных</u> целых чисел несущественно отличается от деления натуральных чисел.
2. Работа посвящена описанию явлений природы, относящихся к наукам о <u>происхождении</u>, эволюции и строении нашей планеты Земля.
3. <u>Первый</u> из двух подходов к решению задачи представляется более обоснованным, чем <u>второй</u>.
4. Полагают, что количественный язык и методы математики <u>в лучшем случае</u> содействуют объяснению явлений неорганической природы, но не могут дать ничего ценного в понимании процессов культурно-исторической и духовной жизни.
5. <u>Отчасти</u> Кант повторяет Декарта, представляя естествознание <u>следующим образом</u>: всякий предмет конструируется прежде всего как геометрическая фигура или тело.
6. Выдающийся немецкий математик Давид Гильберт, <u>в особенности</u> в своем докладе «Математические проблемы», делает <u>упор на</u> целостном характере математики как основе всего точного естественнонаучного познания.
7. Еще никогда ни одно открытие в области физики <u>не вызывало</u> такого <u>всеобщего интереса,</u> как открытие Рентгеном нового, до той поры неизвестного рода лучей.

<u>V. Use the following expressions with the noun "time" instead of the underlined phrases in the sentences below.</u>

- at times
- at a time
- at one time
- at the time
- at no time
- at all times

1. In the absence of algebraic symbolism, diagrams might have served as a combination of a statement and its proof in the most concise form available <u>at that particular point</u>.

2. Indeed physics, engineering, mathematics and statistics are <u>sometimes</u> so intertwined that they are virtually inseparable.
3. It may be best to only give the students one or two problems <u>on each occasion</u> to solve on their own.
4. The Hindus <u>during one period in the past</u> used a cross placed beside a number to indicate a negative quantity, as in the Bakhshali manuscript of possibly the 10th century.
5. Radial and tangential components <u>always</u> remain orthogonal to the axis of rotation.
6. <u>Never</u> are students let in on the secret that mathematics, like any literature, is created by human beings for their own amusement.

■ Grammar

I. Construction: "Subject + Active Verb + the Infinitive"

E.g. A problem regarded in medieval times as very difficult *may prove to be quite simple* when attacked by the mathematical methods of today. = *It may turn out that a problem* regarded in medieval times as very difficult *is quite simple* when attacked by the mathematical methods of today.
E.g. Thus, many mathematical recreations of early origin that have reappeared from time to time in new dress *seem to have survived* chiefly because they appeal to man's sense of curiosity or mystery. = Thus, *it seems that* many mathematical *recreations* of early origin that have reappeared from time to time in new dress *have survived* chiefly because they appeal to man's sense of curiosity or mystery.

a) <u>Replace the subordinate clause with the construction "Subject + Active Verb + the Infinitive".</u>

1. It turned out that the motion of celestial bodies was more complicated than Copernicus had proposed.
2. It seems that Pythagoras' theorem is one of the most important propositions in the realm of history.
3. It appears that mathematics is the language which describes reality's consistency in some sense.

4. It happens that now different branches of mathematics are relevant to different areas of physics but many mathematical tools can be applied to a wide class of problems.

b) Translate the sentences into English using the construction "Subject + Active Verb + the Infinitive".

1. Похоже, мы допустили много ошибок при наших подсчетах.
2. Если единственный закон, объясняющий какие-то данные, оказывается слишком сложным, то рассматриваемые данные на самом деле не подчиняются никакому закону.
3. Кажется, наука в полном смысле слова возникла не в XVII веке, а существовала еще в Древней Греции, начиная с Пифагора и заканчивая V веком.
4. Масштаб оказывается слишком малым, чтобы проверить, какую же геометрию имеет реальное пространство.
5. Упрощение дизъюнктивных форм может показаться кропотливым делом, особенно запоминание многочисленных правил.

Text №5: What is Experimental Mathematics? (Part I)

by Keith Devlin[8]

Read and translate the text.

In my last column I gave some examples of mathematical hypotheses that, while supported by a mass of numerical evidence, nevertheless turn out to be false. Mathematicians know full well that numerical evidence, even billions of cases, does not amount to conclusive proof. No matter how many zeros of the Riemann Zeta function are computed and observed to have real-part equal to 1/2, the

[8] http://www.maa.org/devlin/devlin_03_09.html

Riemann Hypothesis will not be regarded as established until an analytic proof has been produced.

But there is more to mathematics than proof. Indeed, the vast majority of people who earn their living "doing math" are not engaged in finding proofs as all; their goal is to solve problems to whatever degree of accuracy or certainty is required. While proof remains the ultimate, "gold standard" for mathematical truth, conclusions reached on the basis of assessing the available evidence have always been a valid part of the mathematical enterprise. For most of the history of the subject, there were significant limitations to the amount of evidence that could be gathered, but that changed with the advent of the computer age.

For instance, the first published calculation of zeros of the Riemann Zeta function dates back to 1903, when J.P. Gram computed the first 15 zeros (with imaginary part less than 50). Today, we know that the Riemann Hypothesis is true for the first ten trillion zeros. While these computations do not prove the hypothesis, they constitute information about it. In particular, they give us a measure of confidence in results proved under the assumption of RH.

Experimental mathematics is the name generally given to the use of a computer to run computations - sometimes no more than trial-and-error tests - to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions, that may themselves arise by computational means, including search.

Had the ancient Greeks (and the other early civilizations who started the mathematics bandwagon) had access to computers, it is likely that the word "experimental" in the phrase "experimental mathematics" would be superfluous; the kinds of activities or processes that make a particular mathematical activity "experimental" would be viewed simply as mathematics. On what basis do I make this assertion? Just this: if you remove from my above description the requirement that a computer be used, what would be left accurately describes what most, if not all, professional mathematicians have always spent much of their time doing!

Many readers, who studied mathematics at high school or university but did not go on to be professional mathematicians, will find that last remark surprising. For that is not the (carefully crafted) image of mathematics they were presented with. But take a look at the private notebooks of practically any of the mathematical greats and you will find page after page of trial-and-error experimentation (symbolic or numeric), exploratory calculations, guesses formulated, hypotheses examined, etc.

The reason this view of mathematics is not common is that you have to look at the private, unpublished (during their career) work of the greats in order to find this stuff (by the bucketful). What you will discover in their published work are precise statements of true facts, established by logical proofs, based upon axioms (which may be, but more often are not, stated in the work).

Because mathematics is almost universally regarded, and commonly portrayed, as the search for pure, eternal (mathematical) truth, it is easy to understand how the published work of the greats could come to be regarded as constitutive of what mathematics actually is. But to make such an identification is to overlook that key phrase "the search for". Mathematics is not, and never has been, merely the end product of the search; the process of discovery is, and always has been, an integral part of the subject. As the great German mathematician Carl Friedrich Gauss wrote to his colleague Janos Bolyai in 1808, "It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment."

In fact, Gauss was very clearly an "experimental mathematician" of the first order. For example, his analysis - while still a child - of the density of prime numbers, led him to formulate what is now known as the Prime Number Theorem, a result not proved conclusively until 1896, more than 100 years after the young genius made his experimental discovery.

For most of the history of mathematics, the confusion of the activity of mathematics with its final product was understandable: after all, both activities were done by the same individual, using what to an outside observer were essentially the same activities - staring at a sheet of paper, thinking hard, and scribbling on that paper. But as soon as mathematicians started using computers to carry out the exploratory work, the distinction became obvious, especially when the mathematician simply hit the ENTER key to initiate the experimental work, and then went out to eat while the computer did its thing. In some cases, the output that awaited the mathematician on his or her return was a new "result" that no one had hitherto suspected and might have no inkling how to prove.

What makes modern experimental mathematics different (as an enterprise) from the classical conception and practice of mathematics is that the experimental process is regarded not as a precursor to a proof, to be relegated to private notebooks and perhaps studied for historical purposes only after a proof has been obtained. Rather, experimentation is viewed as a significant part of mathematics in its own right, to be published, considered by others, and (of particular importance) contributing to our overall mathematical knowledge. In particular, this gives an epistemological status to assertions that, while supported by a considerable body of experimental results, have not yet been formally proved, and in some cases may never be proved. (It may also happen that an experimental process itself yields a formal proof. For example, if a computation determines that a certain parameter p, known to be an integer, lies between 2.5 and 3.784, that amounts to a rigorous proof that p = 3.)

PRACTICE

■ Discussion

<u>Provide additional information on the following points.</u>

1. What is the Riemann Zeta function and the Riemann Hypothesis?
2. What is Carl Friedrich Gauss famous for?
3. What does the Prime Number Theorem state?

■ Phonetics

<u>Read the words below and define them in English.</u>

[vɑːst]	[ˈtraɪəl]	[ɪkˈsplɒrətri]
[ˈʌltɪmət]	[ˈerə]	[ˌhɪðəˈtuː]
[ˈentəpraɪz]	[əˈsɜːʃn]	[ˌəʊvərˈɔːl]
[ˈædvent]	[suːˈpɜːfluəs]	[ˈrɪgərəs]
[əˈsʌmpʃn]	[kəˈrɪə]	

■ **Vocabulary**

I. Study the list of the words and expressions from the text.

to amount to sth.
conclusive proof
no matter how
there is more to sth. than sth.
to be engaged in (doing) sth.
to assess
significant
the advent of sth.
to date back to sth.
under the assumption of sth.
trial-and-error (process)
to remove from sth.
to come to do sth.
to overlook
an integral part of sth.
outside observer
to be relegated to sth.
rather...
to yield
rigorous

II. Translate the following groups of words into English using their derivative forms.

- оценивать — оценка — оценщик
- значительный — незначительный — обозначать — знак
- допущение (предположение) — допускать — допускаемый

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. Stimulated by the introduction of modern computers the developments in the field of numerical mathematics have been numerous in the past decades.
2. The faculty is actively involved in research in a wide range of areas including algebra, analysis, applied mathematics, geometry and topology and probability and statistics.
3. The analytic method Newton invented far exceeded the more philosophical and less scientifically strict approaches of Aristotle and Aquinas.
4. A number of studies have been conducted to evaluate the impact of innovative science curricula.

5. Time travel, teleportation, parallel universes — notions once moved to the realm of science fiction are now considered quite plausible.
6. A mathematical statement is equivalent to a proposition or assertion of some mathematical fact, formula, or construction.
7. What would you take away from the other side to keep the scale balanced?
8. Some scientists claim that what is essentially a discipline of pure mathematics has started to be called the theory of computer science.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Математики древности накапливали знания методом проб и ошибок.
2. Аристотель разрешил парадоксы, которые возникли в физике, при допущении атомарности пространства и времени.
3. Системы символьной математики способны давать результаты вычислений в виде специальных функций.
4. В атомной физике ученый не сторонний наблюдатель. Скорее, он участник.
5. Эйнштейн нашел убедительное доказательство реальности существования атомов и молекул и даже дал оценку их массы.
6. Система символических обозначений современной алгебры ведет свое начало от Франсуа Виета.
7. Программирование - это далеко не только алгоритмика.
8. Не важно, как химики определяют биссектрису, важно, как её определяют математики.
9. Построение красивых графиков — неотъемлемая часть любой системы аналитических вычислений.
10. Для того чтобы показать, что чистая математика сводится к логике, Рассел берет систему аксиом арифметики, сформулированную Пеано, и пытается их логически доказать.

V. Fill in the gaps with the following synonymous verbs.

**to overlook – to neglect/to ignore/to disregard – to miss –
to omit/to leave out — to discard**

1. If you … air resistance than the flight of the golf ball follows a symmetric parabolic path.
2. Newton is so famous for his calculus, optics and laws of motion that it is easy to … he was also one of the greatest geometers.
3. Mathematicians often analyse models of reality, which first require that they determine what variables to consider and what to … .
4. To criticize mathematics for its abstraction is to … the point entirely.
5. 10% of the data was … as unreliable.

■ Grammar

I. Time clauses with conjunctions "till/until, as soon as, once, before, after, when"

E.g. No matter how many zeros of the Riemann Zeta function are computed and observed to have real-part equal to 1/2, the Riemann Hypothesis *will not be regarded* as established *until* an analytic proof *has been produced*.

a) Translate the sentences into English.

1. Когда эта задача будет решена, мы сможем двигаться дальше в нужном направлении.
2. Алгебраический метод подразумевает прохождение большого числа промежуточных ступеней перед тем, как будет получен искомый конечный результат.
3. Как только бозоны Хиггса будут обнаружены, мы сможем наблюдать, как они взаимодействуют с другими частицами.

4. После того, как программа выведет график, щелкните по нему мышью, и панели программы изменят свой вид: появятся кнопки форматирования графика.
5. Ни одна практическая задача не решается математическими средствами до того времени, пока она не будет сведена к соответствующей математической задаче и не преобразуется, таким образом, в факт, соотнесенный с некоторой математической теорией.

b) Turn a simple sentence into a complex one using an empathic construction "it was not until that".

E.g. Gauss' proof was completed only after 1920. = *It was not until* 1920 that Gauss' proof was completed.

1. Algebraic problems began to be considered in a form similar to those studied today only after the 3rd century.
2. The West was introduced to these new scientific concepts only after Al-Khwarizmi's books on astronomy were translated into Latin.
3. The concept of matrices was further developed into the more complete theory of linear algebra only in the 20-th century.
4. The *Elements* by Euclid were translated into Latin and Arabic, but they became important in European education only after the first printed edition, published in 1482.
5. It became clear that non-commutative Noetherian rings constitute an interesting class of rings in their own right only in the 1950s, with the appearance of Goldie's theorem.

II. "What"-clause

Grammar note:
"What"-clauses are formed with the pronoun "what" (=the thing(s) that/which) or "all" (= the only thing(s) that/which) and have a clause that substitutes a noun phrase and acts as the subject of the whole sentence.

"What"-clause is used:

1) To focus attention on the information following the "what"-clause.

   Structure: "What"-clause + be + X:

   E.g. *What* you will discover in their published work *are precise statements of true facts…*
   E.g. *What* makes modern experimental mathematics different (as an enterprise) from the classical conception and practice of mathematics *is that the experimental process is regarded not as a precursor to a proof…*

2) To emphasise an action performed by someone.

   Structure: "What"-clause + do/be+(to)-infinitive clause.
   "To" can be omitted.

   E.g. *What Fermat did was (to) invent* an entirely new way of approaching geometric problems—what today we call analytic geometry. *What he did was (to) take* a geometric figure and reinterpret it algebraically.

a) <u>Turn two sentences into one using "what"-clause.</u>

1. In modern terminology 'relative motion' used by Newton is called 'uniform motion'. This is motion with a constant velocity in a constant direction.
2. The basic building blocks of arithmetic, numbers, arise naturally in the world around us, when we count things, measure things, buy things, etc. They make arithmetic possible.
3. The sum of an arithmetic series is the number of terms multiplied times the average of the first and last term. This fact was observed by Gauss.
4. Fractals are the best existing mathematical descriptions of many natural forms, such as coastlines, mountains or parts of

living organisms. This thing makes them even more interesting.

III. Noun+participle

Grammar note:
We can use some participles immediately after nouns in order to identify or define the noun. This use is similar to defining relative clauses.

E.g. But take a look at the private notebooks of practically any of the mathematical greats and you will find page after page of trial-and-error experimentation (symbolic or numeric), exploratory calculations, *guesses formulated, hypotheses examined*, etc. = ... guesses *that were formulated*, hypotheses *that were examined.*

a) Translate the sentences into English using the construction "noun+participle".

1. Необходимость повторения изученного материала вызвана самой структурой программы учебного курса математики.
2. Сформулированная задача представляет собой задачу Лагранжа на условный экстремум.
3. Применявшийся метод основывался на следующих принципах.
4. Найденные волновые функции используются для того, чтобы определить взаимодействие электрона с другими электронами и ядрами, уточняя потенциал.
5. Это рассуждение, основанное на делении пополам, отличается от предыдущего рассуждения тем, что взятая величина делится не на две равные части.

Text №6: What is Experimental Mathematics? (Part II)

by Keith Devlin[9]

Read and translate the text.

When experimental methods (using computers) began to creep into mathematical practice in the 1970s, some mathematicians cried foul, saying that such processes should not be viewed as genuine mathematics - that the one true goal should be formal proof. Oddly enough, such a reaction would not have occurred a century or more earlier, when the likes of Fermat, Gauss, Euler, and Riemann spent many hours of their lives carrying out (mental) calculations in order to ascertain "possible truths" (many but not all of which they subsequently went on to prove). The ascendancy of the notion of proof as the sole goal of mathematics came about in the late nineteenth and early twentieth centuries, when attempts to understand the infinitesimal calculus led to a realization that the intuitive concepts of such basic concepts as function, continuity, and differentiability were highly problematic, in some cases leading to seeming contradictions. Faced with the uncomfortable reality that their intuitions could be inadequate or just plain misleading, mathematicians began to insist that value judgments were hitherto to be banished to off-duty chat in the university mathematics common room and nothing would be accepted as legitimate until it had been formally proved.

What swung the pendulum back toward (openly) including experimental methods, was in part pragmatic and part philosophical. (Note that word "including". The inclusion of experimental processes in no way eliminates proofs.)

The pragmatic factor behind the acknowledgment of experimental techniques was the growth in the sheer power of computers, to search for patterns and to amass vast amounts of information in support of a hypothesis.

At the same time that the increasing availability of ever cheaper, faster, and more powerful computers proved irresistible for some mathematicians, there was a significant, though gradual, shift in

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9 http://www.maa.org/devlin/devlin_03_09.html

the way mathematicians viewed their discipline. The Platonistic philosophy that abstract mathematical objects have a definite existence in some realm outside of Mankind, with the task of the mathematician being to uncover or discover eternal, immutable truths about those objects, gave way to an acceptance that the subject is the product of Mankind, the result of a particular kind of human thinking.

The shift from Platonism to viewing mathematics as just another kind of human thinking brought the discipline much closer to the natural sciences, where the object is not to establish "truth" in some absolute sense, but to analyze, to formulate hypotheses, and to obtain evidence that either supports or negates a particular hypothesis.

In fact, as the Hungarian philosopher Imre Lakatos made clear in his 1976 book *Proofs and Refutations,* published two years after his death, the distinction between mathematics and natural science - as practiced - was always more apparent than real, resulting from the fashion among mathematicians to suppress the exploratory work that generally precedes formal proof. By the mid 1990s, it was becoming common to "define" mathematics as a science - "the science of patterns".

The final nail in the coffin of what we might call "hard-core Platonism" was driven in by the emergence of computer proofs, the first really major example being the 1974 proof of the famous Four Color Theorem, a statement that to this day is accepted as a theorem solely on the basis of an argument (actually, today at least two different such arguments) of which a significant portion is of necessity carried out by a computer.

The degree to which mathematics has come to resemble the natural sciences can be illustrated using the example I have already cited: the Riemann Hypothesis. As I mentioned, the hypothesis has been verified computationally for the ten trillion zeros closest to the origin. But every mathematician will agree that this does not amount to a conclusive proof. Now suppose that, next week, a mathematician posts on the Internet a five-hundred page argument that she or he claims is a proof of the hypothesis. The argument is very dense and contains several new and very deep ideas. Several years go by, during which many mathematicians around the world pore over the proof in every detail, and although they discover (and continue to discover) errors, in each case they or someone else (including the original author) is able to find a correction. At what point does the mathematical community as a whole declare that the hypothesis has indeed been proved? And even then, which do you find more convincing, the fact that there is an argument - which you have never read, and have no intention of reading - for which none of the hundred or so errors found so far have proved to be fatal, or the fact that the hypothesis has been verified computationally (and, we shall assume, with total certainty) for 10 trillion cases? Different mathematicians will give differing answers to this question, but their responses are mere opinions.

With a substantial number of mathematicians these days accepting the use of computational and experimental methods, mathematics has indeed grown to resemble much more the natural sciences. Some would argue that it simply is a natural science. If so, it does however remain, and I believe ardently will always remain, the most secure and precise of the sciences. The physicist or the chemist must rely ultimately on observation, measurement, and experiment to determine what is to be accepted as "true," and there is always the possibility of a more accurate (or different) observation, a more precise (or different) measurement, or a new experiment (that modifies or overturns the previously accepted "truths"). The mathematician, however, has that bedrock notion of proof as the final arbitrator. Yes, that method is not (in practice) perfect, particularly when long and complicated proofs are involved, but it provides a degree of certainty that the natural sciences rarely come close to.

So what kinds of things does an experimental mathematician do? (More precisely, what kinds of activity does a mathematician do that classify, or can be classified, as "experimental mathematics"?) Here are a few:

- Symbolic computation using a computer algebra system such as Mathematica or Maple
- Data visualization methods
- Integer-relation methods, such as the PSLQ algorithm
- High-precision integer and floating-point arithmetic
- High-precision numerical evaluation of integrals and summation of infinite series
- Iterative approximations to continuous functions

- Identification of functions based on graph characteristics.

Want to know more? As a mathematician who has not actively worked in an experimental fashion (apart from the familiar trial-and-error playing with ideas that are part and parcel of any mathematical investigation), I did, and I recently had an opportunity to learn more by collaborating with one of the leading figures in the area, the Canadian mathematician Jonathan Borwein, on an introductory-level book about the subject. The result was published recently by A.K. Peters: *The Computer as Crucible: An Introduction to Experimental Mathematics.* This month's column is abridged from that book.

We both hope you enjoy it.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. How is the Platonistic philosophy connected with mathematics?
2. What is the idea behind the Four Colour Theorem?
3. What do you know about a computer algebra system such as Mathematica or Maple?
4. What does the PSLQ algorithm deal with?

■ Phonetics

Read the words below and define them in English.

[faʊl]	[ˌhɪðəˈtuː]	[ˈɑːbɪtreɪtə]
[ˌæsəˈteɪn]	[ˈdɪsəplɪn]	[ˌvɪʒuəlaɪˈzeɪʃn]
[ˈsʌbsɪkwəntli]	[ɪkˈsplɒrətri]	[njuːˈmerɪkl]
[əˈsendənsi]	[ˈɔːθə]	[ˈɪtərətɪv]
[ˌɪnfɪnɪˈtesɪml]	[səbˈstænʃl]	[ˈkruːsɪbl]
[ɪnˈtjuːɪtɪv]	[sɪˈkjʊə]	[əˈbrɪdʒ]

■ Vocabulary

I. Study the list of the words and expressions from the text.

to view sth. as sth.
contradiction
to be faced with sth.
misleading
to insist on (doing) sth.
pendulum
to eliminate
the acknowledgment of sth.
to give way to sb./sth.
to establish
to make sth. clear
to result from (doing) sth.
to precede
the emergence of sth.
to resemble
to cite
to verify
to declare
to argue that
apart from sth.
to be part and parcel of sth.
to collaborate with sb.
to abridge

II. Translate the following groups of words into English using their derivative forms.

- противоречие — противоречить — противоречивый
- обманчивый (вводящий в заблуждение) — обманывать — руководить (вести) — руководитель — руководство
- предшествовать — предшествующий — прецедент — беспрецедентный
- цитировать (упоминать) — цитируемый — цитата
- проверять — проверенный — проверка

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. Since mathematical theories are a necessary feature of scientific theories, they too are confirmed by experience.
2. Pythagoras' theory was that pleasing sounds, which we now call octaves perfect fifths and major thirds, were caused by frequencies with simple ratios.
3. Even Aristotle regarded logic as an independent subject that should go before science and mathematics.
4. Random errors can be minimized but not removed.

5. There have been claims that Einstein's wife, Mileva Marić, worked with him on his celebrated 1905 papers, but historians of physics who have studied the issue find no evidence that she made any substantive contributions.
6. Much of Egyptian mathematics is similar to the Babylonian in that it seeks directly for the solution, rather than creating rigorous methods, though the level of sophistication is much less.
7. In both traditions, algebra of logic was invented within the enterprise to reform basic notions of mathematics which led to the arrival of structural abstract mathematics.
8. To discover a valid proof, we have to understand the concept of mathematical logic.
9. We can choose an arbitrary pair of natural numbers a and b, and we can check that $a+b = b+a$.
10. This physics course is shortened covering the applications of mechanics, fluids, heat and electromagnetism.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Иногда результат может вводить в заблуждение, как в следующем примере.
2. Нередко мы сталкиваемся с противоречием между реалистичностью модели и возможностью оперировать ею.
3. Помимо большого исторического интереса, анализ эволюции математики представляет огромную важность для развития философии и методологии математики.
4. Математический маятник — материальная точка, подвешенная на невесомой нерастяжимой нити.
5. Пифагорейцы настаивали на идеализации бесконечной делимости геометрических величин.
6. Лобачевский рассматривает аксиому параллельности Евклида как слишком жесткое требование, ограничивающее возможности теории, описывающей свойства пространства.

7. Николай Коперник, цитируя древних, утверждает, что гелиоцентрическая система не стала его открытием, а была давно известна.
8. Индийские математики и специалисты в области компьютерного обеспечения заявляют, что разработали метод, позволяющий безошибочно и быстро определять, простым ли является то или иное число.
9. Присуждение самой престижной математической премии специалисту по теории вероятностей — это признание важности этой области для всей нашей науки.
10. Настоящее издание представляет собой краткий курс, составленный в основном путем тщательного сокращения текста «Механики» тех же авторов.
11. Одна из трудностей при изучении трехмерных многообразий состоит в том, что наглядные образы частично должны уступить место абстрактным представлениям.
12. В своей работе итальянский математик Джероламо Кардано прояснил значение нового решения и обобщил его.

V. Fill in the gaps with the following synonymous adjectives,

misleading — confusing — ambiguous — deceptive

1. The name "elliptic geometry" is …, for it has no direct connection with the curve called ellipse.
2. A mathematical statement cannot be …: in mathematics, a statement is only acceptable or valid, if it is either true or false.
3. The math puzzles presented here are selected for the … simplicity of their statement, or the elegance of their solution.
4. To most people, one of the most intriguing and … branches of physics is Einstein's Theory of Relativity.
5. Boolean algebra is based on propositions, which are non-… sentences that can be either true or false.
6. The term 'mathematical induction' is somewhat … because it is actually a case of logical deduction.

■ **Grammar**

I. Conditional Mood

E.g. Oddly enough, such a reaction *would not have occurred* a century or more earlier…

a) ***Translate the text into English paying attention to the Conditional Mood.***

Ленглендс охотно обсуждал свой план построения математики будущего и пытался привлечь других математиков к участию в доказательстве множества своих гипотез. Никаких путей, ведущих к цели не было видно, но если бы мечта Ленглендса все же осуществилась, то награда было бы грандиозной. Любую неразрешимую проблему в одной области математики можно было бы трансформировать в аналогичную проблему из другой области, где для ее решения имелся бы целый новый арсенал методов. В случае неудачи эту проблему можно было бы перенести еще в какую-нибудь другую область математики, и так далее — до тех пор, пока наконец она не будет решена. … Важные следствия программа Ленглендса могла бы иметь и для прикладных наук и техники…. Если бы математики могли доказать «мостообразующие» гипотезы из программы Ленглендса, то появились бы пути решения не только абстрактных, но и практических проблем реального мира…. Никто не имел ни малейшего представления о том, как можно было бы доказать любую из гипотез Ленглендса. Первым шагом к осуществлению программы Ленглендса могло бы стать доказательство гипотезы Таниямы–Шимуры, но и оно пока было неосуществимо.

II. Constructions “go on doing sth.” and “go on to do sth.”

Grammar Note:

to go on + the Gerund to go on doing sth. — to continue happening or doing something as before	*to go on + the Infinitive* to go on to do sth. — to do something after doing something else
E.g. *The* spiral is not a true mathematical spiral (since it is made up of fragments which are parts of circles and does not *go on getting* smaller and smaller) but it is a good approximation to a kind of spiral that does appear often in nature.	***E.g.*** Oddly enough, such a reaction would not have occurred a century or more earlier, when the likes of Fermat, Gauss, Euler, and Riemann spent many hours of their lives carrying out (mental) calculations in order to ascertain "possible truths" (many but not all of which they subsequently *went on* to prove).

a) <u>Fill in the gaps with the Gerund or the Infinitive.</u>

1. John W. Milnor went on (to become) one of the most influential mathematicians of the 20th century.
2. Kepler went on (to produce) calendars for three decades, from 1595 to 1624.
3. Newton demonstrated that the motion of objects on the Earth could be described by three new laws of motion, and then he went on (to show) that Kepler's three Laws of Planetary Motion were but special cases of these three laws.
4. In the early 20th century scientists went on (to experiment) and finally made the gross atom burst into pieces under the impact of particle physics and chemistry.
5. The best-known Indian mathematician Srinivasa Ramanujan went on (to form) one of the most remarkable collaborations in mathematical history with Cambridge mathematician G. H. Hardy.

Text №7: The Origins and Foundations of Mechanics[10]

Read and translate the text.

The discovery of classical mechanics was made necessary by the publication, in 1543, of the book *De revolutionibus orbium coelestium libri VI* ("Six Books Concerning the Revolutions of the Heavenly Orbs") by the Polish astronomer Nicolaus Copernicus. The book was about revolutions, real ones in the heavens, and it sparked the metaphorically named scientific revolution that culminated in Newton's *Principia* about 150 years later. The scientific revolution would change forever how people think about the universe.

In his book, Copernicus pointed out that the calculations needed to predict the positions of the planets in the night sky would be somewhat simplified if the Sun, rather than the Earth, were taken to be the centre of the universe (by which he meant what is now called the solar system). Among the many problems posed by Copernicus' book was an important and legitimate scientific question: if the Earth is hurtling through space and spinning on its axis as Copernicus' model prescribed, why is the motion not apparent?

To the casual observer, the Earth certainly seems to be solidly at rest. Scholarly thought about the universe in the centuries before Copernicus was largely dominated by the philosophy of Plato and Aristotle. According to Aristotelian science, the Earth was the centre of the universe. The four elements—earth, water, air, and fire—were naturally disposed in concentric spheres, with earth at the centre, surrounded respectively by water, air, and fire. Outside these were the crystal spheres on which the heavenly bodies rotated. Heavy, earthy objects fell because they sought their natural place. Smoke would rise through air, and bubbles through water for the same reason. These were natural motions. All other kinds of motion were violent motion and required a proximate cause. For example, an oxcart would not move without the help of an ox.

When Copernicus displaced the Earth from the centre of the universe, he tore the heart out of Aristotelian mechanics, but he did not suggest how it might be replaced. Thus, for those who wished to

[10] From *Encyclopaedia Britannica* 2006 Ultimate Reference Suite DVD

promote Copernicus' ideas, the question of why the motion of the Earth is not noticed took on a special urgency. Without suitable explanation, Copernicanism was a violation not only of Aristotelian philosophy but also of plain common sense.

The solution to the problem was discovered by the Italian mathematician and scientist Galileo Galilei. Inventing experimental physics as he went along, Galileo studied the motion of balls rolling on inclined planes. He noticed that, if a ball rolled down one plane and up another, it would seek to regain its initial height above the ground, regardless of the inclines of the two planes. That meant, he reasoned, that, if the second plane were not inclined at all but were horizontal instead, the ball, unable to regain its original height, would keep rolling forever. From this observation he deduced that bodies do not need a proximate cause to stay in motion. Instead, a body moving in the horizontal direction would tend to stay in motion unless something interfered with it. This is the reason that the Earth's motion is not apparent; the surface of the Earth and everything on and around it are always in motion together and therefore only seem to be at rest.

This observation, which was improved upon by the French philosopher and scientist René Descartes, who altered the concept to apply to motion in a straight line, would ultimately become Newton's first law, or the law of inertia. However, Galileo's experiments took him far beyond even this fundamental discovery. Timing the rate of descent of the balls (by means of precision water clocks and other ingenious contrivances) and imagining what would happen if experiments could be carried out in the absence of air resistance, he deduced that freely falling bodies would be uniformly accelerated at a rate independent of their mass. Moreover, he understood that the motion of any projectile was the consequence of simultaneous and independent inertial motion in the horizontal direction and falling motion in the vertical direction. In his book *Dialogues Concerning the Two New Sciences* (1638), Galileo wrote,

It has been observed that missiles and projectiles describe a curved path of some sort; however, no one has pointed out the fact that this path is a parabola. But this and other facts, not few in number or less worth knowing, I have succeeded in proving . . . .

Just as Galileo boasted, his studies would encompass many aspects of what is now known as classical mechanics, including not only discussions of the law of falling bodies and projectile motion but also an analysis of the pendulum, an example of harmonic motion. His studies fall into the branch of classical mechanics known as kinematics, or the description of motion. Although Galileo and others tried to formulate explanations of the causes of motion, the focus of the field termed dynamics, none would succeed before Newton.

Galileo's fame during his own lifetime rested not so much on his discoveries in mechanics as on his observations of the heavens, which he made with the newly invented telescope about 1610. What he saw there, particularly the moons of Jupiter, either prompted or confirmed his embrace of the Copernican system. At the time, Copernicus had few other followers in Europe. Among those few, however, was the brilliant German astronomer and mathematician Johannes Kepler.

Kepler devoted much of his scientific career to elucidating the Copernican system. Although Copernicus had put the Sun at the centre of the solar system, his astronomy was still rooted in the Platonic ideal of circular motion. Before Copernicus, astronomers had tried to account for the observed motions of heavenly bodies by imagining that they rotated on crystal spheres centred on the Earth. This picture worked well enough for the stars but not for the planets. To "save the appearances" (fit the observations) an elaborate system emerged of circular orbits, called epicycles, on top of circular orbits. This system of astronomy culminated with the *Almagest* of Ptolemy, who worked in Alexandria in the 2nd century AD. The Copernican innovation simplified the system somewhat, but Copernicus' astronomical tables were still based on circular orbits and epicycles. Kepler set out to find further simplifications that would help to establish the validity of the Copernican system.

In the course of his investigations, Kepler discovered the three laws of planetary motion that are still named for him. Kepler's first law says that the orbits of the planets are ellipses, with the Sun at one focus. This observation swept epicycles out of astronomy. His second law stated that, as the planet moved through its orbit, a line joining it to the Sun would sweep out equal areas in equal times. For Kepler, this law was merely a rule that helped him make precise calculations for his astronomical tables. Later, however, it would be understood to be a direct consequence of the law of conservation of angular momentum. Kepler's third law stated that the period of a planet's orbit depended only on its distance from the Sun. In particular, the square of the period is proportional to the cube of the semimajor axis of its elliptical orbit. This observation would suggest to Newton the inverse-square law of universal gravitational attraction.

By the middle of the 17th century, the work of Galileo, Kepler, Descartes, and others had set the stage for Newton's grand synthesis. Newton is thought to have made many of his great discoveries at the age of 23, when in 1665–66 he retreated from the University of Cambridge to his Lincolnshire home to escape from the bubonic plague. However, he chose not to publish his results until the *Principia* emerged 20 years later. In the *Principia*, Newton set out his basic postulates concerning force, mass, and motion. In addition to these, he introduced the universal force of gravity, which, acting instantaneously through space, attracted every bit of matter in the universe to every other bit of matter, with a strength proportional to their masses and inversely proportional to the square of the distance between them. These principles, taken together, accounted not only for Kepler's three laws and Galileo's falling bodies and projectile motions but also for other phenomena, including the precession of the equinoxes, the oscillations of the pendulum, the speed of sound in air, and much more. The effect of Newton's *Principia* was to replace the by-then discredited Aristotelian worldview with a new, coherent view of the universe and how it worked. The way it worked is what is now referred to as classical mechanics.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. How did Nicolaus Copernicus influence the world of physics?
2. What is Newton's *Principia* concerned with?
3. How did Galileo Galilei and René Descartes contribute to the idea of motion?
4. What do you know about the *Almagest* of Ptolemy?
5. What is an inclined plane?
6. What are the basic principles characterising the law of conservation of angular momentum?

■ **Phonetics**

Read the words below and define them in English.

[ˌmetəˈfɒrɪkli]	[ɪnˈdʒiːniəs]	[ɪˈlɪps]
[ˈkæʒuəl]	[əkˈseləreɪt]	[ˈæŋgjələ]
[ˈdɒmɪneɪt]	[prəˈdʒektaɪl]	[ˈiːkwɪnɒks/ˈekwɪnɒks]
[ˈprɒksɪmət]	[ˈpendjələm]	[ˌɒsɪˈleɪʃn]
[ˌɪntəˈfɪə]	[iˈluːsɪdeɪt]	
[ɪˈnɜːʃə]	[iˈmɜːdʒ]	

■ **Vocabulary**

I. Study the list of the words and expressions from the text.

to spin	by means of sth.
casual observer	in the absence of sth.
to be largely dominated by sth.	moreover
respectively	to fall into sth.
to displace	not so much…as
to promote an idea	to be devoted to sth.
to take on sth.	to elucidate
violation	to be rooted in sth.
common sense	to account for sth.
to regain	to set out to do sth.
regardless of sth.	the validity of sth.
to reason	in the course of (doing) sth.
to interfere with sth.	concerning sth.
to improve	

II. Translate the following groups of words into English using their derivative forms.

- нарушение — нарушать — нарушитель — насилие
- вмешиваться — вмешивающийся — вмешательство
- улучшать — улучшенный — улучшение
- обоснованность (справедливость, действительность, пригодность) — обоснованный — необоснованный

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. It is only by treating mathematical sentences like other sentences of our language that we are able to clarify the role of mathematics in scientific arguments.
2. One of the largest branches of physics deals with studying our vast universe.
3. Galois theory gives a clear insight into questions related to problems in compass and straightedge construction.
4. Another method for finding solutions of equations is with the help of geometric construction.
5. Certain structural features of enveloping algebras explain various phenomena in Lie theory.
6. Time is an evolution parameter that is used in physics to denote change, without being affected by the type or rate of change.
7. In *The Sand Reckoner*, Archimedes specially began to calculate the number of grains of sand that the universe could contain.
8. Strong winds that may cause movement of the link transmitter or receiver (or both) may also prevent us from conducting accurate measurements.
9. The history of mathematics in the 18th century is mainly marked by the development of the methods of calculus.
10. A body which restores its original shape when the deformation force is removed is called an elastic body.
11. Research in this area is divided into two categories: the development of mathematics according to constructive principles, and the study of constructive theories in general.
12. In 1671, Leibniz began to invent a machine that could execute all four arithmetical operations, gradually making it better over a number of years.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Обычный наблюдатель может увидеть на небе невооруженным глазом около 3000 звезд.

2. Вращающееся вокруг своей оси тело при отсутствии тормозящих вращение сил так и будет продолжать вращаться.
3. Для обозначения истинности или ложности высказываний используют символы 1 или 0 соответственно.
4. В ходе своих экспериментов Ньютон совершил поистине великий научный подвиг, изумивший не столько его современников, сколько людей последующих двух столетий.
5. Мы не можем доказать истинность математического предложения, основываясь лишь только на практике, как во многих других науках.
6. Асимметрия реального мира не есть нарушение симметрии законов физики.
7. В 1979 году американский физик-теоретик Джон Уилер стал продвигать идею создания института теоретической физики.
8. Вопрос о происхождении инерции очень труден и коренится в тех изначальных свойствах материи, которые физика только-только начинает познавать.
9. Аристотель рассуждает, что в процессе абстрагирования мы можем мыслить тела в двух измерениях.
10. Сила Архимеда, действующая на тело, погруженное в жидкость, равна весу вымещаемого им объема жидкости.
11. В конце 20 века математика достигла такого уровня абстрактности, что здравый смысл отступил на второй план.
12. Математические объекты рассматривались пифагорейцами как первосущность мира, и, кроме того, математика превращена ими в составляющую религии.

V. Fill in the gaps with the following synonymous verbs.

to remove — to replace — to displace — to discard — to dismiss

1. Do not … math symbols with words, to avoid confusion in formulas.
2. Use the distributive property to … the parentheses in the following expression.

3. The mathematician Benoît Mandelbrot had long been interested in unsmooth shapes, but his mathematical colleagues had … his ideas as exotic curiosities.
4. A force is applied to a cart to … it up the incline at constant speed.
5. This conjecture should not be … out of hand.

■ Grammar

I. The verb "would"

Grammar note:
Functions of the verb "would":

1. Auxiliary verb in 'Future-in-the-Past':
   E.g. Einstein concluded that quantum mechanics *would solve* the main problem of classical physics, the specific heat anomaly.
2. Auxiliary verb in conditional sentences:
   E.g. If one travelled in a straight line through the universe, perhaps one *would* eventually revisit one's starting point.
3. Modal verb to describe habitual behaviour in the past:
   E.g. Einstein and Kurt Gödel *would take* long walks together discussing their work.
4. Modal verb to show willingness/unwillingness to perform an action:
   E.g. Cars *wouldn't* work without friction, nor *would* conveyor belts, nor even our muscle tissue.

a) Specify the meaning of "would" in the sentences from the text.

1. The scientific revolution *would change* forever how people think about the universe.
2. In his book, Copernicus pointed out that the calculations needed to predict the positions of the planets in the night sky *would be* somewhat simplified if the Sun, rather than the Earth, were taken to be the centre of the universe.

3. Heavy, earthy objects fell because they sought their natural place. Smoke *would rise* through air, and bubbles through water for the same reason.
4. For example, an oxcart *would not* move without the help of an ox.
5. Galileo noticed that, if a ball rolled down one plane and up another, it *would seek* to regain its initial height above the ground, regardless of the inclines of the two planes.
6. That meant that, if the second plane were not inclined at all but were horizontal instead, the ball, unable to regain its original height, *would keep* rolling forever.
7. Instead, a body moving in the horizontal direction *would tend to* stay in motion unless something interfered with it.
8. This observation *would* ultimately *become* Newton's first law, or the law of inertia.
9. Timing the rate of descent of the balls and imagining what *would happen* if experiments could be carried out in the absence of air resistance, he deduced that freely falling bodies would be uniformly accelerated at a rate independent of their mass.
10. Just as Galileo boasted, his studies *would encompass* many aspects of what is now known as classical mechanics.
11. Although Galileo and others tried to formulate explanations of the causes of motion, the focus of the field termed dynamics, none *would succeed* before Newton.
12. Kepler set out to find further simplifications that *would help* to establish the validity of the Copernican system.
13. Kepler's second law stated that, as the planet moved through its orbit, a line joining it to the Sun *would sweep out* equal areas in equal times.
14. Later, however, it *would be understood* to be a direct consequence of the law of conservation of angular momentum.
15. This observation *would suggest* to Newton the inverse-square law of universal gravitational attraction.

b) ***Translate the sentences into Russian using the verb “would”.***

1. Сам Ньютон отказывался даже от попыток объяснить природу гравитационной силы.
2. Если бы Земля притягивала все тела с одинаковой силой, то самая большая масса двигалась бы медленнее при падении, чем любая другая.
3. Эйнштейн обычно говорил, что все существенные идеи в науке родились в драматическом конфликте между реальностью и нашими попытками ее понять.
4. Еще совсем юным человеком Анри Пуанкаре сформулировал основы того, что стало впоследствии теорией хаоса.

II. Construction: “Subject + Passive Verb + the Infinitive”

E.g. *Newton is thought to have made* many of his great discoveries at the age of 23. = *It is thought that Newton made* many of his great discoveries at the age of 23.

a) ***Replace the subordinate clause with the construction “Subject + Passive Verb + the Infinitive”.***

1. It is considered that Galileo is the father of modern physics.
2. It is said that two models are isomorphic if a one-to-one correspondence can be found between their elements, in a manner that preserves their relationship.
3. It is known that there exist millions of prime numbers, and more are being added by mathematicians and computer scientists.
4. It is thought that Greek mathematics began with Thales of Miletus and Pythagoras of Samos.
5. It is believed that Archimedes fell asleep in his bathtub where he had interesting insights into the properties of liquids.
6. It was expected that the most practical benefit would come from the investigation of minimal surfaces, which have been of particular interest to mathematicians in the areas of differential geometry and topology.

7. It was supposed that Kepler's laws applied only to the motions of the planets; they said nothing about any other motion in the universe.
8. It was reported that Thales measured the height of the pyramids by comparing the length of their shadows to that of a vertical stick.

b) ***Translate the sentences into English using the construction "Subject + Passive Verb+ the Infinitive".***

1. Полагают, что Пифагор мог познакомиться с вавилонской и египетской математикой во время своих долгих странствий.
2. О двух величинах говорят, что они одного порядка, если отношение большего к меньшему из них меньше 10.
3. Предполагалось, что логика и математика в принципе однородны.
4. До 1975 г. считалось, что физика элементарных частиц и космология — разные области науки.
5. Известно, что в городе Троя жили три великих математика: Архимед, Пифагор и Евклид.
6. В середине 1990-х ожидалось, что в ближайшее время на основе теории струн будет сформулирована так называемая «единая теория», или «теория всего».

III. Construction "was/were to do sth."

Grammar note:
was/were to do sth. — used about someone or something in the past to say what would happen at a later time
E.g. The effect of Newton's Principia *was to replace* the by-then discredited Aristotelian worldview with a new, coherent view of the universe and how it worked.

a) ***Translate the sentences into Russian.***

1. Курт Гедель, ученый, которому было суждено осуществить переворот в точной науке, математике, родился в 1906 году в Австро-Венгерской империи.

2. Лагранж писал, что Гюйгенсу «было суждено усовершенствовать и развить важнейшие открытия Галилея».
3. В середине 19 столетия возникло совершенно новое течение в геометрии, топология, которому было суждено вслед за тем стать одной из главных движущих сил современной математики.

Text № 8: What is Financial Mathematics?

by Tim Johnson[11]

Read and translate the text.

If I tell someone I am a financial mathematician, they often think I am an accountant with pretensions. Since accountants do not like using negative numbers, one of the oldest mathematical technologies, I find this irritating.

A roll of the dice

I was drawn into financial maths not because I was interested in finance, but because I was interested in making good decisions in the face of uncertainty. Mathematicians have been interested in the topic of decision-making since Girolamo Cardano explored the ethics of gambling in his *Liber de Ludo Aleae* of 1564, which contains the first discussion of the idea of mathematical probability. Cardano, famously, commented that knowing that the chance of a fair dice coming up with a six is one in six is of no use to the gambler since probability does not predict the future. But it is of interest if you are trying to establish whether a gamble is fair or not; it helps in making good decisions.

With the exception of Pascal's wager (essentially that you've got nothing to lose by betting that God exists), the early development of probability, from Cardano, through Galileo and Fermat and Pascal up to Daniel Bernoulli, was driven by considering gambling problems.

[11] http://plus.maths.org/content/what-financial-mathematics. Issue 52. September 1, 2009

These ideas about probability were collected by Jacob Bernoulli (Daniel's uncle), in his work *Ars Conjectandi*. He introduced the *law of large numbers*, proving that if you repeat the same experiment (say rolling a dice) a large number of times, then the observed mean (the average of the scores you have rolled) will converge to the expected mean. (For a fair dice each of the six scores is equally likely, so the expected mean is (1+2+3+4+5+6)/6 = 3.5.)

Measure theory

Building on Jacob Bernoulli's work, *probability theory* was developed by the likes of Laplace in the eighteenth century and the Fisher, Neyman and Pearson in the twentieth. In conjunction with statistics, probability theory became an essential tool of the scientist. For the first third of the twentieth century, probability was associated with inferring results, such as the life expectancy of a person, from observed data. But as an inductive science (i.e. the results were inspired by experimental observations, rather than the *deductive* nature of mathematics built on axioms), probability was not fully integrated into maths until 1933 when Andrey Kolmogorov identified probability with measure theory. Kolmogorov defined probability to be any measure on a collection of events — not necessarily based on the frequency of events.

This idea is counter-intuitive if you have been taught to calculate probabilities by counting events, but can be explained with a simple example. If I want to measure the value of a painting, I can do this by measuring the area that the painting occupies, base it on the price an auctioneer gives the painting or base it on my own subjective assessment. For Kolmogorov, these are all acceptable measures which could be transformed into probability measures. The measure you choose to help you make decisions will depend on the problem you are addressing: if you want to work out how to cover a wall with pictures, the area measure would be best; if you are speculating, the auctioneer's would be better.

Kolmogorov formulated the axioms of probability that we now take for granted. Firstly, that the probability of an event happening is a non-negative real number ($P(E) \geq 0$). Secondly, that you know all the possible outcomes, and the probability of one of these outcomes occurring is 1 (e.g. for a six-sided dice, the probability of rolling a 1, 2, 3, 4, 5, or 6 is $P(1,2,3,4,5,6) = 1$). And finally, that you can sum the probability of mutually exclusive events (e.g. the probability of rolling an even number is $P(2,4,6) = P(2) + P(4) + P(6) = 1/2$).

Deciding a fair price

Why is the measure theoretic approach so important in finance? Financial mathematicians investigate markets on the basis of a simple premise; when you price an asset it should be impossible to make money without the risk of losing money, and by symmetry, it should be impossible to lose money without the chance of making money. If you stop and think about this premise you should quickly realise it has little to do with the practicalities of business, where the objective is to make money without the risk of losing it, which is called an *arbitrage*, and financial institutions invest millions in technology that helps them identify arbitrage opportunities.

An asset should be priced so as to prevent such arbitrages. Financial mathematicians realised that an asset's price can be represented as an expectation under a special probability measure, called a *risk-neutral measure*, which bears no direct relation to the 'natural' probability of the asset price rising or falling based on past observations.

However, as with much of probability, what seems simple can be very subtle. A no-arbitrage price is not simply an expectation using a special probability; it is only arbitrage-free if it is risk neutral and will not result in the possibility of making or losing money. And you have to undertake an investment strategy, known as hedging, that removes these possibilities. In the real world, which involves awkward things like taxes and transaction costs, it is impossible to find a unique risk-neutral measure that will ensure all these risks can be hedged away. One of the key objectives of financial maths is to understand how to construct the best investment strategies that minimises risks in the real world.

In good company

Financial mathematics is interesting because it synthesizes a highly technical and abstract branch of maths, measure theoretic probability, with practical applications that affect peoples' everyday lives. Financial mathematics is exciting because, by employing advanced mathematics, we are developing the theoretical foundations of finance and economics. To appreciate the impact of this work, we need to realise that much of modern financial theory, including Nobel

Prize winning work, is based on assumptions that are imposed, not because they reflect observed phenomena but because they enable mathematical tractability. Just as physics has motivated new maths, financial mathematicians are now developing new maths to model observed economic, rather than physical, phenomena.

Financial innovation currently has a poor reputation and some might feel that mathematicians should think twice before becoming involved with "filthy lucre". However, Aristotle tells us that Thales, the father of western science, became rich by applying his scientific knowledge to speculation, Galileo left the University of Padua to work for Cosimo II de Medici, and wrote *On the Discoveries of Dice*, becoming the first quant. Around a hundred years after Galileo left Padua, Sir Isaac Newton, left Cambridge to become warden of the Royal Mint, and lost the modern equivalent of £3,000,000 in the South Sea Bubble. Personally, what was good enough for Newton is good enough for me. Moreover, interesting things happen when maths meets finance: the concept of probability emerged out of the interface. And looking at the 23 DARPA Challenges for mathematics, several of these — the mathematics of the brain, the dynamics of networks and capturing and harnessing stochasticity in nature, beyond convex optimization — are all highly relevant to finance.

The Credit Crisis did not affect all banks in the same way. Some banks, like J.P. Morgan, engaged with mathematics and made good decisions, while others did not and caused mayhem (see Gillian Tett's book *Fools' Gold* for more information). Since Cardano, financial maths has been about understanding how humans make decisions in the face of uncertainty and then establishing how to make good decisions. Making, or at least not losing, money is simply a by-product of this knowledge. As Xunyu Zhou, who is developing the rigorous mathematical basis for behavioural economics at Oxford, recently commented:

Financial mathematics needs to tell not only what people ought to do, but also what people actually do. This gives rise to a whole new horizon for mathematical finance research: can we model and analyse ... the consistency and predictability in human flaws so that such flaws can be explained, avoided or even exploited for profit?

This is the theory. In practice, in the words of one investment banker:

Banks need high level maths skills because that is how the bank makes money.

PRACTICE

■ Discussion

<u>Provide additional information on the following points.</u>

1. Which role did Girolamo Cardano play in the development of mathematics?
2. What does Pascal's Wager (or Pascal's Gambit) state?
3. What do you know about Daniel and Jacob Bernoulli?
4. How did Laplace, Fisher, Neyman, Pearson, and Kolmogorov contribute to probability theory?
5. Why is Thales referred to as the father of western science?
6. What is The South Sea Company and the South Sea Bubble?
7. Which mathematical challenges does the Defense Advanced Research Projects Agency (DARPA) deal with?
8. What is the Credit Crisis?

■ Phonetics

<u>Read the words below and define them in English.</u>

[ˈfaɪnæns/ faɪˈnæns]	[ˈdeɪtə/dɑːtə]	[ˈnjuːtrəl]
[ˈkɒment]	[ˈfriːkwənsi]	[trænˈzækʃn]
[rəʊl]	[ˌkaʊntəɪnˈtjuːɪtɪv]	[ˌstɒkəˈstɪsɪtɪ]
[ˈævərɪdʒ]	[ˌɔːkʃəˈnɪə/ˌɒkʃəˈnɪə]	[ˌɒptɪmaɪˈzeɪʃn]
[kənˈdʒʌŋkʃn]	[ˈspekjuleɪt]	[bɪˈheɪvjərəl]
[ɪnˈfɜː]	[ˈpremɪs]	
[ɪkˈspektənsi]	[ˈɑːbɪtrɑːʒ/ˈɑːbɪtrɪdʒ]	

■ Vocabulary

I. ***<u>Study the list of the words and expressions from the text.</u>***

in the face of sth.	with the exception of sth.
decision-making	mean (n.)
gambling	to converge to sth.
to be of no use to sb./sth.	to build on sth.

in conjunction with sth.
to infer
to be inspired by sth./sb.
to be integrated into sth.
to take sth. for granted
mutually exclusive
premise
to have much/little to do with sth.
to result in (doing) sth.
to undertake
to affect
to appreciate
to impose sth. on sth.
consistency
flaw

II. Translate the following groups of words into English using their derivative forms.

- азартная игра — играть на деньги — азартный игрок
- вдохновленный — вдохновлять — вдохновение
- влиять (воздействовать) — привязанность — эффект (влияние)
- ценить (быть признательным) — признательность — признательный — ощутимый (заметный)

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. The fact that every composite number has a unique prime decomposition, a concept known as the fundamental theorem of arithmetic, is often believed to be true.
2. We study homological methods together with several concepts from commutative algebra, which are central to algebraic geometry.
3. The use of algebra has led to many great technological innovations, which have contributed to the improvement of the quality of life around the world.
4. These ideas can and should be part of physics courses at every level, instead of being reserved for specialized upper level courses.
5. We do research into the fundamental macroscopic quantum physics of superconductors and superfluids.
6. How did Bernhard Riemann influence the non-Euclidean system of geometry?

7. We shall call such a set with operations an algebra, but keep in mind that the structure may not be connected with traditional algebra.
8. It is very difficult to enjoy or recognize the value of mathematics in a purely passive way—in mathematics there is no real analogy of the role of the spectator, audience, or viewer.
9. Despite fierce resistance from most of his contemporaries and his own battle against mental illness, Cantor explored new mathematical worlds where there were many different infinities, some of which were larger than others.
10. A number of proofs were given, all containing more or less serious mistakes.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Курт Гедель доказал, что непротиворечивость аксиом арифметики нельзя доказать, исходя из самих аксиом арифметики.
2. Вероятность объединения взаимоисключающих событий равна сумме их вероятностей.
3. Возникновение теории вероятностей как науки относят к средним векам и первым попыткам математического анализа азартных игр.
4. Ряд Фурье в точках разрыва сходится к среднему арифметическому левого и правого предельных значений.
5. Продолжая рассуждения, методом математической индукции заключаем справедливость утверждения теоремы.
6. Конформная симметрия налагает ряд ограничений на локальную операторную алгебру в двумерном пространстве.
7. Геометрия Лобачевского — одна из неевклидовых геометрий, основанная на тех же основных посылках, что и обычная евклидова геометрия, за исключением аксиомы о параллельных, которая заменяется на аксиому о параллельных Лобачевского.

8. Теория <u>принятия решений</u> активно использует методы философии, математики, психологии, информатики.
9. Вычисление вероятности совершенно <u>бесполезно для</u> игрока, так как является еще одним способом математического или логического рассуждения.
10. Луи де Бройль в 1923-1924 годах написал работы (<u>вдохновленный</u> во многом своим братом Морисом, физиком-экспериментатором), утверждающие волновую природу электрона и других частиц.

<u>V. Fill in the gaps with the following synonymous verbs.</u>

to integrate — to combine — to join — to include — to unite

1. Examples … groups of permutations and groups of non-singular matrices.
2. In this paper, we discuss a teaching experiment designed to … algebra and proof in the high school curriculum.
3. Bayesian methods can be used to … results from different experiments, for example.
4. No more than one straight line can … two points.
5. One of the goals of physics is to find a single theory that … all of the four forces of nature.

■ Grammar

I. Articles "a", "the", and the "zero" article "ø"

a) ***<u>Comment on the use of the articles in the following sentences from the text:</u>***

the	ø
He introduced *the law of* large numbers…	…probability was not fully integrated into maths until 1933 when Andrey Kolmogorov identified probability with *measure theory.*

Firstly, that *the probability of* an event happening is a non-negative real number ($P(E) \geq 0$).	We need to realise that much of modern *financial theory*… is based on assumptions that are imposed, not because they reflect observed phenomena but because they enable mathematical tractability.
And looking at the 23 DARPA Challenges for mathematics, several of these — *the mathematics of* the brain, *the dynamics of* networks and capturing and harnessing stochasticity in nature, beyond convex optimization — are all highly relevant to finance.	

b) Insert "the" or "ø".

1. … mathematical theory of … probability has its roots in attempts to analyze games of chance by Gerolamo Cardano in the 16th century, and by Pierre de Fermat and Blaise Pascal in the 17th century.
2. In … probability theory, … law of large numbers is a theorem that describes … result of performing the same experiment a large number of times.
3. It follows from … law of large numbers that … empirical probability of success in a series of Bernoulli trials will converge to the theoretical probability.
4. For finite sets X, … axiom of choice follows from the other axioms of … set theory.
5. Within … electromagnetic theory generally, there are numerous hypotheses about how … electromagnetism applies to specific situations.

c) Comment on the use of the underlined articles in the following sentences from the text:

1. Mathematicians have been interested in the topic of_decision-making since Girolamo Cardano explored the ethics of gambling in his *Liber de Ludo Aleae* of 1564, which contains the first discussion of the idea of__mathematical probability.
2. This idea is counter-intuitive if you have been taught to calculate probabilities by counting events, but can be explained with a simple example. If I want to measure the value of a painting, I can do this by measuring the area that the painting occupies, base it on the price an auctioneer gives the painting or base it on my own subjective assessment.
3. Kolmogorov formulated the axioms of__probability that we now take for granted. Firstly, that the probability of an event happening is a non-negative real number.

d) Insert "a", "the", or "ø".

1. In …mathematics we often stress getting … exact answer.
2. …. fractal is … mathematical set that has … fractal dimension.
3. Newton was the first to apply …calculus to …general physics and Leibniz developed much of … notation used in …calculus today.
4. … curve in … plane can be approximated by connecting … finite number of points on …curve using line segments to create … polygonal path.
5. …Pythagorean equation relates … sides of … right triangle in … simple way, so that if …lengths of any two sides are known … length of … third side can be found.

Text № 9: Development of Mathematical Logic[12]

Read and translate the text.

Since the time of Aristotle, logic has been allied to philosophy. Until the late 19th century, however, logic was largely confined to formulating elaborate rules for one fairly simple form of argument—the syllogism; and there was a lack of systematic development of the subject along lines that had been taken in mathematics since early times.

Almost from the beginning, mathematicians had rigorously exploited two important techniques: (1) the use of the axiomatic method (as in Euclid's geometry) in developing the subject; and (2) the use of schematic letters or variables for stating general truths in the subject (thus, one can write "$A + B = B + A$", in which any names or numbers whatsoever can be substituted for *A* and *B*, and the result will still be true).

It is surprising that logicians through the ages failed to grasp the power of the use of schematic letters. When they finally began to employ these and other mathematical techniques, they made great contributions to man's understanding of the subject.

Among the developments that occurred in the 19th century, primarily through the work of mathematicians, those of the Englishman George Boole, creator of Boolean algebra, and of Georg Cantor, the Russian-born creator of set theory, are especially important inasmuch as they gave promise of bringing logic and mathematics closer together. The one figure who was both a mathematician and a philosopher and so might be credited with the marriage of logic as a philosophical subject with the techniques of mathematics was Gottlob Frege (died 1925), of the University of Jena in Germany. Historically, Frege, whose works are now appreciated in their own right, was important principally for his influence on Bertrand Russell, whose monumental work, *Principia Mathematica* (1910–13), written in collaboration with Alfred North Whitehead, together with Russell's earlier *Principles of Mathematics* (1903), awakened philosophers to the fact that the use of mathematical techniques in logic might prove to be of great importance for

[12] Encyclopaedia Britannica 2006 Ultimate Reference Suite DVD

philosophy. Its symbolism had the advantage of being closely connected with ordinary language, whereas its rules can be precisely formulated. Moreover, work in symbolic logic has produced many distinctions and techniques that can be applied to ordinary language.

Divergence of ordinary language from formal logic

Ordinary language, however, seems to differ from the artificial language of symbolic logic in more respects than its lack of precisely stated rules. On the surface, it often appears to violate the rules of symbolic logic. In the English statement "If this is gold [symbolized by *p*], then this will dissolve in aqua regia [symbolized by *q*]," for example, which in symbolic logic is expressed in a form known as the material conditional, *p* □ *q* (in which □ means "If . . . then . . . "), one of the rules is that the statement is true whenever "This is gold" is false. In ordinary language, on the contrary, one would not count the statement as true merely on formal logical grounds but only if there were some real connection in the world of chemical reactions between being gold and dissolving in aqua regia—a connection that plays no role in symbolic logic.

Among Analytic philosophers the existence of many such apparent divergences between symbolic logic and ordinary language has generated attitudes ranging from complete mistrust of symbolic logic as relevant to nonartificial languages to the position that ordinary language is not a proper vehicle for the rigorous statement of scientific truths.

Interpretations of the relation of logic to language

Symbolic logic has been viewed by many analytic philosophers as providing the framework for an ideal or perfect language. This statement can be taken in two ways:

1. Russell and the early Wittgenstein thought of logic as revealing, in a precise fashion, the real structure of any language. Any seeming departure from this structure in ordinary language must therefore be attributed to the fact that its surface grammar fails to reveal its real structure and is apt to be misleading. As a corollary, philosophers who have held this view have often explained philosophical problems as arising from being taken in by the surface features of the language. Because of the similarity of sentences such as "Tigers bite" and "Tigers exist," for example, the verb "to exist" may seem to function, as other verbs do, to predicate something of the

subject. It may seem, then, that existence is a property of tigers just as their biting is. In symbolic logic, however, the symbolic equivalent of the two sentences would be quite different; existence would not be represented by a symbol for a predicate but by what is called the existential quantifier, (□ *x*), which means "There exists at least one *x* such that . . . ."

2. The other sense in which symbolic logic has been seen as the framework of an ideal language is exemplified in the work of Rudolf Carnap, a 20th-century semanticist, who was concerned with what the best language—especially the best for the purposes of science—is.

One distinctive feature of the formal language of *Principia Mathematica* is that it becomes, when interpreted, a language of true-or-false statements. In ordinary language, on the contrary, one is not restricted to statements of truths; in it one can also issue commands, ask questions, make promises, express beliefs, give permission, and assert necessities and possibilities. Consequently, many philosophers have developed nonstandard logics that incorporate the nonassertoric features of language. Thus, various systems of logic have been formulated and studied.

On the other side of the coin, many philosophers—most notably the later Wittgenstein and those influenced by him—have thought that attempting to put language into the straitjacket of a formal system is to falsify the way that language works. Language performs a multitude of tasks, and even among expressions that seem to be alike in the way they function—those sentences, for example, that one might think are used simply for expressing facts—examination of their actual use reveals many differences: differences, for instance, in what is counted as showing them to be true or false and in their relationships to other parts of language. Formal systems, according to this view, at best oversimplify and at worst can lead to philosophical problems generated by supposing that all language operates strictly according to a simple set of rules. Accordingly, far from settling philosophical disputes by getting underneath the misleading exterior of ordinary language, formal systems add their own share of confusion.

PRACTICE

■ Discussion

Provide additional information on the following points.

1. What were Aristotle's views on logic?
2. What do you know about George Boole, Georg Cantor, Gottlob Frege, and Alfred North Whitehead?
3. What is analytic philosophy and who are analytic philosophers?
4. How did Bertrand Russell and Ludwig Wittgenstein connect philosophy with mathematics?
5. How did Rudolf Carnap develop logic?

■ Phonetics

Read the words below and define them in English.

[ɪˈlæbərət]	[əˈpriːʃieɪt]	[əˈtrɪbjuːt]
[ləˈdʒɪʃn]	[daɪˈvɜːdʒəns]	[kəˈrɒləri]
[skiːˈmætɪk]	[ˈviːəkl]	[praɪˈmerəli/ˈpraɪmərəli]
[ˈfɔːlsɪfaɪ]		

■ Vocabulary

I. Study the list of the words and expressions from the text.

to be largely confined to sth.	to violate the rules of sth.
a lack of sth.	on the contrary
along the lines of sth.	the framework for sth.
to be substituted for sth.	to attribute sth. to sth./sb.
to employ	distinctive feature
to bring sth. together	on the other side of the coin
in one's own right	at best/at worst
the advantage of (doing) sth.	to be far from (doing) sth.
in (some/many) respect(s)	to settle a dispute

II. Translate the following groups of words into English using their derivative forms.

- замененный — заменять — замена
- нанимать (использовать) — работодатель — сотрудник — занятость — безработица
- преимущество — недостаток — выгодный

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. The study of mathematics as a subject as it is begins in the 6th century BC with the Pythagoreans, who coined the term "mathematics" from the ancient Greek μάθημα (mathema), meaning "subject of instruction".
2. Mathematics provides a universal structure for innovation, but the interaction between mathematics and industry is certainly not optimal.
3. The physical world is structured in a way similar to fractal geometry.
4. To evaluate an algebraic expression, replace a number with the variable.
5. In spherical geometry angles are defined between great circles, resulting in a spherical trigonometry that differs from ordinary trigonometry in many ways.
6. A collection of personal probabilities is called coherent if it does not act against the principles of mathematical probability.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Первое доказательство существования иррациональных чисел обычно приписывается Гиппасу.
2. Арабские математики соединили древнегреческие понятия «числа» и «величины» в единую, более общую идею вещественных чисел.
3. Геометрия пифагорейцев в основном ограничивалась планиметрией.

4. Основным препятствием для дальнейшего развития геометрии после эпохи Возрождения было отсутствие общих методов геометрического исследования.
5. Некоторые теории, которые лежат в основе большинства западных философских учений, в действительности оказываются в лучшем случае неподтвержденными, а в худшем – просто предрассудками.
6. Преимущества алгебраических методов обусловлены использованием достаточно компактных символических систем, что внешне выглядит как самая характерная их черта.
7. В аффинной геометрии используется обычное понятие параллельности прямых; в проективной геометрии, наоборот, любые две прямые пересекаются в единственно возможной точке, и потому параллельных прямых не существует.
8. Циклоида должна была решить спор между Ферма и Декартом о преимуществах предлагавшихся ими методов проведения касательных.

V. Fill in the gaps with the following synonymous nouns.

corollary — consequence — effect — result — aftermath

1. The fact that any convergent sequence in a metric space is a Cauchy sequence is a direct … of the triangle inequality.
2. The geometric operations, their arguments, and their … are summarized in the table below.
3. In mathematics a … typically follows a theorem.
4. In the … of an intense earthquake, the earth as a whole “rings” with a period of 54 minutes.
5. In chaos theory, the butterfly … is the sensitive dependence on initial conditions; where a small change at one place in a nonlinear system can result in large differences to a later state.

■ **Grammar**

I. Construction "it + to be + adjective + that"

Grammar note:

Adjective	**Adverb**
It is interesting that	Interestingly
It is surprising that	Surprisingly
It is strange/odd that	Strangely/Oddly
It is ironical that	Ironically

E.g. *It is surprising that* logicians through the ages failed to grasp the power of the use of schematic letters. = *Surprisingly*, logicians through the ages failed to grasp the power of the use of schematic letters.

a) ***Translate the sentences into English using adjectives or adverbs in place of the underlined phrases.***

1. Удивительно, что на основе простого и даже чуть наивного принципа Дирихле математикам удается решать весьма трудные задачи, доказывать красивые теоремы, причем не только элементарные.
2. По иронии судьбы Фибоначчи стал известным в современной математике только лишь как автор интересной числовой последовательности, называемой числами Фибоначчи.
3. Странно, что физики, зная о Большом взрыве, начале существования мира, и о возможности создания человеком систем управления, нарушающих естественный ход процессов в природе, так боятся допустить мысль о создании этого мира и его законов.
4. Интересно, что Гаусса и Лобачевского учил в молодости один и тот же учитель — Мартин Бартельс (который, впрочем сам неевклидовой геометрией не занимался).

II. Pronouns “whose”, “which”, “that”

Grammar note:

whose — possessive pronoun for people and things **whose/of which (fml.)** — possessive pronoun for things only	**which/that** — relative pronoun for things only **who/that** — relative pronoun for people only
1. Historically, Frege, *whose* works are now appreciated in their own right, was important principally for his influence on Bertrand Russell… 2. A self-similar object is one *whose* component parts resemble the whole = A self-similar object is one *the component parts of which* resemble the whole	1. An axiom is a statement *which*/*that* is taken to be true without proof. 2. The one figure *who*/*that* was both a mathematician and a philosopher and so might be credited with the marriage of logic as a philosophical subject with the techniques of mathematics was Gottlob Frege.

a) Translate the sentences into English paying attention to the underlined words.

1. Часто задачи по геометрии приходится решать не только конструкторам, географам, астрономам или архитекторам, т.е. людям, чья работа связана с пространственным измерением или проектированием.
2. Если пятый постулат Евклида ложен, то сфера, радиус которой стремиться к бесконечности, приближается к предельной поверхности, чья внутренняя геометрия совпадает с геометрией евклидовой плоскости.
3. В действительности орицикл – это предельная форма окружности, центр которой уходит в бесконечность (так, что диаметры окружности становятся параллельными).
4. Математики стран ислама не только сохранили античные достижения, но и смогли осуществить их синтез с открытиями индийских математиков, которые в теории чисел продвинулись дальше греков.

5. Египтяне писали на папирусе, <u>который</u> сохраняется плохо, и поэтому в настоящее время знаний о математике Египта существенно меньше, чем о математике Вавилона или Греции.

III. Constructions with the Gerund

Grammar note:

Verb + preposition + the Gerund	**Noun/Adverb + preposition + the Gerund**
1. Until the late 19th century, however, logic was largely *confined to formulating* elaborate rules for one fairly simple form of argument—the syllogism. 2. As a corollary, philosophers who have held this view have often explained philosophical problems as *arising from being taken* in by the surface features of the language 3. … those sentences, for example, that one might think *are used simply for expressing* facts…	1.…as they gave *promise of bringing* logic and mathematics closer together. 2. Its symbolism had *the advantage of being closely connected with* ordinary language 3.…(1) *the use* of the axiomatic method (as in Euclid's geometry) *in developing* the subject; and (2) *the use* of schematic letters or variables *for stating* general truths in the subject 4.…*far from settling* philosophical disputes formal systems add their own share of confusion

a) <u>Translate the sentences into English using expressions with the Gerund in place of the underlined phrases .</u>

1. Этот список математических задач <u>далеко не полный</u>.
2. Практически, начертательная геометрия <u>ограничивается исследованием</u> объектов трехмерного евклидова пространства.
3. Вы уже убедились, какие <u>преимущества при решении</u> задач дает алгебра в сравнении с арифметикой.
4. Законы алгебры логики и свойства логических операций <u>используются для упрощения</u> логических выражений.

5. Линейное программирование – направление математики, изучающее методы решения экстремальных задач, которые характеризуются линейной зависимостью между переменными и линейным критерием оптимальности.
6. Многие распространенные заблуждения в отношении математики возникают в результате смешения этих двух толкований «прикладной математики».
7. Регулярность множества Мандельброта вселяет надежду на то, что в мире нелинейных явлений будут найдены более характерные сценарии.

Text № 10: *From* **The Unreasonable Effectiveness of Mathematics in the Natural Sciences**

by Eugene Wigner[13]

Read and translate the text.

Is the Success of Physical Theories Truly Surprising?

A possible explanation of the physicist's use of mathematics to formulate his laws of nature is that he is a somewhat irresponsible person. As a result, when he finds a connection between two quantities, which resembles a connection well-known from mathematics, he will jump at the conclusion that the connection is that discussed in mathematics simply because he does not know of any other similar connection. It is not the intention of the present discussion to refute the charge that the physicist is a somewhat irresponsible person. Perhaps he is. However, it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language. Let us consider a few examples.

[13] http://www.dartmouth.edu/~matc/MathDrama/reading/Wigner.html

The first example is the oft-quoted one of planetary motion. The laws of falling bodies became rather well established as a result of experiments carried out principally in Italy. These experiments could not be very accurate in the sense in which we understand accuracy today partly because of the effect of air resistance and partly because of the impossibility, at that time, to measure short time intervals. Nevertheless, it is not surprising that, as a result of their studies, the Italian natural scientists acquired a familiarity with the ways in which objects travel through the atmosphere.

It was Newton who then brought the law of freely falling objects into relation with the motion of the moon, noted that the parabola of the thrown rock's path on the earth and the circle of the moon's path in the sky are particular cases of the same mathematical object of an ellipse, and postulated the universal law of gravitation on the basis of a single, and at that time very approximate, numerical coincidence.

Philosophically, the law of gravitation as formulated by Newton was repugnant to his time and to himself. Empirically, it was based on very scanty observations. The mathematical language in which it was formulated contained the concept of a second derivative and those of us who have tried to draw an osculating circle to a curve know that the second derivative is not a very immediate concept. The law of gravity which Newton reluctantly established and which he could verify with an accuracy of about 4% has proved to be accurate to less than a ten thousandth of a per cent and became so closely associated with the idea of absolute accuracy that only recently did physicists become again bold enough to inquire into the limitations of its accuracy. Certainly, the example of Newton's law, quoted over and over again, must be mentioned first as a monumental example of a law, formulated in terms which appear simple to the mathematician, which has proved accurate beyond all reasonable expectations.

Let us just recapitulate our thesis on this example: first, the law, particularly since a second derivative appears in it, is simple only to the mathematician, not to common sense or to non-mathematically-minded freshmen; second, it is a conditional law of very limited scope. It explains nothing about the earth, which attracts Galileo's rocks, or about the circular form of the moon's orbit, or about the planets of the sun. The explanation of these initial conditions is left to the geologist and the astronomer, and they have a hard time with them.

The second example is that of ordinary, elementary quantum mechanics. This originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices, established a long time before by mathematicians. Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. They applied the rules of matrix mechanics to a few highly idealized problems and the results were quite satisfactory. However, there was, at that time, no rational evidence that their matrix mechanics would prove correct under more realistic conditions. Indeed, they say "if the mechanics as here proposed should already be correct in its essential traits."

As a matter of fact, the first application of their mechanics to a realistic problem, that of the hydrogen atom, was given several months later, by Pauli. This application gave results in agreement with experience. This was satisfactory but still understandable because Heisenberg's rules of calculation were abstracted from problems which included the old theory of the hydrogen atom. The miracle occurred only when matrix mechanics, or a mathematically equivalent theory, was applied to problems for which Heisenberg's calculating rules were meaningless. Heisenberg's rules presupposed that the classical equations of motion had solutions with certain periodicity properties; and the equations of motion of the two electrons of the helium atom, or of the even greater number of electrons of heavier atoms, simply do not have these properties, so that Heisenberg's rules cannot be applied to these cases. Nevertheless, the calculation of the lowest energy level of helium, as carried out by Kinoshita at Cornell and by Bazley at the Bureau of Standards, agrees with the experimental data within the accuracy of the observations, which is one part in ten million. Surely, in this case we "got something out" of the equations that we did not put in…

…The last example is that of quantum electrodynamics, or the theory of the Lamb shift. Whereas Newton's theory of gravitation still had obvious connections with experience, experience entered the formulation of matrix mechanics only in the refined or sublimated form of Heisenberg's prescriptions. The quantum theory of the Lamb shift, as conceived by Bethe and established by Schwinger, is a purely mathematical theory and the only direct contribution of experiment was to show the existence of a measurable effect. The agreement with calculation is better than one part in a thousand.

The preceding three examples, which could be multiplied almost indefinitely, should illustrate the appropriateness and accuracy of the mathematical formulation of the laws of nature in terms of concepts chosen for their manipulability, the "laws of nature" being of almost fantastic accuracy but of strictly limited scope. I propose to refer to the observation which these examples illustrate as the empirical law of epistemology. Together with the laws of invariance of physical theories, it is an indispensable foundation of these theories. Without the laws of invariance the physical theories could have been given no foundation of fact; if the empirical law of epistemology were not correct, we would lack the encouragement and reassurance which are emotional necessities, without which the "laws of nature" could not have been successfully explored…

PRACTICE

■ Discussion

<u>Provide additional information on the following points.</u>

1. What is the second derivative?
2. What is an osculating circle to a curve?
3. Which discoveries did physicists Max Born, Werner Heisenberg, Pascual Jordan, Wolfgang Pauli, Hans Bethe, and Julian Schwinger make?
4. What does the National Institute of Standards and Technology (known between 1901 and 1988 as the National Bureau of Standards) deal with?
5. How is the Lamb shift defined and what does it mean in physics?
6. What is epistemology?

■ **Phonetics**

Read the words below and define them in English.

[ˈfɪzɪsɪst]	[ˈθiːsɪs]	[ˈbjʊərəʊ]
[kwəʊt]	[ɪˈnɪʃl]	[ɪˌlektrəʊdaɪˈnæmɪks]
[əˈkwaɪə]	[ˈkwɒntəm]	[kənˈsiːv]
[ɪmˈpɪrɪkli]	[ɪˈsenʃl]	[əˈprəʊpriətnəs]
[ɪnˈkwaɪə]	[ˈhaɪdrədʒən]	[ˌɪndɪˈspensəbl]
[ˌriːkəˈpɪtʃuleɪt]	[ˈætəm]	[ɪnˈveəriəns]

■ **Vocabulary**

I. Study the list of the words and expressions from the text.

to jump to/at the conclusion
oft-quoted
to acquire
to bring sth. into relation with sth.
coincidence
to be associated with sth.
to inquire into sth.
to be beyond all expectations
to recapitulate
limited scope
initial
under conditions
trait
to be in agreement with sth.
to presuppose
refined
to conceive (of sth.)
preceding
to refer to sth. as sth.
indispensable
to lack sth.

II. Translate the following groups of words into English using their derivative forms.

- приобретать (получать) — приобретенный — приобретение
- совпадение — совпадать — совпадающий
- ассоциирующийся — ассоциироваться — ассоциативный — партнер
- осведомляться — интересующийся (пытливый) — запрос
- предполагать — полагать — предположение — предположительный

III. Replace the underlined phrases with the words and expressions from the list and translate the sentences into Russian.

1. When we are looking for patterns we use inductive reasoning to develop our conjectures, but we also need to remember that we can't always make a decision too quickly without knowing all the facts.
2. We can imagine a chemistry that is different from ours or a biology, but we cannot imagine a different mathematics of numbers.
3. Syncopated algebra does not have the compact form of contemporary algebraic equations.
4. Babylonian astronomy was the first and highly successful attempt at giving an improved mathematical description of astronomical phenomena.
5. Imaginary numbers are absolutely necessary in such fields as quantum mechanics, electrical engineering, computer programming, signal processing, and cartography.
6. Symbolic logic is more commonly called mathematical logic today.
7. Pythagoras looked into the ultimate nature of the world of the senses and concluded that it is mathematical.
8. Let us briefly review some basic results from algebra.
9. Descartes finally established the possibility of obtaining knowledge about the world based on deduction and perception.
10. The operations of addition and multiplication may be connected with one another by means of the concept of complementary sets.

IV. Translate the following sentences into English using the words and expressions from the list.

1. Нужно найти частное решение дифференциального уравнения, удовлетворяющее начальным условиям.
2. За вероятность события принимают относительную частоту события при достаточно большом количестве испытаний (проводящихся при одних и тех же условиях) или число близкое к ней.

3. Только в начале девяностых годов теория множеств вошла в моду и стала, сверх всяких ожиданий, широко применяться в анализе и геометрии.
4. Достаточно вспомнить часто цитируемые слова Галилея: «Природа – открытая книга, написанная на языке математики».
5. Аристарх отмечал, что гелиоцентрическая модель лучше объясняет видимое движение планет по круговым орбитам и согласуется с результатами наблюдений.
6. Книга «Краткая история времени» Стивена Хокинга объясняет создание Вселенной как цепочку совпадений.
7. Изучение математики предполагает не только запоминание и воспроизведение, но и узнавание, понимание и анализ фактов.
8. К концу жизни в характере Ньютона появились такие черты, как добродушие, снисходительность и общительность, ранее ему не свойственные.
9. Для большинства людей геометрия ассоциируется только с геометрией Евклида.
10. В честь Фибоначчи назван числовой ряд, в котором каждое последующее число равно сумме двух предыдущих.

V. Fill in the gaps with the following synonymous verbs.

to acquire — to obtain — to gain — to earn

1. The famous text "The Elements", used in schools for about 2,000 years, … Euclid the name "the father of geometry."
2. As civilizations developed, instincts for judging distances, angles, and height were augmented by observations and procedures … from experience, experimentation, and intuition.
3. Games are also a good way to … understanding of mathematical principles.
4. Then results that have been … are applied to the Hopf algebra itself.

■ **Grammar**

I. Emphatic construction "It-clause"

Grammar note:

Emphasis on the subject	***E.g.*** *It was Newton who* then *brought* the law of freely falling objects into relation with the motion of the moon… *It was Newton and Leibniz who developed* calculus into a systematic mathematical method.
Emphasis on the object	***E.g.*** *It was geometry that* Archimedes found most fascinating.
Emphasis on the adverbial	***E.g.*** *It is* only *by treating* mathematical sentences like other sentences of our language *that we are able* to make clear the role of mathematics in scientific arguments.
Emphasis on the prepositional phrase	***E.g.*** *It was in mathematics that* Hamilton did his most important work.

a) Turn the underlined words or phrases into emphatic constructions.

1. Einstein thought that Newtonian mechanics was no longer enough to reconcile the laws of classical mechanics with the laws of the electromagnetic field.
2. Early in the 20th century, algebraic geometry underwent a significant overhaul.
3. By solving an equation, we mean to find a proper number for x that makes the given expression true.
4. Lobachevsky's main achievement is the development of a non-Euclidean geometry.
5. Gauss and Weber founded the "Magnetischer Verein" ("magnetic club" in German), which supported measurements of the Earth's magnetic field in many regions of the world.

6. In one of his earliest papers, Cantor proved that the set of real numbers is "more numerous" than the set of natural numbers.
7. Gödel is best known for his two incompleteness theorems, published in 1931 when he was 25 years old.

II. Inversion

1) Inversion after adverbs

Grammar note:

Inversion is used mostly with a restrictive or negative sense after the following adverbs and adverb phrases:

– never, rarely, seldom
– hardly, barely, no sooner, scarcely
– only after, only when, only then, etc.
– little
– so, such

E.g. The law of gravity became so closely associated with the idea of absolute accuracy that ***only recently*** *did physicists become* again bold enough to inquire into the limitations of its accuracy.

a) Use inversion in the following sentences.

1. Mathematics seldom helps economists predict irrational human behaviour.
2. The importance of Riemann in the history of algebraic geometry can hardly be overestimated.
3. The possibility of establishing a fruitful correspondence between geometry and algebra became evident only when algebra had become a more complete and useful subject in its own right.
4. We know little about the life and work of Thales.
5. Max Planck's work was so important that his research is considered the pivotal point where "classical physics" ended and modern physics began.

2) ***Inversion in conditional sentences with no "if"***

Grammar note:
E.g. *Had the ancient Greeks had access* to computers, it is likely that the word "experimental" in the phrase "experimental mathematics" would be superfluous = *If ancient Greeks had had access* to computers, it is likely that the word "experimental" in the phrase "experimental mathematics" would be superfluous.
E.g.: *Should you neglect* air resistance than the flight of the golf ball follows a symmetric parabolic path = *If you neglect* air resistance than the flight of the golf ball follows a symmetric parabolic path.

a) ***Use inversion in the following conditional sentences.***
1. If Poincaré had lived longer, he would have reached the conclusion that Newton's emission theory of light is the only theory compatible with the Michelson-Morley experiment.
2. It is true that if there were no phenomena which are independent of all but a manageably small set of conditions, physics would be impossible.
3. Certain structural features of enveloping algebras explain various phenomena in Lie theory that remain elusive if one confines one's attention to the category of Lie algebras.
4. If the second plane were not inclined at all but were horizontal instead, the ball, unable to regain its original height, would keep rolling forever.

III. Pronouns *many, much, few, little*
Grammar note:

With countable nouns	**With uncountable nouns**
"A large number/amount of"	
Many —many more ***E.g.:*** *Many* fractals possess the property of self-similarity	*Much — much more* ***E.g.:*** An appreciation of mathematics often comes after *much* intuitive knowledge is acquired.

"A small number/amount; not enough; not many/much"	
Few — fewer — much fewer ***E.g.:*** At the time, Copernicus had *few* other followers in Europe. Among those *few*, however, was the brilliant German astronomer and mathematician Johannes Kepler.	*little — less —much less* **E.g.:** The problems in general were of two kinds: those involving the manipulation of objects, and those requiring computation. The first required *little* or no mathematical skill.
"Some"	
a few **E.g.:** They applied the rules of matrix mechanics to *a few* highly idealized problems and the results were quite satisfactory.	*a little* ***E.g.:*** *A little* knowledge of low-dimensional topology would be helpful for the course but not necessary.

a) <u>Fill in the gaps with the pronouns "many, much, few, little".</u>

1. With … effort, derivative, integral and expansion of complex functions can be understood using the analogy with real functions.
2. There are … mathematical models used to introduce negative numbers to children but the most commonly used one is most certainly the number line.
3. … information is available about Chinese mathematical thought before 206 BC.
4. Newton is thought to have made … of his great discoveries at the age of 23.
5. After …. work on the very large number of theorems in the subject, the basics of projective geometry became understood.
6. In ancient times, … people could understand even the simplest arithmetic and geometry, and the confusion of mathematics with magic has a long history.

b) _Translate the sentences into English using the comparative form of the pronouns “many, much, few, little”._

1. Компьютеры могут последовательно производить большое количество математических действий и решать сложные задачи, затрачивая на это намного меньше времени, чем человек.
2. Расчет траектории ближайшего спутника Земли, Луны, требует гораздо больше вычислений, чем расчет траекторий самых отдаленных звезд.
3. Эпоха возрождения в науке потребовала гораздо больше усилий и даже жертв, нежели возрождение в искусстве или литературе.
4. При строгом математическом исследовании оказывается, что для достаточно точного описания поведения системы нужно значительно меньше уравнений, чем можно было бы представить.