# Mathematical language

## Introduction

When we try to use ordinary language to explore mathematics, the words involved may not have a precise meaning, or may have more than one meaning. Many words have meanings that evolve as people adapt their understanding of them to accord with new experiences and new ideas. At any given time, one person's interpretation of language may differ from another person's interpretation, and this can lead to misunderstandings and confusion.

In mathematics we try to avoid these difficulties by expressing our thoughts in terms of well-defined mathematical objects. These objects can be anything from numbers and geometrical shapes to more complicated objects, usually constructed from numbers, points and functions. We discuss these objects using precise language which should be interpreted in the same way by everyone. In this unit we introduce the basic mathematical language needed to express a range of mathematical concepts.

## 1. Sets

In Section 1 we discuss the idea of a *set* and describe some ways to define sets. We illustrate our discussion with sets of numbers and with geometrical sets of points in the plane. We also explain how to check whether two given sets are equal and whether one set is a subset of another. Finally, we introduce the set operations of *union*, *intersection* and *difference*.

## 2. Functions

In Section 2 we give the general definition of a *function*, and illustrate how functions can be used to describe a variety of mathematical concepts, such as transformations of the plane. We discuss the idea of *composing* two functions, and the idea of forming the *inverse* of a function.

## 3. The language of proof

In Section 3 we examine the language used to express mathematical statements and proofs, and discuss various techniques for proving that a mathematical statement is true. These techniques include *direct proof, proof by mathematical induction, proof by contradiction* and *proof by contraposition*. We also illustrate the use of *counter-examples* to show that a statement is false.

# 4. Two identities

Section 4 introduces some important mathematical theorems.

# 5. Solutions to the exercises

Section 5 contains solutions to the exercises that appear throughout sections 1-4.