***Model Answer***

This model answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

## The Shortest Nets

The article headlined “The Shortest Nets” published in “Kvant” magazine touches on/upon (concerns, is concerned with, looks at, examines, studies, deals with, is about) Steiner’s utmost problem: there are three villages, that want to build the roads so, that any villager can reach any other of them, provided the total distance he covers  is the least possible. The author considers two solutions for/to the task: empirical (experimental) and geometric (hypothetical, theoretical).

The article goes on to say that there is, actually, a simple way of ultimate connection. In fact, if the angles in the triangles formed by the villages are all less than 120°, then one has to draw a point, forming a vertex of the triangles that equal 120°. Anyway, to prove the assumption let’s put a local geographic map on the table and drill some holes in the area, where the villages are situated. So, all we have to do next is to pass three strings through the holes and tie the top ends into a knot, while attaching 1kg weights to the bottom ones. Therefore, the total length of the strings under the table will be the ultimate one, and the strings on the surface of the table will determine the necessary shortest intersection of the three given points, called Fermat’s point.

Naturally, the question arises why the knot will certainly mark the point, where the vertex of the angles equals 120°. Apparently, one can’t help noticing that the knot is affected by three equally matched forces, which follows that the latter can balance each other only if the three angles they form are equal.

However, the table could have been saved if one found a pure geometric solution. Surely, we have to build outside equilateral triangles on the sides of the ABC triangle. The point in question will be the intersection of the AA1, BB1, CC1 segments.

In conclusion (Finally, To sum up, To summarise) I‘d like to stress (point out, emphasise) that the shortest net is like a tree in the plane that is made up of unclosed curves, forming a homogeneous structure, where a curve connects any two ends of the segment.

***Model Answer***

This model answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

**Plane above Fire**

The article headlined “Plane above Fire” published in the 6th issue of “Kvant” magazine in 2010 looks at ways of calculating the differential (amount) of water poured from the plane and reaching the ground. To solve the task the author resorts to (makes use of/uses) both theoretical and experimental researches/investigations/data.

Speaking about forest fires the author quotes two writers N. Gumelev and V.Y. Shishkov respectively as saying: “That summer witnessed thunder storms, unbearable stuffiness and heat /…/So that at midday the sun was ever crimson lit./”and “The fire advanced as a rapid and roaring avalanche slaying whatever came its way…further and further on the frantic and mighty lava sails on and no power can stop it.” The article goes on to strongly oppose the idea described (given/expressed)in the latter extract from the novella “Forest” and argues that there surely is a way out.

Naturally, that is aircraft. On the one hand it’s obvious that the higher the aircraft the safer. However, the amount of water reaching the ground seems much less then, doesn’t it? In fact, numerous analyses have proved to the fact that to put down a fierce wood fire there needs to be a front line of irrigation 10m in width absorbing at least 1l every $1m^2$. Evidently, the optimal height for an aircraft should be good enough for the water width to grow up to 10m after the deformation of its initial volumes providing (provided) the water mass shouldn’t turn completely into drops easily carried away by the wind.

Anyway, the width of the irrigation line can be figured out. Say, we have 10 tons of water ($3{\cdot}10^3l$) so, that the uniform (normal/usual) surface density is $1lm^2$

and the desired (necessary/required) width is equal to 10m we get the line about 300m long. Ideally, that would be the case if the whole water amount got to the line and was equally spread. What if only half the volume or its third was? That's where physics and maths come along.

Apparently, the water falling from the craft is dispersed into particles compelled by the air stream to move at a $u_0$ speed, flattening the water in the perpendicular way, whereas the liquid is dispersed vertically at a height of 10m. Meanwhile, the velocity of the falling water equals about 10ms and its horizontal velocity relative to the earth matches that of the craft itself.

Presumably, the application of the numerical research will yield the following Cartesian system of reference, where $u_0$=70ms is the velocity of the flight and t=0,27s is the period of time. The given picture illustrates the symmetry of the vertical plane. However, if we consider the model of consistent dispersion we'll get j as the number of the dispersion series and $r_j$ as the radius of the liquid spherical element. The air stream compels it to turn the following ratio of the axes into a pancake ("ellipsoid of rotation" in scientific means) Δ x/y=1/10. Hereafter, the liquid element is split into two parts, which gain a spherical shape. But now the radius equals $r_{j+1}=\frac{\mathrm{rj}}{\sqrt[3]{2}}$and so on.

In conclusion the author stresses that if a body of uniform mass falls at a zero initial velocity (u≡0 V≡υ) then while t→∞ the terminal velocity is $\upsilon_\infty$= -$\sqrt{\frac{ma}{CS P\mathrm{air}}}$. Therefore, no wonder parachutes should cover a vast area in the denominator.

*Model Answer*

This model answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

**Domino Tilings and Height Functions**

The article headlined "Domino Tilings and Height Functions" studies the distribution of "checked" figures or "dominoes" containing adjoined black and white elements as dealt with in spherical geometry. By way of introducing a new terminology such as triangular lattice dominoes, diamonds and flips the author presumes that one can split any F figure containing triangular dominoes which form a hexagon into n others by flipping or rotating it 60° and considers the number of flips for the longest solution.

The author goes on to assert that the horizontal shift to the right and downwards to the left and upwards to the left does gain height, while the horizontal shift to the left, upwards to the right and downwards to the right loses height. Any tiling of a hexagon with diamonds looks like a 3D picture in a plane: "a box half full of bricks". Therefore a flip is just adding or removing one brick from the box.

Actually, we can illustrate the picture with the help of the height function. So, let's put a hexagon in the horizontal Cartesian OXY plane, then let's draw straight vertical segments from each node of the lattice, that equal the value of the height function, in the direction of 0Z axis. Connecting the upper ends of the segments placed at the adjoined nodes of the tiled frame will yield the basis of a polygonal surface, which is the diagram of the height function, actually. The height values of the two tilings differ in one flip in the centre of the hexagon. The similar part having been removed, the remaining sides determine a polyhedron above the hexagon. That is the brick in the box that needs adding or removing. Evidently, the height function h(υ) is the actual sum of the vertex v coordinates or, similarly, the distance from the v node to the x+y+z=0 plane multiplied by √3. Thus, the quantity of bricks in the box equals $n^3$ flips and one needs no more than $n^3$ flips to turn one tiling into another.

Speaking about the minimum height function min (f,g) the author goes on to prove the lemma: the most number of bricks in a checked frame formed as a result of tiling the square 2n×2n into dominoes equals n(2n-1)(2n+1)/3 or flips which rotate dominoes or rectangles, containing two squares with one common side, 90°. It's noteworthy that historically, the height function was first described by W.P. Thurston, while Bo-Yin Young was the first to research into ways of tiling the N×N square with outside square quarters drawn on each second border segment.

Interestingly, there are only two kinds of Young's fortresses if n is an odd number whereas only one kind is possible if n is an even number. Strictly speaking, the fortress as big as 2N×2N can be tiled with dominoes in $5^{N^2}$ ways. However, odd-sized fortresses have a long solution. For example, 2011×2011 fortress without corner bastions can split into dominoes in $2 \cdot 5^{1006 \cdot 1005}$ ways.

To sum up (summarise), the article sets forth the following lemma to count the number of flips: 1(n-1)+2(n-2)+3(n-3)+…+(n-2)2+(n-1)1=(n-1) n(n+1)/6 and to prove the theorem that any way of tiling Young's fortress can turn into any other one by means of no more than $(n+1)(2N^2-2N+3)/6$ flips.

***Model Answer***

This model  answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

**On Moires, Animated Illustrations and Use of Models"**

The article headlined "On Moires, Animated Illustrations and Use of Models" examines the concepts of coherence and electromagnetic interference and their application in modern super high resolution microscopy. The author argues that deeper understanding of the moire effects will enable the reader to have a better insight into the complicated physical phenomena.

As IT resources are becoming more and more popular they outnumber paper ones. However, there has appeared a new type of book illustrations. The authors go on to cut the upper layer of such an illustration featuring the "Star Wars" series to reveal a transparent film with a vertical black ribbed pattern and a picture concealing the sketch of the original image. Presumably, once the film covers the picture again, the image springs into motion.

Strictly speaking, the above mentioned phenomenon has a French name "moires", which are a cloth with different tints glittering in the light. Actually, there can be various moire structures such as cloth threads, lattice hedges or combs with a row of thin teeth on one side. Naturally, PC users often come across moires when examining stripy pictures on a small scale. However, moires do pose a big problem for photographers always seeking to avoid them.

In fact, the nature of moires is close to the origin of electromagnetic waves. They have one thing in common, which is their regularity. Whereas electromagnetic fields keep on varying in space and time, moires can vary in space. Therefore, if two waves meet somewhere in space, there occurs their interference, likewise two combs placed upon each other interrelate.

True, speaking about interference one cannot help admitting that light is characterized by harmonious or sinusoidal relation of the field tension that depends on time and coordinates. Though the grids have sharp edges and vary in space, drawing the sinusoidal distribution will turn moires into a blurred picture. Among the main properties of moires as compared to waves there is the fact that the closer there are the directions in which the waves spread and their sources the wider the interference stripes are.

Surprisingly, moires look larger than their original structure. No wonder, the latter property is widely used in microscopy to study surfaces with a thin layer, which cannot be examined through a common microscope. After all, in contrast to electromagnetic waves that vary in time and space, moirés do only in space. Nevertheless, moirés can help to understand one of the most complicated issues in optics, that is coherence. Strictly speaking, waves are coherent if the difference of the phases caused by oscillation waves remains permanent in time. Generally, there are two kinds of coherence: in space and time. The former deals with waves produced by different points of an extensive (lengthy, prolonged) source at one and the same time, while their distributive difference remains permanent in time. The latter means that the difference of wave distribution produced by one point of the source at different periods of time remains permanent in time. As for moires, the first type of coherence is illustrated by random distribution of the distance between grid systems, while the second type is characterized by random distribution of grid angles.

In conclusion, it's worth stressing that the time coherence of a point source equals the length of one impulse of radiation , whereas space coherence $r=\lambda/\varphi$, where $\lambda$ is the length of the radiated wave and $\varphi$ is the rate of angle change (rotation). Thus, the less the lengths of coherence in space and periods of coherence in time are, there occurs an interference.

Conversely, a uniform distribution of energy is observed if the distances or intervals are increased as in the case with the time of coherence for a lamp, which is $10^{-8}$, while the human eye reacts to $10^{-1}$s. That difference accounts for the fact that we can’t notice any interference of independent sources.

*Model Answer*

This model  answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

**Fruit Electricity**

The article headlined "Fruit Electricity" compares chemical sources of fruit electricity to choose the most efficient one and to determine the relation between tension in terminals, element power and the surface area of electrodes. By revising the basics of chemistry the author attempts to give a popular explanation of such sophisticated phenomena as potential difference, oxidization, reduction, anodes and cathodes.

Speaking about the most popular chemical sources of electricity the author reminds the reader of the first chemical source of energy that was invented in 1800 by the Italian scientist Alexander Volta. It goes without saying that the chemical energy units have a vast area of application from microelements of energy in pacemakers to hydrogen energy batteries charging spacecraft. In fact, many modern electronic appliances cannot do without chemical sources of electricity. After all, even some kinds of weapons are activated by an impulse from a chemical charge.

It's a well known fact that the basis of the chemical source of electricity are two metal electrodes: the cathode containing the oxidizer and the anode carrying the reductant, that reacts with the electrolyte. Eventually, the difference of potentials between the electrodes makes for the electrically driven force that equals the amount of free energy provided by the oxidization and reduction. Strictly speaking, the work of the chemical sources inducing current is based on two spatially divided processes: oxidisation, which occurs in the cathode of a closed external circuit, and the free electrons that form hereafter and travel through the external circuit to the anode, where they take part in reduction.Anyway, there are numerous ways to make a fruit battery since juice contains a lot of organic acids, their oxygen and hydrogen are good oxidisers, for they take away electrons from metal.

The article goes on to carry out an experiment to prove the fact that the best electrodes are made of metals that possess different oxidization and reduction properties. Inserting various pairs of electrodes made of copper, stainless steel, steel, or zinc into apples, lemons, oranges and potatoes to measure the amount of electricity we'll see that the highest tension is produced by copper and zinc pairs of terminals.

Surprisingly, the most quantity of different acids can be found in apples and not in lemons. However, the tension in terminals is not the only characteristic of a charge. A more important quality is the maximum tension of the source, which depends on its internal resistance. Actually, there is one more surprise since the tension of terminals doesn't change regardless of their distance. That phenomenon can be accounted for by the fact that there is no current inside the sources. Therefore the tension in terminals depends on the velocity of chemical reactions in anodes and cathodes, which is characterized by the choice of electrode elements and the electrolyte.

According to Ohm's law the maximum current power is generated when the external resistance tends to zero. The milliamperemetre has the lowest internal resistance, which enables the reader to measure the power during the electrical fault. Hence, the maximum power of common fruit sources equals Pmax=UImax. However, there is no current in the fruit battery but in the electrode, which is the result of the oxidization and reduction processes. While the iones leave zinc electrodes, the latter are destroyed and cannot be restored. Surely, no accumulators can be made from fruit batteries, which can never be recharged. Though the power of the fruit batteries is as low as one tenth of the milliwatt, you can still charge such low current devices as watches, calculators or light-emitting diodes (luminodiodes, LEDs).

To summarise, I’d like to point out that more powerful devices will need several fruit sources of current connected into one battery, as the area of the electrode surface certainly affects the tension in the terminals.

***Model Answer***

This model  answer has been written by an examiner as an example of a very good answer. However, please note that this is just one approach out of many possible ways.

**Margaret Thatcher**

The article headlined “Margaret Thatcher” and published in “The Guardian” gives an exclusive profile of the Conservative ex-Prime Minister of the UK. Interviewing the “champion of free market economies” the reporter traces the major stages of the politician’s career, revealing their pros and cons.

Defining the basic characteristics of conservatism the article goes to compare Margaret Thatcher to an evangelist, who is faithfully devoted to her policy. Getting ready to speaking about the ex-Prime Minister’s career, the correspondent worries she can get him wrong. However, proceeding with facts from her Grantham background, discussing her successful studies at grammar school as a bursary winner and university chemist research at Lyons the journalist concludes that Margaret Thatcher must have had a better raft to float up from.

Anyway, Margaret Thatcher proves to be always on her guard and disapproves of any criticism merely admitting that “the unjust amount of epithets” signify more about the author than about the subject. Nevertheless, among the controversial political topics dealt with by Conservatives the author mentions education, children nutrition, and, consequently, egalitarianism, which appears to be interpreted unfairly by the rich and poor. Opposing the journalist’s opinion Margaret Thatcher strongly disapproves of the egalitarianism in terms of “everyone-shall-have-the-same” thinking. To support her opinion she goes on to remind the reader that she has known to stretch her money and used to earn £8-10s a week when she started work after Oxford. After all, she seems fairly annoyed and wonders why the journalist has been going for her much more than for her Labour predecessors. Eventually, Margaret Thatcher goes on admitting she has expanded education, which the author can’t help agreeing, is certainly her greatest achievement.

Finally, foreseeing her image of an ordinary woman who is fundamentally interested in education, is going to be misunderstood again she seems utterly dissatisfied to appear as the typical middle-class Tory woman. Fortunately, the journalist assures her that the reader will, surely, retain the image of the woman capable of going back to almost any standard of living, having been through most, who is living in a small house while desperately trying to sell her other, larger one. Obviously, the author doesn’t think she is being a bit too cynical, does he?