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им. М. В. Ломоносова
механико-математический факультет
кафедра английского языка

Л. Н. Выгонская

Focus on Scientific English

Учебное пособие

Москва
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Цель пособия — совершенствование навыков чтения как самостоятельного вида речевой деятельности и как средства обучения другим языковым и речевым навыкам. Оно предназначено для аспирантов-математиков и механиков.

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Данное пособие предназначено для аспирантов, готовящихся сдавать кандидатский экзамен по английскому языку. Его основная цель — совершенствование навыков чтения, что предполагает умение извлекать информацию в необходимом объеме, делать адекватный перевод, передавать содержание текста в устной и письменной форме, участвовать в беседе. Иначе говоря, мы рассматриваем чтение и как самостоятельный вид речевой деятельности, и как средство обучения другим языковым и речевым навыкам.

Совершенствование навыков чтения является важной составляющей обучения иностранному языку людей, занимающихся наукой, поскольку письменный текст по-прежнему остается ценным источником научной информации.

В зависимости от поставленной задачи чтение бывает *просмотровым*, предполагающим ознакомление с текстом с последующей краткой его научной характеристикой, *ознакомительным*, когда необходимо умение проследить развитие научной темы и, наконец, *изучающим*, дающим полное понимание текста. Умение понять и адекватно перевести текст на родной язык непосредственно связано с умением разобраться в тонкостях языкового выражения, поэтому трудно переоценить роль обучения *осмыслению* и *анализу* языковых явлений в изучаемом тексте. Этим, в основном, и занимается данное пособие.

В качестве анализируемого материала используются две статьи из научно-популярного журнала *Scientific American*. Думается, что тематика этих статей будет интересна и понятна как математикам, так и механикам. Выбор научно-популярного жанра объясняется тем, что именно он, как показали исследования, является наиболее удачным материалом (в силу своих особенностей), позволяющим подготовиться к практическому использованию английского языка в реальной ситуации общения[1].

Текст каждой статьи разделен на части, которые сопровождаются заданиями (часто с включенными в них комментариями), их цель — привлечь внимание к наиболее частотным и важным лексическим, грамматическим и стилистическим особенностям научного текста. Этим объясняется не сплошной, а выборочный анализ языковых явлений. Для закрепления пройденного материала даются

[1] А.Л. Назаренко. Научно-популярная литература как объект функциональной стилистики и лингводидактики.— Автореф. дисс. … д-ра филол. наук — М., 2000.

упражнения на перевод с русского языка на английский. И наконец, в качестве дополнительного материала с целью выработки навыков восприятия и понимания научной речи со слуха (аудирования) предлагаются видеосюжеты, один из них — *Fermat's Last Theorem* — тематически полностью совпадает с анализируемым текстом *Fermat's Last Stand?* и создан теми же авторами. Таким образом, появляется возможность сопоставления письменной и устной форм выражения на одну и ту же тему. В приложении имеется сценарий. Также в приложении даются образцы текстов для письменного и устного перевода, подобные тем, что предлагаются на экзамене.

Выражаем искреннюю благодарность рецензентам — доктору филологических наук, профессору А. Л. Назаренко и доктору физико-математических наук, профессору Н. Н. Смирнову, а также выпускнику механико-математического факультета МГУ Р. Ю. Рогову за помощь, оказанную при подготовке пособия к печати.

Contents

Fermat's Last Theorem

Reading

In 1994 Andrew J. Wiles of Princeton University announced that he had discovered a proof of Fermat's last theorem.

1. What does this theorem state?
2. What do you know about attempts to prove it?

Now you are going to read an article on the subject. The text is divided into six parts, each of which is followed by exercises. Some of them contain notes to simplify a task. After reading Part I for the first time to get the general idea, read it again more carefully and do the exercises. The same procedure should be followed when you pass on to the next part of the text and so on.

1. Look through the title and the annotation of the article below. You may not know the verbs *to baffle* and *to crack.* Try to predict their meanings. While translating this part of the text, concentrate on the superlatives: *His most notorious theorem* and *the greatest minds.* Do you remember the rule of the comparison of adjectives and adverbs?

FERMAT'S LAST STAND?

His most notorious theorem baffled the greatest minds for more than three centuries. But after 10 years of work, one mathematician cracked it

by Simon Singh and Kenneth A. Ribet

This past June, 500 mathematicians gathered in the Great Hall of Göttingen University in Germany to watch Andrew J. Wiles of Princeton University collect the prestigious Wolfskehl Prize. The reward—established in 1908 for whoever proved Pierre de Fermat's famed last theorem—was originally worth \$2 million (in today's dollars). By the summer of 1997, hyperinflation and the devaluation of the mark had reduced it to a mere \$50,000. But no one cared. For Wiles, proving Fermat's 17th-century conundrum had realized a childhood dream and ended a decade of intense effort. For the assembled guests, Wiles's proof promised to revolutionize the future of mathematics.

Indeed, to complete his 100-page calculation, Wiles needed to draw on and further develop many modern ideas in mathematics. In particular, he had to tackle the Shimura-Taniyama conjecture, an important 20th-century insight into both algebraic geometry and complex analysis. In doing so, Wiles forged a link between these major branches of mathematics. Henceforth, insights from either field are certain to inspire new results in the other. Moreover, now that this bridge has been built, other connections between distant mathematical realms may emerge.

2. a) Infinitive clauses can act as an adverbial, especially of purpose (answering the question *why?* or *what … for?*). They are translated into Russian with the help of the conjunctions: *для того чтобы* or *чтобы.*

   To pass this exam you must work hard.
   or
   In order to pass …
   Чтобы сдать этот экзамен, нужно хорошо потрудиться.

   Identify two sentences with infinitive clauses of the above type and translate them into Russian.

   b) Infinitive clauses can follow the object after some verbs (Complex Object), e. g. *ask*, *tell*, *expect*, *consider*:

   He asked them to come back later.

   These verbs can be followed by a noun or pronoun object + the bare infinitive or the –ing form: *feel*, *bear*, *listen* *to*, *look at*, *notice*, *observe*, *see*, *match*, etc.

I watched him draw/drawing a portrait.

In translating this construction into Russian we nearly always use a subordinate clause (using *чтобы*, *что* or *как*).

Я видел, что он рисовал/как он рисовал какой-то портрет.

Find the sentence with Complex Object and translate it into Russian.

3. What is the function of the pronoun *one* (l. 7)? Translate this sentence.
4. Identify the part of speech and its function of the –ing form *proving* (l. 8) and translate it into Russian.
5. The Past Simple is used to express a finished action in the past. What do we use the Past Perfect for? Analyse ll. 6–9.
6. Identify the sentence with the Present Perfect. Explain why this form is used here. Translate it. Compare the Present Perfect and the Past Simple. Give your examples.
7. *Must* and *have (got) to* are used to express obligation. *Must* usually expresses the feelings and wishes of the speaker. *Have (got) to* often expresses obligation that comes from somewhere else. Translate: *In particular, he had to …* (l. 14) Explain why *had to* is used here.
8. The expression *both … and* is translated into Russian in the following way: как … так и, и … и. Consider ll. 14–16.
9. Compare the uses of *other* (l. 18 and l. 19). What is the difference?
10. We use *do so* instead of repeating a *verb* + *object* or *verb* + *complement* when it is clear from the context what we are talking about. What is meant by *In doing so …* (l. 16)?
11. Find three words in the text with the same or similar meaning to the following: one part of large subject of study or knowledge.

Now read Part II. Use your dictionary to check new words and expressions, then pass on to the exercises.

THE PRINCE OF AMATEURS

Pierre de Fermat was born on August 20, 1601, in Beaumont-de-Lomagne, a small town in southwest France. He pursued a career in local government and the judiciary. To ensure impartiality, judges were discouraged from socializing, and so each evening Fermat would retreat to his study and concentrate on his hobby, mathematics. Although an amateur, Fermat was highly accomplished and was largely responsible for probability theory and the foundations of calculus. Isaac Newton, the father of modern calculus, stated that he had based his work on "Monsieur Fermat's method of drawing tangents."

Above all, Fermat was a master of number theory—the study of whole numbers and their relationships. He would often write to other mathematicians about his work on a particular problem and ask if they had the ingenuity to match his solution. These challenges, and the fact that he would never reveal his own calculations, caused others a great deal of frustration. René Descartes, perhaps most noted for inventing coordinate geometry, called Fermat a braggart, and the English mathematician John Wallis once referred to him as "that damned Frenchman."

Fermat penned his most famous challenge, his so-called last theorem, while studying the ancient Greek mathematical text *Arithmetica,* by Diophantus of Alexandria. The book discussed positive whole-number solutions to the equation $a^2 + b^2 = c^2$, Pythagoras's formula describing the relation between the sides of a right triangle. This equation has infinitely many sets of integer solutions, such as $a = 3$, $b = 4$, $c = 5$, which are known as Pythagorean triples. Fermat took the formula one step further and concluded that there are no nontrivial solutions for a whole family of similar equations, $a^n + b^n = c^n$, where n represents any whole number greater than 2.

It seems remarkable that although there are infinitely many Pythagorean triples, there are no Fermat triples. Even so, Fermat believed he could support his claim with a rigorous proof. In the margin of *Arithmetica,* the mischievous genius jotted a comment that taunted generations of mathematicians: "I have a truly marvelous demonstration of this proposition, which this margin is too narrow to contain." Fermat made many such infuriating notes, and after his death his son published an edition of Arithmetica that included these teases. All the theorems were proved, one by one, until only Fermat's last remained.

Numerous mathematicians battled the last theorem and failed. In 1742 Leonhard Euler, the greatest number theorist of the 18th century, became so frustrated by his inability to prove the last theorem that he asked a friend to search Fermat's house in case some vital scrap of paper was left behind. In the 19th century Sophie Germain—who, because of prejudice against women mathematicians, pursued her studies under the name of Monsieur Leblanc—made the first significant breakthrough. Germain proved a general theorem that went a long way toward solving Fermat's equation for values of n that are prime numbers greater than 2 and for which $2n + 1$ is also prime. (Recall that a prime number is divisible only by 1 and itself.) But a complete proof for these exponents, or any others, remained out of her reach.

At the start of the 20th century Paul Wolfskehl, a German industrialist, bequeathed $100,000$ marks to whoever could meet Fermat's challenge. According to some historians, Wolfskehl was at one time almost at the point of suicide, but he became so obsessed with trying to prove the last theorem that his death wish disappeared. In light of what had happened, Wolfskehl rewrote his will. The prize was his way of repaying a debt to the puzzle that saved his life.

Ironically, just as the Wolfskehl Prize was encouraging enthusiastic amateurs to attempt a proof, professional mathematicians were losing hope. When the great German logician David Hilbert was asked why he never attempted a proof of Fermat's last theorem, he replied, "Before beginning I should have to put in three years of intensive study, and I haven't that much time to squander on a probable failure." The problem still held a special place in the hearts of number theorists, but they regarded Fermat's last theorem in the same way that chemists regarded alchemy. It was a foolish romantic dream from a past age.

1. *Would* is a very common modal auxiliary. In this part of the text it is used to talk about past habit and translated обычно and to express past refusal (in the negative form). Find the sentences with *would* and translate them. Do you remember other functions of *would*?
2. Find a word in the text with the same or similar meaning to the following: someone who does an activity for pleasure or interest, not as a job.
3. Explain the meaning of the word combinations in a–h below.
   a) … were discouraged from socializing … (l. 4)
   b) Although an amateur, … (l. 6)
   c) … was highly accomplished and was largely responsible for probability theory … (ll. 6–7)
   d) Above all, … (l. 11)
   e) … was a master of number theory … (l. 11)
   f) … caused others a great deal of frustration. (l. 16)
   g) … his most famous challenge … (l. 20)
   h) … to whoever could meet Fermat's challenge … (ll. 55–56)
4. Look at lines 8–10 (Isaac Newton …) and comment on the use of tenses. In connection with this grammar area review the rule of sequence of tenses.
5. In indirect speech general questions (which invite *yes* or *no* as an answer) begin with *if*, *whether* or *whether or not*.
   E. g. 'Can I borrow your dictionary?'→ *He asked* her *if* he could borrow her dictionary. Он спросил ее, может *ли* он воспользоваться ее словарем.
   Notice that in *direct speech* the questions have inversion, but that in *indirect speech* the word order is normal: if + subject + verb …
   Find the sentence of the above type in the text and translate it into Russian. What can you say about special questions in indirect speech? Is there any example here?
6. Find two words in the text with the same or similar meaning to *name* v.
7. Consider the functions of the −ing forms:

   a) drawing (l. 10)
   b) inventing (l. 17)
   c) studying (l. 21)
   d) describing (l. 24)
   e) infuriating (l. 37)
   f) solving (l. 49)
   g) trying (l. 57)
   h) repaying (l. 60)
   i) encouraging (l. 61)
   j) loosing (l. 63)
   k) beginning (l. 65)

8. Having reviewed the rule of *sequence of tenses*, you know that if the reporting verb (e. g. *said*) is in the past, the verb in the reported clause will usually be in the past form as well.
   E. g. *She said* she *had read* the book. (she *had read* the book = the reported clause)
   Sometimes the present tense is retained when the reported sentence deals with a general truth.
   E. g. Copernicus *concluded* that the earth *goes* round the sun.
   Compare: Fermat ... *concluded* that there *are* no nontrivial solutions ... (ll. 27–28)
   ... Fermat *believed* he *could support* his claim with a rigorous proof. (ll. 32–33)
   Find quotations in the text and change to indirect speech.
9. *Even so* is a prepositional phrase used to introduce a fact that is surprising in the light of what was just said. It connects ideas between sentences.
   Look at lines 31–40 and find sentences which express contrasting ideas or concession. What is the function of *although* (l. 6, l. 31)?
10. What does *one by one* mean? (l. 39)
11. Here, *until* (l. 40) means 'up to the time'. What other time conjunctions do you know?
12. What do the following prefixes add?
    a) **in**ability (l. 43)
    b) **re**write (l. 59)

c) **non**trivial (l. 28)
d) **dis**courage (l. 4)
e) **en**courage (l. 61)
f) **co**ordinate (l. 17)

13. The word combination *in case* is usually translated *если.* What do you think about its meaning in the sentence *In 1742* ... (ll. 42–45)?
14. Explain the difference in meaning.
    a) David Hilbert asked why he never attempted a proof of Fermat's last theorem.
    b) David Hilbert was asked why he never attempted a proof of Fermat's last theorem.
15. Insert prepositions:
    to be discouraged ... , to concentrate ... , to be responsible ... , to base sth ... sth, to write ... sb ... sth, a work ... a particular problem, to refer ... sb ... , a solution ... an equation, ... the margin, prejudice ... sth, to be divisible ... 1, according ... some historians, ... light of what had happened.

Read Part III. Use your dictionary to check new words and expressions, then pass on to the exercises.

THE CHILDHOOD DREAM

Children, of course, love romantic dreams. And in 1963, at age 10, Wiles became enamored with Fermat's last theorem. He read about it in his local library in Cambridge, England, and promised himself that he would find a proof. His schoolteachers discouraged him from wasting time on the impossible. His college lecturers also tried to dissuade him. Eventually his graduate supervisor at the University of Cambridge steered him toward more mainstream mathematics, namely into the fruitful research area surrounding objects called elliptic curves. The ancient Greeks originally studied elliptic curves, and they appear in *Arithmetica.* Little did Wiles know that this training would lead him back to Fermat's last theorem.

Elliptic curves are not ellipses. Instead they are named as such because they are described by cubic equations, like those used for calculating the perimeter of an ellipse. In general, cubic equations for elliptical curves take the form $y^2 = x^3 + ax^2 + bx + c$, where a, b and c are whole numbers that satisfy some simple conditions. Such equations are said to be of degree 3, because the highest exponent they contain is a cube.

Number theorists regularly try to ascertain the number of so-called rational solutions, those that are whole numbers or fractions, for various equations. Linear or quadratic equations, of degree 1 and 2, respectively, have either no rational solutions or infinitely many, and it is simple to decide which is the case. For complicated equations, typically of degree 4 or higher, the number of solutions is always finite—a fact called Mordell's conjecture, which the German mathematician Gerd Faltings proved in 1983. But elliptic curves present a unique challenge. They may have a finite or infinite number of solutions, and there is no easy way of telling.

To simplify problems concerning elliptic curves, mathematicians often reexamine them using modular arithmetic. They divide x and y in the cubic equation by a prime number p and keep only the remainder. This modified version of the equation is its "mod p" equivalent. Next, they repeat these divisions with another prime number, then another, and as they go, they note the number of solutions for each prime modulus. Eventually these calculations generate a series of simpler problems that are analogous to the original.

The great advantage of modular arithmetic is that the maximum values of x and y are effectively limited to p, and so the problem is reduced to something finite. To grasp some understanding of the original infinite problem, mathematicians observe how the number of solutions changes as p varies. And using that information, they generate a so-called L-series for the elliptic curve. In essence, an L-series is an infinite series in powers, where the value of the coefficient for each pth power is determined by the number of solutions in modulo p.

In fact, other mathematical objects, called modular forms, also have L-series. Modular forms should not be confused with modular arithmetic. They are a certain kind of function that deals with complex numbers of the form $(x + iy)$, where x and y are real numbers, and i is the imaginary number (equal to the square root of -1).

What makes modular forms special is that one can transform a complex number in many ways, and yet the function yields virtually the same result. In this respect, modular forms are quite remarkable. Trigonometric functions are similar inasmuch as an angle, q, can be transformed by adding π, and yet the answer is constant: $\sin q = \sin(q+\pi)$. This property is termed symmetry, and trigonometric functions display it to a limited extent. In contrast, modular forms exhibit an immense level of symmetry. So much so that when the French polymath Henri Poincaré discovered the first modular forms in the late 19th century, he struggled to come to terms with their symmetry. He described to his colleagues how every day for two weeks he would wake up and search for an error in his calculations. On the 15th day he finally gave up, accepting that modular forms are symmetrical in the extreme.

A decade or so before Wiles learned about Fermat, two young Japanese mathematicians, Goro Shimura and Yutaka Taniyama, developed an idea involving modular forms that would ultimately serve as a cornerstone in Wiles's proof. They believed that modular forms and elliptic curves were fundamentally related—even though elliptic curves apparently belonged to a totally different area of mathematics. In particular, because modular forms have an L-series—although derived by a different prescription than that for elliptic curves—the two men proposed that every elliptic curve could be paired with a modular form, such that the two L-series would match.

Shimura and Taniyama knew that if they were right, the consequences would be extraordinary. First, mathematicians generally know more about the L-series of a modular form than that of an elliptic curve. Hence, it would be unnecessary to compile the L-series for an elliptic curve, because it would be identical to that of the corresponding modular form. More generally, building such a bridge between two hitherto unrelated branches of mathematics could benefit both: potentially each discipline could become enriched by knowledge already gathered in the other.

The Shimura-Taniyama conjecture, as it was formulated by Shimura in the early 1960s, states that every elliptic curve can be paired with a modular form; in other words, all elliptic curves are modular. Even though no one could find a way to prove it, as the decades passed the hypothesis became increasingly influential. By the 1970s, for instance, mathematicians would often assume that the Shimura-Taniyama conjecture was true and then derive some new result from it. In due course, many major findings came to rely on the conjecture, although few scholars expected it would be proved in this century. Tragically, one of the men who inspired it did not live to see its ultimate importance. On November 17, 1958, Yutaka Taniyama committed suicide.

1. In lines 1-11, does *would* refer to past habit, the future in the past, unreal meaning, or anything else? Translate these sentences into Russian.
2. Explain the meaning of the word combination *more mainstream mathematics* (l. 7).
3. In statements it is usual for the verb to follow the subject. Sometimes, however, this word order is reversed. We can refer to this as *inversion*. It is used to give emphasis, especially when the statement begins whith a negative word or idea, or with *so*.

   E. g. *Not a single word had* one *written* since the exam had started.

   Look at the sentence *Little did Wiles know that ...* (l. 10) Notice how the subject *Wiles* comes after the auxiliary *did*. Translate the sentence into Russian.
4. Consider some of many ways of translating *as*: 1) так как, поскольку; 2) как, в качестве; 3) по мере того как.

   Explain the meaning of *as* and translate it in 1–5 below:

   1) named *as* such (l. 12)
   2) and *as* they go (l. 34)
   3) serve *as* a cornerstone (l. 70)
   4) *as* the decades passed (l. 90)
   5) *as* it was formulated (l. 87)
5. *Those* is the plural of the demonstrative *that*. It can be a replacement for an earlier phrase. It means 'the ones ... '

E. g. … they are described by cubic equations, like *those* (=the ones=cubic equations) used for calculating … (l. 13)

In the word combination *those used for calculating*, consider the function of the past participle *used.* Note that a participle placed after a noun often has the same 'identifying' function as a relative clause. Compare:

a) the only *place left*

b) the only *place that was left*

In the above example, *those* is used instead of a noun. Reconstruct the whole phrase and translate it into Russian.

6. Consider the sentence *Such equations are said to be of degree* 3 … (l. 16) We often use a passive to report what people say, think, etc., particularly if we want to avoid mentioning who said or thought what we were reporting:

   Everyone *was asked* to bring some food to the party.

   A common way of reporting what is said by people in general is to use *it + passive verb + that + clause*:

   It is said that this problem has already been solved.

   An alternative to *it + passive verb + that + clause* is to use *subject + passive verb + to-infinitive* (Complex Subject).

   This problem is said to have already been solved.

   Come back to the sentence *Such equations* … (l. 16), paraphrase it using *it + passive verb + that + clause* and translate it into Russian. What other verbs can be used in *Complex Subject*?

7. Compare the following sentences:

   1) In general, cubic equations for elliptic curves take the form $y^2 = x^3 + ax^2 + bx + c$, where a, b and c are whole numbers *that satisfy some simple conditions.*
   2) Such equations are said to be of degree 3, because the highest exponent *they contain* is a cube.
   3) For complicated equations, typically of degree 4 or higher, the number of solutions is always finite—a fact called Mordell's conjecture, *which the German mathematician Gerd Faltings proved in 1983.*

All these sentences contain relative clauses. *A relative clause* gives more information about one of the nouns in the main clause. Some relative clauses *(defining relative clauses)* are used to specify *which* person or thing we are referring to, or which *type* of person or thing we are referring to. Notice that we don't put a comma between the noun and a defining relative clause. Relative clauses begin with *a relative pronoun: a wh-word (who, which, etc.) or that* (1). The relative pronoun can be omitted (2).

Some relative clauses are used to add extra information about a noun, but this information is not necessary to explain which person or thing we are referring to *(non-defining relative clauses).* Don't use *that* at the beginning of a non-defining clause. Use *who (whom, whose)* or *which* instead. Notice that we put a comma between the noun and a non-defining relative clause and another comma at the end of this clause if it is not also the end of a sentence (3).

Identify the noun to which the italicized clauses refer. Find in the text other examples of the above type of clauses.

8. Note the translation of *either/neither*: either—любой, either ... or—или ... или (либо ... либо), neither—1) ни один (из) + не (к сказуемому); 2) а также + не (к сказуемому), neither ... nor—ни ... ни. Translate the sentence *Linear or quadratic ...* (ll. 21–23).
9. Consider the word combination *to be the case* (l. 23). It is translated иметь место, происходить.
10. One of the meanings of the word *challenge* (n.) is something that tests strength, skill, or ability especially in a way that is interesting, e. g. I liked the speed and challenge of racing. Translate the sentence: *But elliptic curves present a unique challenge.* (l. 26).
11. Use your dictionary to check pronunciation: *finite—infinite.*
12. Compare the functions of −ing forms in:

    a) there is no easy way of *telling* (l. 28)
    b) ... problems *concerning* elliptic curves (l. 29)

c) ... mathematicians often reexamine them *using* modular arithmetic (l. 29)
d) ... the *corresponding* modular forms (l. 83)

13. *To simplify problems ... , mathematicians often reexamine ...* (l. 29). *To grasp some understanding of the original infinite problem, mathematicians observe ...* (l. 40). What is the function of the infinitive in these sentences? Cf. also 2, p. 6.
14. Read lines 29–37 and state the function of the following linking words: *next*, *then*, *eventually*.
15. Translate the sentence *What makes* (ll. 53–55) into Russian, paying attention to the functions of the subordinate clauses *What makes ...* and ... *that one can ...* .
16. What does the phrasal verb *to give up* (l. 65) mean?
17. Look at the following word combinations and think about the functions of past participles:
    a) objects called elliptic curves
    b) they are named
    c) so-called rational solutions
    d) he became enamored with
    e) complicated equations
    f) to a limited extent
18. In the sentence *In particular, because modular forms ...* (ll. 73–77), the word *that* is used three times. Identify where *that* replaces a noun. What is the noun? Translate the sentence into Russian. In lines 78–86 find the sentences with *that* in the same function and translate them.
19. Learn to distinguish between three types of conditional sentences shown in the following examples. Translate them into Russian.
    1) If I **work** hard, I **will pass** my exams. (real conditions mainly for future possibilities)
    2) If I **worked** hard, I **would pass** my exams. ('unreal' conditions or improbable conditions in the present or future)

3) If I **had worked** hard, I **would have passed** my exams. ('unreal' conditions or impossible conditions in the past)

Now, come back to the text (ll. 78–79). What type of conditional sentence is this? Translate the sentence into Russian. Note that a conditional clause can come before or after the main clause. We often use a comma when the if-clause comes first. What other conjunctions can be used to introduce conditional clauses?

20. Find a word in the text (ll. 78–86) with the meaning *up to this time.*
21. Compare the uses of *other* (l. 86 and l. 89). What is the difference?
22. *In due course* means at some time in the future when it is the right time, but not before: *The committee will consider your application in due course.* Find it in the text (l. 94) and translate it into Russian.
23. Give equivalents of the following:
    a) другими словами
    b) совершенно другая область математики
    c) на пятнадцатый день
    d) в этом отношении
    e) до определенной степени
    f) эта задача сводится к
    g) по существу
    h) в действительности
    i) не следует путать с
    j) искать ошибку в вычислениях
24. Discuss the following questions in pairs:
    1) "At age 10, Wiles became enamored with Fermat's last theorem. ... His schoolteachers discouraged him from wasting time on the impossible." But later he came back to the theorem again. How did it happen?
    2) "To simplify problems concerning elliptic curves, mathematicians often reexamine them using modular arithmetic." What is the method they follow? What advantage does modular arithmetic have?

3) " ... as the decades passed the hypothesis became increasingly influential." Which hypothesis is meant?

Read Part IV at a quick comfortable pace to answer the questions: What was a new strategy for attacking Fermat's last theorem? Then read it again more carefully to do the exercises.

THE MISSING LINK

In the fall of 1984, at a symposium in Oberwolfach, Germany, Gerhard Frey of the University of Saarland gave a lecture that hinted at a new strategy for attacking Fermat's last theorem. The theorem asserts that Fermat's equation has no positive whole-number solutions. To test a statement of this type, mathematicians frequently assume that it is false and then explore the consequences. To say that Fermat's last theorem is false is to say that there are two perfect nth powers whose sum is a third nth power.

Frey's idea proceeded as follows: Suppose that A and B are perfect nth powers of two numbers such that $A + B$ is again an nth power—that is, they are a solution to Fermat's equation. A and B can then be used as coefficients in a special elliptic curve: $y^2 = x(x - A)(x + B)$. A quantity that is routinely calculated whenever one studies elliptic curves is the "discriminant" of the elliptic curve, $A^2B^2(A + B)^2$. Because A and B are solutions to the Fermat equation, the discriminant is a perfect nth power.

The crucial point in Frey's tactic is that if Fermat's last theorem is false, then whole-number solutions such as A and B can be used to construct an elliptic curve whose discriminant is a perfect nth power. So a proof that the discriminant of an elliptic curve can never be an nth power would contain, implicitly, a proof of Fermat's last theorem. Frey saw no way to construct that proof. He did, however, suspect that an elliptic curve whose discriminant was a perfect nth power-if it existed-could not be modular. In other words, such an elliptic curve would defy the Shimura-Taniyama conjecture. Running the argument backwards, Frey pointed out that if someone proved that the Shimura-Taniyama conjecture is true and that the elliptic equation $y^2 = x(x - A)(x + B)$ is not modular, then they would have shown that the elliptic equation cannot exist.

In that case, the solution to Fermat's equation cannot exist, and Fermat's last theorem is proved true.

Many mathematicians explored this link between Fermat and Shimura-Taniyama. Their first goal was to show that the Frey elliptic curve, $y^2 = x(x - A)(x + B)$, was in fact not modular. Jean-Pierre Serre of the College of France and Barry Mazur of Harvard University made important contributions in this direction. And in June 1986 one of us (Ribet) at last constructed a complete proof of the assertion. It is not possible to describe the full argument in this article, but we will give a few hints.

To begin, Ribet's proof depends on a geometric method for "adding" two points on an elliptic curve. Visually, the idea is that if you project a line through a pair of distinct solutions, P_1 and P_2, the line cuts the curve at a third point, which we might provisionally call the sum of P_1 and P_2. A slightly more complicated but more valuable version of this addition is as follows: first add two points and derive a new point, P_3, as already described, and then reflect this point through the x axis to get the final sum, Q.

This special form of addition can be applied to any pair of points within the infinite set of all points on an elliptic curve, but this operation is particularly interesting because there are finite sets of points having the crucial property that the sum of any two points in the set is again in the set. These finite sets of points form a group: a set of points that obeys a handful of simple axioms. It turns out that if the elliptic curve is modular, so are the points in each finite group of the elliptic curve. What Ribet proved is that a specific finite group of Frey's curve cannot be modular, ruling out the modularity of the whole curve.

For three and half centuries, the last theorem had been an isolated problem, a curious and impossible riddle on the edge of mathematics. In 1986 Ribet, building on Frey's work, had brought it center stage. It was possible to prove Fermat's last theorem by proving the Shimura-Taniyama conjecture. Wiles, who was by now a professor at Princeton, wasted no time. For seven years, he worked in complete secrecy. Not only did he want to avoid the pressure of public attention, but he hoped to keep others from copying his ideas. During this period, only his wife learned of his obsession—on their honeymoon.

1. *Fall* (l. 1) means autumn in *AmE*.
2. Identify the part of speech of the −ing forms *attacking* (l. 3), *proving* (l. 62) and translate them into Russian.
3. The infinitive is used in different syntactic functions. Some of them we have already dealt with (see 2, p. 8–9). To review the others, consult any grammar book, then consider the following sentences and translate them into Russian:
   a) To test a statement of this type, mathematicians frequently assume that . . . . (l. 5)
   b) To say that Fermat's theorem is false is to say that . . . . (l. 7)
   c) . . . then whole-number solutions such as A and B can be used to construct an elliptic curve . . . . (l. 18)
   d) Frey saw no way to construct that proof. (l. 22)
   e) Their first goal was to show that . . . . (l. 33)
   f) It is not possible to describe the full argument . . . . (l. 38)
   g) To begin, Ribet's proof depends on geometric method . . . . (l. 40)
4. Note the position for adverbs of frequency (e. g. sometimes, occasionaly, always, often, usually, never) that go with the verb:
   a) after am/is/are/was/were
      She is *never* at home these days.
   b) after auxiliary verbs
      He can *never* come in time.
   c) before other verbs
      You *usually* read better than this.

   In this part of the text, find some sentences where the adverbs of frequency are used.
5. Find the English equivalent of *i. e.* (ll. 9–16)
6. Note that there are two ways to use proper nouns as modifiers: *Fermat's equations* (l. 11); *the Fermat equation* (l. 16)
   Look through the text again to find some other examples.
   Can you work out the rule?

7. Consider the sentence: *So a proof that ...* (l. 20). In the main clause, find its subject and verb.
8. In the sentence, *He did, however, suspect ...* (l. 23), identify the function of the verb *to do* and translate the sentence into Russian.
9. Give Russian equivalents of: *the idea proceeded as follows, whenever one studies, the crucial point, running the argument backwards, a slightly more complicated but more valuable version, to keep others from copying his ideas, a lecture that hinted at a new strategy, by now, a curious and impossible riddle on the edge of mathematics.*
10. Go through the paragraph (ll. 48–57). This time, comment on the use of articles (a, the, zero) and translate the paragraph into Russian.[2]
11. Find the emphatic structure (ll. 58–67) and translate the whole sentence into Russian. (See also 3, p. 17)
12. Write a summary of this part of the text.

Read Part V without consulting any dictionary. Try to guess the meaning of the unknown words by thinking about the context in which they are found. Here Wiles "describes his experience of doing mathematics as a journey through a dark, unexplored mansion." Reproduce it.

SEVEN YEARS OF SECRECY

Wiles had to pull together many of the major findings of 20th-century number theory. When those ideas were inadequate, he was forced to create other tools and techniques. He describes his experience of doing mathematics as a journey through a dark, unexplored mansion: "You enter the first room of the mansion, and it's completely dark. You stumble around bumping into the furniture, but gradually you learn where each piece of furniture is. Finally, after six months or so, you find the light switch. You turn it on, and suddenly it's all illuminated. You can see exactly where you were.

[2]Note that the meaning of the English article may sometimes be expressed in Russian by means of the words *какой-нибудь, один, любой* (the indefinite article), *этот, тот самый* (the definite article), *вообще* (zero).

Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of, and couldn't exist without, the many months of stumbling around in the dark that precede them."

As it turned out, Wiles did not have to prove the full Shimura-Taniyama conjecture. Instead he had to show only that a particular subset of elliptic curves—one that would include the hypothetical elliptic curve Frey proposed, should it exist—is modular. It wasn't really much of a simplification. This subset is still infinite in size and includes the majority of interesting cases. Wiles's strategy used the same techniques employed by Ribet, plus many more. And as with Ribet's argument, it is possible to give only a hint of the main points involved.

The difficulty was to show that every elliptic curve in Wiles's subset is modular. To do so, Wiles exploited the group property of points on the elliptic curves and applied a theorem of Robert P. Langlands of the Institute for Advanced Study in Princeton, N.J., and Jerrold Tunnell of Rutgers University. The theorem shows, for each elliptic curve in Wiles's set, that a specific group of points inside the elliptic curve is modular. This requirement is necessary but not sufficient to demonstrate that the elliptic curve as a whole is modular.

The group in question has only nine elements, so one might imagine that its modularity represents an extremely small first step toward complete modularity. To close this gap, Wiles wanted to examine increasingly larger groups, stepping from groups of size 9 to 9^2, or 81, then to 9^3, or 729, and so on. If he could reach an infinitely large group and prove that it, too, is modular, that would be equivalent to proving that the entire curve is modular.

Wiles accomplished this task via a process loosely based on induction. He had to show that if one group was modular, then so must be the next larger group. This approach is similar to toppling dominoes: to knock down an infinite number of dominoes, one merely has to ensure that knocking down any one domino will always topple the next. Eventually Wiles felt confident that his proof was complete, and on June 23, 1993, he announced his result at a conference at the Isaac Newton Mathematical Sciences Institute in Cambridge. His secret research program had been a success, and the mathematical community and the world's press were surprised and delighted by his proof. The front page of the *New York Times* exclaimed, "At Last, Shout of 'Eureka!' in Age-Old Math Mystery."

As the media circus intensified, the official peer-review process began. Almost immediately, Nicholas M. Katz of Princeton uncovered a fundamental and devastating flaw in one stage of Wiles's argument. In his induction process, Wiles had borrowed a method from Victor A. Kolyvagin of Johns Hopkins University and Matthias Flach of the California Institute of Technology to show that the group is modular. But it now seemed that this method could not be relied on in this particular instance. Wiles's childhood dream had turned into a nightmare.

1. *To pull sth together* (phr v) means to improve something by organizing it more effectively.
2. To make *have to* negative, we use an auxiliary verb. Translate the sentence: ... *Wiles didn't have to* ... (l. 15). Explain what is the difference between *must not* and *do not have to.*
3. What does *one* mean (l. 17, l. 33, l. 44 (2))?
4. *Should it exist* (l. 18) means *if it should exist,* here inversion takes place to express emphasis. Translate the sentence into Russian.
5. Give a Russian equivalent of *much of a simplification* (l. 19).
6. Compare −ed forms. Identify the parts of speech and their functions: strategy *used* (l. 20), techniques *employed* (l. 21), points *involved* (l. 23).
7. Identify the words used by the authors as equivalent to *to do so* (l. 25).
8. *The group in question* (l. 33) is the group that is being discussed or talked about. Explain which group is meant. Translate the sentence into Russian.
9. In the sentence *If he could reach* ... (l. 37), comment on the tense forms used and give a Russian equivalent.

10. Find a word in the text with the same or similar meaning to the following: by way of (sth) or through.
11. In this text, the meaning of *circus* (l. 53) can be explained as follows: if someone describes an event as a circus, they mean that they think it is only being done to attract attention or to impress people, and will not achieve anything. Think about its Russian equivalent.
12. *Peer* means 1) someone of the same age, social class etc as you; 2) a member of the British nobility who has the right to sit in the House of Lords. What is the meaning of *peer* (l. 53) in the text? Translate the sentence.
13. Try to explain the authors' use of the tense forms (ll. 53–61).
14. See if you remember: *to pull together major findings, to be forced to do sth, to create tools and techniques, to do mathematics, as a whole, the group in question, to be similar to, to uncover a flaw, to accomplish a task, to borrow a method from sb, to rely on.*

Read Part VI without consulting any dictionary. What is meant by *they found the vital fix*?

FINDING THE FIX

For the next 14 months, Wiles hid himself away, discussing the error only with his former student Richard Taylor. Together they wrestled with the problem, trying to patch up the method Wiles had already used and applying other tools that he had previously rejected. They were at the point of admitting defeat and releasing the flawed proof so that others could try to correct it, when, on September 19, 1994, they found the vital fix. Many years earlier Wiles had considered using an alternative approach based on so-called Iwasawa theory, but it floundered, and he abandoned it. Now he realized that what was causing the Kolyvagin-Flach method to fail was exactly what would make the Iwasawa theory approach succeed.

Wiles recalls his reaction to the discovery: "It was so indescribably beautiful; it was so simple and so elegant. The first night I went back home and slept on it. I checked through it again the next morning, and I went down and told my wife, 'I've got it. I think I've found it.' And it was so unexpected that she thought I was talking about a children's toy or something, and she said, 'Got what?' I said, 'I've fixed my proof. I've got it.'"

For Wiles, the award of the Wolfskehl Prize marks the end of an obsession that lasted more than 30 years: "Having solved this problem, there's certainly a sense of freedom. I was so obsessed by this problem that for eight years I was thinking about it all of the time—when I woke up in the morning to when I went to sleep at night. That particular odyssey is now over. My mind is at rest." For other mathematicians, though, major questions remain. In particular, all agree that Wiles's proof is far too complicated and modern to be the one that Fermat had in mind when he wrote his marginal note. Either Fermat was mistaken, and his proof, if it existed, was flawed, or a simple and cunning proof awaits discovery.

1. Compare: Together they wrestled with the problem, *trying* to patch up the method ... (l. 3) *Having solved* this problem, there's certainly a sense of freedom. (l. 20) Explain the difference in meaning expressed by the participles and their translation.
2. Identify the sentences with the Past Continuous. Explain why this form is used there.
3. What is the tense form in the sentences with the contractions (ll. 12–18)?
4. Say which sentences contain phrasal verbs, explaining why you think they are phrasal verbs.
5. Compare:
   That particular odyssey ... (l. 24)
   In particular, ... (l. 26)
   What are the functions of the word *particular*?
6. Find a word combination in the text with the same or similar meaning to the following: to be thinking about or considering sth for a particular purpose.
7. Give Russian equivalents of: *they wrestled with the problem, trying to patch up the method, tools ... previously rejected, they were at the point of admitting defeat, the flawed proof, the vital fix, the proof is far too complicated, either ... or.*

Video

1. Having read the text, watch the video *Fermat's Last Theorem* co-produced by Simon Singh, one of the authors of the article. In this documentary Andrew Wiles tells his story. As you watch the video, take notes for they will help you in discussion.
2. Having watched the video, choose the topic or topics you would like to discuss in pairs.
3. Choose a student to take the role of A. Wiles and ask him questions.
4. Using your notes, give an oral summary of what you have watched.

What do you think?

1. Why has this theorem been so difficult to prove?
2. Are mathematicians finally satisfied with Andrew Wiles' proof of Fermat's last theorem?

If you are interested in the opinion of experts, go to:
`http://www.sciam.com/askexpert/math/math6.html`

Translation

Translate the following sentences into English:

1. 23 июня 1993 г. Э. Уайлз сообщил математическому сообществу, что он доказал знаменитую теорему Ферма.
2. На поиски доказательства теоремы у него ушло десять лет интенсивного труда. При этом Э. Уайлз работал в полной изоляции, потому что он хотел не только избежать повышенного интереса общественности, но и не дать возможности другим воспользоваться его идеями.
3. Сто страниц текста, которые представляют собой доказательство, соединили в себе достижения разных областей

математики, таких, например, как алгебраическая геометрия и комплексный анализ.

4. В 1950-х годах Г. Шимура и Ю. Танияма выдвинули некую идею, которая впоследствии была использована Э. Уайлзом в его доказательстве.
5. Их предположение касалось модулярных функций, которые имеют дело с комплексными числами вида $(x + iy)$, где x и y—действительные числа, i—мнимое число (равное $\sqrt{-1}$).
6. Осенью 1984 г. Г. Фрей на одной из своих лекций подсказал новый подход к решению теоремы Ферма.
7. Как выяснилось позже, Э. Уайлзу не нужно было доказывать всю гипотезу Г. Шимуры и Ю. Таниямы. Ему нужно было лишь показать, что некое подмножество эллиптических кривых модулярно. Но сделать это было не так-то просто.
8. Э. Уайлз решил задачу методом индукции. Ему нужно было показать, что если одна группа модулярна, то модулярной должна быть и следующая большая группа.
9. Когда Э. Уайлза уже поздравляли с блестящим результатом его многолетней работы, Н.М. Кац обнаружил существенную ошибку в его доказательстве. На ее ликвидацию ушло еще 14 месяцев.
10. Однако, несмотря на то, что полученный Э. Уайлзом результат воспринят всеми как выдающееся достижение математики XX в., многие специалисты считают, что представленное им доказательство основано на достижениях современной науки, поэтому вряд ли это то доказательство, о котором писал Ферма.

Black Holes

Reading

Here is an early history of black holes:

1900 Max Planck discovers black-body radiation.

1905 In a paper on black-body radiation, Albert Einstein shows that light can be viewed as particles (photons).

1915 Through spectroscopic studies, astronomer Walter S. Adams identifies Sirius's faint companion (which causes Sirius to wobble slightly as it moves) as a small, hot, dense star—a white dwarf.

1916 Einstein published his general theory of relativity, producing equations that describe gravity.

1916 Karl Schwarzschild shows that a radius of a collapsing object exists at which Einstein's gravity equations become "singular"—time vanishes, and space becomes infinite.

1924 Einstein publishes Satyendra Nath Bose's work on black-body radiation, developing so-called quantum statistics for one class of particles (such as photons).

1924 Sir Arthur Eddington proposes that gravity can strip away electrons from protons in a white dwarf.

1925 Wolfgang Pauli formulates the exclusion principle, which states that certain particles cannot be in exactly the same quantum-mechanical state.

1926 Enrico Fermi and P.A.M. Dirac develop quantum statistics for particles that obey Pauli's exclusion principle (such as electrons and protons). When compressed, such particles fly away from one another, creating a so-called degeneracy pressure.

1930 Using quantum statistics and Eddington's work on stars, Subrahmanyan Chandrasekhar finds that the upper mass limit for white dwarfs is 1.4 times the mass of the sun, suggesting that more massive stars collapse into oblivion. Eddington makes fun of him.

1932 James Chadwick discovers the neutron. Its existence leads researchers to wonder if "neutron stars" could be an alternative to white dwarfs.

1939 Sparked by conversations with colleagues, Einstein tries to kill off the Schwarzschild radius once and for all: he concludes that black holes are impossible in a paper published in *Annals of Mathematics.*

1939 Using ideas of collapsing neutron stars and white dwarfs, J. Robert Oppenheimer and his student Hartland S. Snyder show how a black hole can form.

Now you are going to read an article about the theory of black holes and the role played in it by Albert Einstein. The text is divided into six parts, each of which is followed by exercises. Some of them contain notes to simplify a task. After reading Part I for the first time to get the general idea, read it again more carefully and do the exercises. The same procedure should be followed when you pass on to the next part of the text and so on.

1. Look through the title and the annotation of the article below. The word *reluctant* means slow and unwilling: *he gave a reluctant smile.* Think of its Russian equivalent and translate the title.

THE RELUCTANT FATHER OF BLACK HOLES

Albert Einstein's equations of gravity are the foundation of the modern view of black holes; ironically, he used the equations in trying to prove these objects cannot exist

by Jeremy Bernstein

Great science sometimes produces a legacy that outstrips not only the imagination of its practitioners but also their intentions. A case in point is the early development of the theory of black holes and, above all, the role played in it by Albert Einstein. In 1939 Einstein published a paper in the journal *Annals of Mathematics* with the daunting title "On a Stationary System with Spherical Symmetry Consisting of Many Gravitating Masses." With it, Einstein sought to prove that black holes—celestial objects so dense that their gravity prevents even light from escaping—were impossible.

The irony is that, to make his case, he used his own general theory of relativity and gravitation, published in 1916—the very theory that is now used to argue that black holes are not only possible but, for many astronomical objects, inevitable. Indeed, a few months after Einstein's rejection of black holes appeared—and with no reference to it—J. Robert Oppenheimer and his student Hartland S. Snyder published a paper entitled "On Continued Gravitational Contraction." That work used Einstein's general theory of relativity to show, for the first time in the context of modern physics, how black holes could form.

Perhaps even more ironically, the modern study of black holes, and more generally that of collapsing stars, builds on a completely different aspect of Einstein's legacy—namely, his invention of quantum-statistical mechanics. Without the effects predicted by quantum statistics, every astronomical object would eventually collapse into a black hole, yielding a universe that would bear no resemblance to the one we actually live in.

2. *A case in point* (l. 3) means a clear example of a situation, problem etc that you are discussing or explaining. The word *case* is also used here in the expression *to make his case* (l. 10). Explain which case is meant and give Russian equivalents of both expressions. Cf. also 9, p. 19.
3. Find a word in the text (ll. 1–19) with the same or similar meaning to the following: to look for, try to get.
4. The adjective *very* is used to emphasize a noun: We climbed to the *very* top of the mountain. Explain its meaning in *the very theory* (l. 11).

5. Identify the parts of speech and their functions of the following −ed forms: *played* (l. 4), *published* (l. 5, l. 11, l. 16), *used* (l. 10, l. 12, l. 17), *appeared* (l. 14), *entitled* (l. 16), *predicted* (l. 23).
6. In lines 10–19, find an infinitive clause of purpose and translate it into Russian.
7. Identify the word (ll. 20–26) used by the author to clarify his ideas.
8. Compare different functions of *that* (l. 10, l. 17, l. 21). What is the plural form of *that*?
9. Consider the functions of the −ing forms: *collapsing* (l. 21) and *yielding* (l. 25).
10. What noun is replaced by *one* to avoid repeating (l. 26)?
11. When relative pronouns are left out, this can make reading difficult. Study the last sentence and find the relative clause of the above type. Make this sentence easier to read by adding a relative pronoun.
12. Put in suitable prepositions:

    a) a case ... point
    b) the role played ... it
    c) ' ... Continued Gravitational Contraction'
    d) to prevent light ... escaping
    e) with no reference ... it
    f) the modern study ... black holes builds ... a completely different aspect ... Einstein's legacy
    g) the effects predicted ... quantum statistics
    h) every astronomical object would eventually collapse ... a black hole
    i) to bear no resemblance ... sth

13. Check up for comprehension: What is the role of Albert Einstein in the early development of the theory of black holes?

Now read Part II. Use your dictionary to check new words and expressions, then move on to the exercises.

BOSE, EINSTEIN AND STATISTICS

Einstein's creation of quantum statistics was inspired by a letter he received in June 1924 from a then unknown young Indian physicist named Satyendra Nath Bose. Along with Bose's letter came a manuscript that had already been rejected by one British scientific publication. After reading the manuscript, Einstein translated it himself into German and arranged to have it published in the prestigious journal *Zeitschrift für Physik*.

Why did Einstein think that this manuscript was so important? For two decades, he had been struggling with the nature of electromagnetic radiation—especially the radiation trapped inside a heated container that attains the same temperature as its walls. At the turn of the century the German physicist Max Planck had discovered the mathematical function that describes how the various wavelengths, or colors, of this "black body" radiation vary in intensity. It turns out that the form of this spectrum does not depend on the material of the container walls. Only the temperature of the radiation matters. (A striking example of black-body radiation is the photons left over from the big bang, in which case the entire universe is the "container." The temperature of these photons was recently measured at 2.726 ± 0.002 kelvins.)

Somewhat serendipitously, Bose had worked out the statistical mechanics of black-body radiation—that is, he derived the Planck law from a mathematical, quantum-mechanical perspective. That outcome caught Einstein's attention. But being Einstein, he took the matter a step further. He used the same methods to examine the statistical mechanics of a gas of massive molecules obeying the same kinds of rules that Bose had used for the photons. He derived the analogue of the Planck law for this case and noticed something absolutely remarkable. If one cools the gas of particles obeying so-called Bose-Einstein statistics, then at a certain critical temperature all the molecules suddenly collect themselves into a "degenerate," or single, state. That state is now known as Bose-Einstein condensation (although Bose had nothing to do with it).

An interesting example is a gas made up of the common isotope helium 4, whose nucleus consists of two protons and two neutrons.

At a temperature of 2.18 kelvins, this gas turns into a liquid that has the most uncanny properties one can imagine, including frictionless flow (that is, superfluidity). U.S. researchers in the past year accomplished the difficult task of cooling other kinds of atoms to several billionths of a kelvin to achieve a Bose-Einstein condensate.

Not all the particles in nature, however, show this condensation. In 1925, just after Einstein published his papers on the condensation, the Austrian-born physicist Wolfgang Pauli identified a second class of particles, which includes the electron, proton and neutron, that obeyed different properties. He found that no two such identical particles—two electrons, for example—can ever be in exactly the same quantum-mechanical state, a property that has since become known as the Pauli exclusion principle. In 1926 Enrico Fermi and P.A.M. Dirac invented the quantum statistics of these particles, making them the analogue of the Bose-Einstein statistics.

Because of the Pauli principle, the last thing in the world these particles want to do at low temperatures is to condense. In fact, they exhibit just the opposite tendency. If you compress, say, a gas of electrons, cooling it to very low temperatures and shrinking its volume, the electrons are forced to begin invading one another's space. But Pauli's principle forbids this, so they dart away from one another at speeds that can approach that of light. For electrons and the other Pauli particles, the pressure created by these fleeing particles—the "degeneracy pressure"—persists even if the gas is cooled to absolute zero. It has nothing to do with the fact that the electrons repel one another electrically. Neutrons, which have no charge, do the same thing. It is pure quantum physics.

1. Write out all the examples where a proper noun is used to modify another noun. E.g. *Einstein's creation.* Compare your examples. Explain the use or omission of the article before proper nouns.

2. Note the difference between *then* as an adverb and as an adjective.

Adverb of time

a) *Then* means '*at that time*' or '*after that*'.
E. g. We met in 1998. I was still a student *then*.

b) *Then* means '*after that*' in a series of points or events, *first* ... *then*.

Linking adverb

a) *Then* can mean '*in that case*' (mainly in spoken English).
E. g. 'They've just telephoned to say John's in hospital.'
'*Then* we'd better go immediately.'

b) We sometimes use *if* ... *then* to emphasize that one thing depends on another.
E. g. If $x = 3$ and $y = 5$ then $xy = 15$
Consider the sentence *If one cools* ... (ll. 29–32). Translate it into Russian.

Adjective 'The *then* President, Director, etc.' means 'the President, Director, etc. at a particular time in the past'. What is the Russian equivalent of *a then unknown young Indian physicist* (l. 2)?

3. In lines 1–7, find a sentence with inversion. Cf. also 3, p. 17. Why is inversion used here?
4. Put in reflexive pronouns.

I — myself	we —
you —	you —
he —	they —
she —	
it —	

Consider the use of reflexive pronouns in the text (l. 6, l. 31).

5. *Have something done*: arrange something to be done. Notice the difference in meaning:
I have typed my report. = I typed it myself.
My report has been typed. = Someone typed it.
I have had my report typed. = I arranged/paid for someone to type it for me.

In lines 1–20, identify the above structure and translate it into Russian.

6. Comment on the use of tenses (ll. 8–20).
7. What part of speech is *matter* in l. 17 and l. 25? Compare the functions, give Russian equivalents.
8. Consider the relative clauses (l. 11, l. 13, l. 18). Comment on the use of a comma or its omission. Cf. also 7, pp. 18–19.
9. *Somewhat* means 'more than a little but not very': *The price is somewhat higher than I expected.*
   Serendipity means 'the natural ability to make interesting or valuable discoveries by accident'.
   What is the Russian equivalent of *somewhat serendipitously* (l. 21)?
10. Identify the function of *that is* (l. 22, l. 38).
11. What is meant by *this case* (l. 28) and *that state* (l. 32)?
12. Consider the sentence *In 1925, just after* ... (l. 43–46). It contains two relative clauses. Is there any difference between them? Identify the nouns to which these clauses refer. Cf. also 7, pp. 18–19.
13. *Friction* is a noun. What part of speech is *frictionless*? (l. 37)
14. Compare:
    They *blamed themselves* for the mistake. = They both took the blame.
    They *blamed each other* for the mistake. = The one blamed the other.
    You can use *one another* instead of *each other*. However, sometimes a distinction is drawn between *each other* (used to refer to two people etc) and *one another* (used to refer to more than two). Find the sentences (3) with *one another* in the text and translate them into Russian.
15. Insert prepositions:
    Along ... Bose's letter came a manuscript; he translated it himself ... German; ... two decades, he had been struggling ... the nature ... electromagnetic radiation; ... the turn of the century; the function describes how the various wavelengths or colors vary ... intensity; it turns ... that; this

spectrum does not depend … the material; he worked … the statistical mechanics … black-body radiation; a gas made … of the common isotope helium 4, whose nucleus consists … two protons and two neutrons; this gas turns … a liquid; his papers … the condensation.

16. Explain the function of the word *say* (l. 54).
17. Put the following phrases from the text into your own words:
    a) … creation … was inspired by a letter … (l. 1)
    b) … a letter … from a then unknown … physicist … (ll. 1–2)
    c) at the turn of the century … (l. 12)
    d) … the temperature … matters (ll. 16–17)
    e) Somewhat serendipitously … (l. 21)
    f) … he took the matter a step further (ll. 24–25)
    g) … although Bose had nothing to do with it (l. 33)
    h) … the last thing in the world these particles want to do … (ll. 52–53)
18. See if you remember:
    to depend on, to derive a law, to obey rules, to have sth to do with sth, to consist of, to become known as, to exhibit a tendency, to do the same thing, a principle forbids, because of this principle, it turns out that, so-called Bose-Einstein statistics.
19. Check up for comprehension:
    1. Why did Einstein think that Bose's manuscript was so important?
    2. What method did Einstein use to examine the statistical mechanics of a gas of massive molecules obeying the same kinds of rules that Bose had used for the photons?
    3. What is the Pauli exclusion principle?

Read Part III. Use your dictionary to check new words and expressions, then pass on to the exercises.

QUANTUM STATISTICS AND WHITE DWARFS

But what has quantum statistics got to do with the stars? Before the turn of the century, astronomers had begun to identify a class of peculiar stars that are small and dim: white dwarfs. The one that accompanies Sirius, the brightest star in the heavens, has the mass of the sun but emits about 1/360 the light. Given their mass and size, white dwarfs must be humongously dense. Sirius's companion is some 61,000 times denser than water. What are these bizarre objects? Enter Sir Arthur Eddington.

When I began studying physics in the late 1940s, Eddington was a hero of mine but for the wrong reasons. I knew nothing about his great work in astronomy. I admired his popular books (which, since I have learned more about physics, now seem rather silly to me). Eddington, who died in 1944, was a neo-Kantian who believed that everything of significance about the universe could be learned by examining what went on inside one's head. But starting in the late 1910s, when Eddington led one of the two expeditions that confirmed Einstein's prediction that the sun bends starlight, until the late 1930s, when Eddington really started going off the deep end, he was truly one of the giants of 20th-century science. He practically created the discipline that led to the first understanding of the internal constitution of stars, the title of his classic 1926 book. To him, white dwarfs were an affront, at least from an aesthetic point of view. But he studied them nonetheless and came up with a liberating idea.

In 1924 Eddington proposed that the gravitational pressure that was squeezing the dwarf might strip some of the electrons off protons. The atoms would then lose their "boundaries" and might be squeezed together into a small, dense package. The dwarf would eventually stop collapsing because of the Fermi-Dirac degeneracy pressure—that is, when the Pauli exclusion principle forced the electrons to recoil from one another.

The understanding of white dwarfs took another step forward in July 1930, when Subrahmanyan Chandrasekhar, who was 19, was on board a ship sailing from Madras to Southampton. He had been accepted by the British physicist R.H. Fowler to study with him at the University of Cambridge (where Eddington was, too). Having read Eddington's book on the stars and Fowler's book on quantum-statistical mechanics, Chandrasekhar had become fascinated by white dwarfs. To pass the time during the voyage, Chandrasekhar asked himself: Is there any upper limit to how massive a white dwarf can be before it collapses under the force of its own gravitation? His answer set off a revolution.

A white dwarf as a whole is electrically neutral, so all the electrons must have a corresponding proton, which is some 2,000 times more massive. Consequently, protons must supply the bulk of the gravitational compression. If the dwarf is not collapsing, the degeneracy pressure of the electrons and the gravitational collapse of the protons must just balance. This balance, it turns out, limits the number of protons and hence the mass of the dwarf. This maximum is known as the Chandrasekhar limit and equals about 1.4 times the mass of the sun. Any dwarf more massive than this number cannot be stable.

Chandrasekhar's result deeply disturbed Eddington. What happens if the mass is more than 1.4 times that of the sun? He was not pleased with the answer. Unless some mechanism could be found for limiting the mass of any star that was eventually going to compress itself into a dwarf, or unless Chandrasekhar's result was wrong, massive stars were fated to collapse gravitationally into oblivion.

Eddington found this intolerable and proceeded to attack Chandrasekhar's use of quantum statistics—both publicly and privately. The criticism devastated Chandrasekhar. But he held his ground, bolstered by people such as the Danish physicist Niels Bohr, who assured him that Eddington was simply wrong and should be ignored.

1. What does the pronoun *one* stand for? (l. 3)
2. In line 5, *given* (prep) is used to say that the fact that 'white dwarfs must be humongously dense' is not surprising when you consider their mass and size. Translate the sentence into Russian.
3. In lines 1–8, find an imperative.
4. Look at the word combination *a hero of mine* (l. 10). *Mine* is a possessive form of the personal pronoun *I*. What is the difference between *my* and *mine*? *My* occurs before a noun, in the position of a word such as *the* or *a(n)*. *Mine* is never

used in front of nouns. It can stand alone as subject, object, etc, as pronouns can.

Compare: Have you seen *my* book?
No. This one is *mine*.

Complete the table of the possessive forms of personal pronouns.

	I	you	he	she	it	we	they
determiner:	my	your	his	her	its	our	their
pronoun:	mine	—	—	—	*	—	—

* *Its* is never used as a pronoun.

5. In line 18, *to go off the deep end* (infml) means to speak very angrily, often without cause.
6. Consider the sentence 'But he studied...' (ll. 23–24). Can you work out the meaning of the phrasal verb?
7. Paying special attention to the use of modal verbs, translate lines 25–31 into Russian.
8. Write out the words equivalent to: получить дальнейшее развитие; самая яркая звезда на небе; все, что важно; подтвердить предположение; по крайней мере; тем не менее.
9. Comment on the use of tenses (ll. 32–42).
10. In line 37, state the function of *having read* and give its Russian equivalent.
11. What part of speech is *limit* in l. 48 and in l. 50? Compare the functions and give Russian equivalents.
12. Explain the difference between *a* and *the*: *a white dwarf* (l. 43)—*the dwarf* (l. 46, l. 49).
13. What is the author referring to when he says *this balance* (l. 48), *this number* (l. 51), *this* (l. 60)?
14. In lines 43–52, find two words meaning 'approximately'.
15. Comment on the use of the following words: *so* (l. 43), *consequently* (l. 45), *hence* (l. 49). Why are they used there?
16. Read the sentence *If the dwarf is not collapsing, ...* (ll. 46–48), state the function of *just* and give its Russian equivalent.
17. Put the following phrase from the text into your own words: *he held his ground* (l. 62).

18. These sentences are all taken from the text. Can you put the adverbs in the right places?
    1. He created the discipline that led to the first understanding of the internal constitution of stars. (practically)
    2. The dwarf would stop collapsing because of the Fermi-Dirac degeneracy pressure. (eventually)
    3. Protons must supply the bulk of the gravitational compression. (consequently)
    4. Chandrasekhar's result disturbed Eddington. (deeply)
    5. Unless some mechanism could be found for limiting the mass of any star that was going to compress itself into a dwarf, or unless Chandrasekhar's result was wrong, massive stars were fated to collapse into oblivion. (eventually, gravitationally)
    6. The Danish physicist Niels Bohr assured him that Eddington was wrong and should be ignored. (simply)
    7. Eddington started going off the deep end. (really)
    8. He was one of the giants of 20th century science. (truly)
19. In line 54, what does *that* stand for?
20. See if your remember:
    before the turn of the century; given their mass and size; some $61,000$ times denser than water; in the late 1940s; for the wrong reasons; his great work in astronomy; his book on the stars; everything of significance about the universe; by examining what went on inside one's head; to confirm sb's prediction; to create a discipline; to lead to sth; at least; to come up with; to be squeezed together into a small, dense package; because of the pressure; to recoil from one another; to take another step forward; to be on board a ship; under the force of its own gravitation; as a whole; it turns out; equals about 1.4 times the mass of the sun; to be pleased with sth; both publicly and privately; to hold one's ground.
21. Check up for comprehension:
    1. The author says, 'Eddington was a hero of mine but for the wrong reasons.' Why?

2. Why do you think white dwarfs were an affront to Eddington?
3. What is Chandrasekhar's contribution to the study of white dwarfs?
4. What did Eddington think of Chandrasekhar's result?

Read Part IV. Use your dictionary to check new words and expressions, then move on to the exercises.

A SINGULAR SENSATION

As researchers explored quantum statistics and white dwarfs, others tackled Einstein's work on gravitation, his general theory of relativity. As far as I know, Einstein never spent a great deal of time looking for exact solutions to his gravitational equations. The part that described gravity around matter was extremely complicated, because gravity distorts the geometry of space and time, causing a particle to move from point to point along a curved path. More important to Einstein, the source of gravity—matter—could not be described by the gravitational equations alone. It had to be put in by hand, leaving Einstein to feel the equations were incomplete. Still, approximate solutions could describe with sufficient accuracy phenomena such as the bending of starlight. Nevertheless, he was impressed when, in 1916, the German astronomer Karl Schwarzschild came up with an exact solution for a realistic situation—in particular, the case of a planet orbiting a star.

In the process, Schwarzschild found something disturbing. There is a distance from the center of the star at which the mathematics goes berserk. At this distance, now known as the Schwarzschild radius, time vanishes, and space becomes infinite. The equation becomes what mathematicians call singular. The Schwarzschild radius is usually much smaller than the radius of the object. For the sun, for example, it is three kilometers, whereas for a one-gram marble it is 10^{-28} centimeter.

Schwarzschild was, of course, aware that his formula went crazy at this radius, but he decided that it did not matter. He constructed a simplified model of a star and showed that it would take an infinite gradient of pressure to compress it to his radius. The finding, he argued, served no practical interest.

But his analysis did not appease everybody. It bothered Einstein, because Schwarzschild's model star did not satisfy certain technical requirements of relativity theory. Various people, however, showed that one could rewrite Schwarzschild's solutions so that they avoided the singularity. But was the result really nonsingular? It would be incorrect to say that a debate raged, because most physicists had rather little regard for these matters—at least until 1939.

In his 1939 paper Einstein credits his renewed concern about the Schwarzschild radius to discussions with the Princeton cosmologist Harold P. Robertson and with his assistant Peter G. Bergmann, who is now professor emeritus at Syracuse University. It was certainly Einstein's intention in this paper to kill off the Schwarzschild singularity once and for all. At the end of it he writes, "The essential result of this investigation is a clear understanding as to why 'Schwarzschild singularities' do not exist in physical reality." In other words, black holes cannot exist.

To make his point, Einstein focused on a collection of small particles moving in circular orbits under the influence of one another's gravitation—in effect, a system resembling a spherical star cluster. He then asked whether such a configuration could collapse under its own gravity into a stable star with a radius equal to its Schwarzschild radius. He concluded that it could not, because at a somewhat larger radius the stars in the cluster would have to move faster than light in order to keep the configuration stable. Although Einstein's reasoning is correct, his point is irrelevant: it does not matter that a collapsing star at the Schwarzschild radius is unstable, because the star collapses past that radius anyway. I was much taken by the fact that the then 60-year-old Einstein presents in this paper tables of numerical results, which he must have gotten by using a slide rule. But the paper, like the slide rule, is now a historical artifact.

1. Some words from foreign languages keep their foreign plural in English.
   E. g.

stimulus	—	*stimuli*
curriculum	—	*curricula*
index	—	*indices*

In this part of the text, there are some nouns of the above type. What is the singular of *phenomena* (l. 12)? What are the plurals of *radius* (l. 19), *analysis* (l. 29) and *formula* (l. 24)?

2. Consider the sentence *More important to Einstein, ...* (l. 8–9). Comment on the use of dashes.
3. Identify the functions of *still* (l. 11), *nevertheless* (l. 12), *in particular* (l. 15), *for example* (l. 22), *of course* (l. 24), *however* (l. 31), *in other words* (l. 45).
4. We usually put zero article in front of academic subjects: *biology, physics, mathematics.* Can you explain the use of the definite article: *the mathematics* (l. 17), *the geometry* (l. 6)?
5. Compare the uses of *as*: *as researchers explored* (l. 1), *as far as I know* (l. 3), *such as* (l. 12), *as to* (l. 43). What are the functions? Translate the phrases into Russian.
6. In lines 1–15, say which sentences contain phrasal verbs and explain why you think they are phrasal verbs.
7. Read lines 37–60 and find the sentences with indirect questions. What are their peculiarities? Change them to direct speech.
8. Identify the parts of speech and their functions of the following −ing forms: looking (l. 4), causing (l. 7), bending (l. 12), orbiting (l. 15), disturbing (l. 16), finding (l. 27), understanding (l. 43), moving (l. 47), resembling (l. 48), collapsing (l. 55), using (l. 59).
9. What is the author referring to when he says:
   a) It had to be put in by hand ... (ll. 9–10)
   b) There is a distance from the center of the star at which the mathematics goes berserk. (ll. 16–18)
   c) Schwarzschild was ... aware that his formula went crazy at this radius ... (ll. 24–25)

d) It would be incorrect to say ... because most physicists had rather little regard for these matters ... (ll. 34–35)
e) ... Einstein credits his renewed concern about ... to discussion with ... Harold P. Robertson ... (ll. 37–39)
f) ... who is now professor emeritus at Syracuse University (l. 40)

10. Compare the functions of *then* (l. 49, l. 57).
11. In lines 46–60, find the infinitives of purpose and give their Russian equivalents.
12. In lines 29–60, find three adjectives with three different negative prefixes.
13. Identify the words used by the author as equivalent to: насколько мне известно, двигаться от точки к точке по кривой линии, достаточно точно описать, приблизительное/точное решение, построить упрощенную модель, удовлетворять определенным требованиям.
14. See if you remember: to appease sb, relativity theory, once and for all, in other words, to make one's point, to focus on, in effect, to be equal to, a work on gravitation, in particular, a great deal of time.
15. Translate into Russian the sentence '*I was much taken ...* ' (ll. 56–59), paying special attention to the modal verb *must*.
16. Write out all the negative structures. What is the rule?
17. These sentences are all taken from the text. Can you put the adverbs in the right places?
    a) The Schwarzschild radius is smaller than the radius of the object. (much, usually)
    b) It was Einstein's intention in this paper to kill off the Schwarzschild singularity. (certainly, once and for all)
    c) He asked whether such a configuration could collapse under its own gravity into a stable star. (then)
    d) But was the result nonsingular? (really)
    e) As far as I know, Einstein spent a great deal of time looking for exact solutions to his gravitational equations. (never)

f) The part that described gravity around matter was complicated. (extremely)

18. What do you think?

1) According to the author, 'Einstein never spent a great deal of time looking for exact solutions to his gravitational equations.' Why?
2) Why did Einstein want to kill off the Schwarzschild singularity?
3) Why is this part of the text titled 'A Singular Sensation'?

19. Write a summary.

Read Part V without consulting any dictionary. Try to guess the meaning of the unknown words by thinking about the context in which they are found. State the main idea.

FROM NEUTRONS TO BLACK HOLES

While Einstein was doing this research, an entirely different enterprise was unfolding in California. Oppenheimer and his students were creating the modern theory of black holes [see "J. Robert Oppenheimer: Before the War," by John S. Rigden; *Scientific American,* July 1995]. The curious thing about the black-hole research is that it was inspired by an idea that turned out to be entirely wrong. In 1932 the British experimental physicist James Chadwick found the neutron, the neutral component of the atomic nucleus. Soon thereafter speculation began—most notably by Fritz Zwicky of the California Institute of Technology and independently by the brilliant Soviet theoretical physicist Lev D. Landau—that neutrons could lead to an alternative to white dwarfs.

When the gravitational pressure got large enough, they argued, an electron in a star could react with a proton to produce a neutron. (Zwicky even conjectured that this process would happen in supernova explosions; he was right, and these "neutron stars" we now identify as pulsars.) At the time of this work, the actual mechanism for generating the energy in ordinary stars was not known. One solution placed a neutron star at the center of ordinary stars, in somewhat the same spirit that many astrophysicists now conjecture that black holes power quasars.

The question then arose: What was the equivalent of the Chandrasekhar mass limit for these stars? Determining this answer is much harder than finding the limit for the white dwarfs. The reason is that the neutrons interact with one another with a strong force whose specifics we still do not fully understand. Gravity will eventually overcome this force, but the precise limiting mass is sensitive to the details. Oppenheimer published two papers on this subject with his students Robert Serber and George M. Volkoff and concluded that the mass limit here is comparable to the Chandrasekhar limit for white dwarfs. The first of these papers was published in 1938, and the second in 1939. (The real source of stellar energy—fusion—was discovered in 1938 by Hans Bethe and Carl Friedrich von Weizsäcker, but it took a few years to be accepted, and so astrophysicists continued to pursue alternative theories.)

Oppenheimer went on to ask exactly what Eddington had wondered about white dwarfs: What would happen if one had a collapsing star whose mass exceeded any of the limits? Einstein's 1939 rejection of black holes—to which Oppenheimer and his students were certainly oblivious, for they were working concurrently, 3,000 miles away—was of no relevance. But Oppenheimer did not want to construct a stable star with a radius equal to its Schwarzschild radius. He wanted to see what would happen if one let the star collapse through its Schwarzschild radius. He suggested that Snyder work out this problem in detail.

To simplify matters, Oppenheimer told Snyder to make certain assumptions and to neglect technical considerations such as the degeneracy pressure or the possible rotation of the star. Oppenheimer's intuition told him that these factors would not change anything essential. (These assumptions were challenged many years later by a new generation of researchers using sophisticated high-speed computers—poor Snyder had an old-fashioned mechanical desk calculator—but Oppenheimer was right. Nothing essential changes.) With the simplified assumptions, Snyder found out that what happens to a collapsing star depends dramatically on the vantage point of the observer.

1. Look at lines 1–3 and comment on the use of tenses.
2. What are the functions of the infinitives?
   a) an idea that turned out to be entirely wrong (ll. 6–7)
   b) an electron in a star could react with a proton to produce a neutron (l. 14)
   c) it took a few years to be accepted (l. 34)
   d) to simplify matters (l. 46)
   e) if one let the star collapse (l. 43)
   f) Oppenheimer told Snyder to make certain assumptions and to neglect technical considerations (ll. 46–47)

   What do you notice in **e**?
3. These sentences are all taken from the text. Can you put the adverbs in the right places?
   a) ... Snyder found out that what happens to a collapsing star depends on the vantage point of the observer. (dramatically)
   b) Oppenheimer went on to ask what Eddington had wondered about white dwarfs. (exactly)
   c) Gravity will overcome this force .... (eventually)
   d) ... an idea ... turned out to be wrong. (entirely)
   e) ... the neutrons interact with one another with a strong force whose specifics we still do not understand. (fully)
   f) Oppenheimer and his students were oblivious to Einstein's 1939 rejection of black holes. (certainly)
4. Describe the different uses of *would* here.
   a) Zwicky even conjectured that this process *would* happen in supernova explosions .... (ll. 15–16)
   b) What *would* happen if one had a collapsing star whose mass exceeded any of the limits? (ll. 37–38)
   c) He wanted to see what *would* happen if one let the star collapse through its Schwarzschild radius. (ll. 43–44)
5. We use the subjunctive in *that*-clauses after some verbs (suggest, propose, recommend, insist, demand) and adjectives (important, vital, necessary, essential). This subjunctive expresses an intention or proposal about the future. E. g. The Minister *insisted* that he *leave* the country immediately.

I *propose* that Ms. Bond *be* elected secretary.
It is *essential* that the committee *resign*.
You can also use *should* or normal present and past tenses:
The Minister *insisted* that he *should leave* the country immediately.
I *propose* that Ms. Bond *is elected* secretary.
In lines 36–45, find the sentence of the above type and translate it into Russian.

6. Explain the meaning of the following:
the *curious* thing, soon *thereafter*, the *black-hole research*, in *somewhat* the same spirit, *specifics*, *concurrently*, to *challenge* assumptions, to use *sophisticated* high-speed computers, an *old-fashioned mechanical desk calculator*, to depend *dramatically*.
7. Supply *do* or *make* in these sentences, then check against the text.
   a) Oppenheimer told Snyder to ... certain assumptions.
   b) While Einstein was ... this research, an entirely different enterprise was unfolding in California.
8. Say which sentence contains a phrasal verb and explain why you think it is a phrasal verb:
   a) The real source of stellar energy—fusion—was discovered in 1938 by Hans Bethe and Carl Friedrich von Weizsäcker, but it took a few years to be accepted, and so astrophysicists continued to pursue alternative theories.
   b) Oppenheimer went on to ask exactly what Eddington had wondered about white dwarfs.

   Look through the text again to find some other examples of phrasal verbs and explain their meaning.
9. What part of speech is *power* (l. 21), *mass* (l. 23), *determining* (l. 23), *finding* (l. 24), *limiting* (l. 27) and what does that imply in terms of usage?
10. What is the meaning of *for* (l. 40)?
11. See if you know what preposition is needed with each of these words:
to be *sensitive* ... sth, to publish *papers* ... this subject, to

be *comparable* ... sth, to be *oblivious* ... sth, to *be* ... no relevance, to *depend* ... sth, to *lead* ... sth, an *alternative* ... sth. Check against the text.

12. Explain why passive verbs rather than active ones are used here:
    a) The real source of stellar energy—fusion—*was discovered* in 1938 by Hans Bethe and Carl Friedrich von Weizsäcker ... (ll. 32–34)
    b) These assumptions *were challenged* many years later by a new generations of researchers using sophisticated high-speed computers .... (ll. 50–54)
13. Here are the answers to some questions. Write the questions.
    a) Oppenheimer and his students.
    b) Fritz Zwicky and Lev D. Landau.
    c) When the gravitational pressure got large enough.
    d) Pulsars.
    e) It took a few years to be accepted.
    f) Oppenheimer and his students were certainly oblivious to it.
    g) To simplify matters.
    h) Many years later.
    i) The vantage point of the observer.

Read Part VI at a quick comfortable pace to answer the question: What are two views of a collapse? Then read it again more carefully to do the exercises.

TWO VIEWS OF A COLLAPSE

Let us start with an observer at rest a safe distance from the star. Let us also suppose that there is another observer attached to the surface of the star—"co-moving" with its collapse—who can send light signals back to his stationary colleague. The stationary observer will see the signals from his moving counterpart gradually shift to the red end of the electromagnetic spectrum. If the frequency of the signals is thought of as a clock, the stationary observer will say that the moving observer's clock is gradually slowing down.

Indeed, at the Schwarzschild radius the clock will slow down to zero. The stationary observer will argue that it took an infinite amount of time for the star to collapse to its Schwarzschild radius. What happens after that we cannot say, because, according to the stationary observer, there is no "after." As far as this observer is concerned, the star is frozen at its Schwarzschild radius.

Indeed, until December 1967, when the physicist John A. Wheeler, now at Princeton University, coined the name "black hole" in a lecture he presented, these objects were often referred to in the literature as frozen stars. This frozen state is the real significance of the singularity in the Schwarzschild geometry. As Oppenheimer and Snyder observed in their paper, the collapsing star "tends to close itself off from any communication with a distant observer; only its gravitational field persists." In other words, a black hole has been formed.

But what about observers riding with collapsing stars? These observers, Oppenheimer and Snyder pointed out, have a completely different sense of things. To them, the Schwarzschild radius has no special significance. They pass right through it and on to the center in a matter of hours, as measured by their watches. They would, however, be subject to monstrous tidal gravitational forces that would tear them to pieces.

The year was 1939, and the world itself was about to be torn to pieces. Oppenheimer was soon to go off to war to build the most destructive weapon ever devised by humans. He never worked on the subject of black holes again. As far as I know, Einstein never did, either. In peacetime, in 1947, Oppenheimer became the director of the Institute for Advanced Study in Princeton, N. J., where Einstein was still a professor. From time to time they talked. There is no record of their ever having discussed black holes. Further progress would have to wait until the 1960s, when discoveries of quasars, pulsars and compact x-ray sources reinvigorated thinking about the mysterious fate of stars.

1. Do you remember the meanings of the prefixes *co* and *re*? Give Russian equivalents of *to co-move* (l. 3) and *to reinvigorate* (l. 41). To help you, here is the meaning of the verb *to*

invigorate: to make (sb) feel more lively and healthy. Here are some more examples with different prefixes. What are their meanings?

Biannual, **ex**tract, **mis**translate, **multi**-purpose, **over**do, **post**-graduate, **under**used, **dis**prove, **ir**replaceable, **il**legal.

2. In lines 10–15, comment on the use of tenses and translate this part of the text into Russian.
3. What type of subordinate clause is *he presented* (l. 18)?
4. Compare −ing forms. Identify the parts of speech and their functions: *riding* (l. 25) and *collapsing* (l. 25).
5. What can you say about the sentence: *But what about observers riding with collapsing stars?* (l. 25)?
6. What is the singular form of *these*?
7. Compare the uses of *subject* (l. 30 and l. 35). What is the difference?
8. Like all the verbals, the gerund can form predicative constructions, i. e. constructions in which the verbal element expressed by the gerund is in predicate relations to the nominal element expressed by a noun or pronoun.

   E. g. Do you mind *my asking you one or two more quesitons*?
   Вы ничего не имеете против того, чтобы я задал вам еще один-два вопроса?

   Here the gerund *asking* is in predicate relation to the pronoun *my*, which denotes the doer of the action expressed by the gerund.

   A gerundial constuction is nearly always rendered by то, что; тем, что; как, etc.

   Identify the sentence in the text with the construction of the above type and translate it into Russian.
9. Explain the meaning of the verb phrase in bold italics as accurately as you can: Further progress ***would have to wait*** until the 1960's ... (ll. 39–42)
10. Say which sentences contain phrasal verbs and explain why you think they are phrasal verbs.
11. Give English equivalents of the following:
    a) в состоянии покоя на безопасном расстоянии от звезды

   b) на это потребуется много времени
   c) что касается наблюдателя
   d) создать новое слово
   e) называться
   f) другими словами
   g) подвергаться чему-то
   h) разорвать на куски
   i) насколько мне известно
   j) время от времени
   k) совершенно другое представление о вещах
   l) самое разрушительное оружие, которое было когда-либо создано человечеством
   m) давайте начнем с …

12. Write a summary of this part of the text.

What do you think?

What do you think?

1. What part of the article do you find most interesting?
2. Which facts do you find most remarkable?

Translation

1. В своей статье, опубликованной в 1939 году, Альберт Эйнштейн попытался доказать, что черные дыры невозможны.
2. Для этой цели он использовал свою собственную теорию — общую теорию относительности.
3. По иронии судьбы сейчас эта же теория используется для доказательства того, что черные дыры не только возможны, но иногда и неизбежны. Впервые это сделал Роберт Оппенгеймер.
4. Современные воззрения на черные дыры базируются на совсем ином фундаменте — квантово-статистической механике.

5. Без эффектов, предсказанных именно квантовой статистикой, каждый астрономический объект мог бы случайно свалиться в черную дыру и мир был бы совсем не таким, каков он на самом деле.

6. На создание квантовой статистики Эйнштейна натолкнуло письмо, которое он получил в июне 1924 года от совсем неизвестного тогда молодого индийского физика — Сатьендра Нат Бозе.

7. Внимание Эйнштейна привлек подход Бозе: рассматривать квантовые свойства фотонов статистически.

8. Оказалось, что таким образом можно получить знаменитую формулу Планка для излучения абсолютно черного тела.

9. Одни исследователи разрабатывали проблемы квантовой статистики, а другие занялись тщательной проработкой статей Эйнштейна о гравитации.

10. Часть уравнений Эйнштейна, которая описывает поле вокруг вещества, очень сложна.

11. Однако такие явления, как искривление светового луча, можно объяснить в некотором приближении.

12. Точное решение для достаточно реалистической ситуации — планеты, вращающейся вокруг звезды,— нашел немецкий астроном Карл Шварцшильд.

13. Эйнштейн серьезно обеспокоился открытием немецкого астронома, поскольку полученное решение не удовлетворяло некоторым техническим требованиям теории относительности.

14. Свой вывод о том, что черных дыр нет, Эйнштейн сделал на основе анализа системы небольших частиц, движущихся по круговым орбитам.

15. Буквально в те же дни Роберт Оппенгеймер со своим студентом создали современную теорию черных дыр.

Video

1. Watch the video 'A Life of Time'. As you watch it take notes, they will help you in discussion.
2. Having watched the video, work with a partner and discuss the following:
   a) Speed slows down time.
   b) Time is different to everybody.
   c) Space and time change together.
   d) The past and the future coexist with the present.
   e) Time, speed, gravity — what is the correlation?
   f) What is dark matter?

Appendix I: Texts for Translation

1 Read and translate into Russian two extracts taken from the article *Black Holes and the Centrifugal Force Paradox* by Marek Artur Abramowicz (*Scientific American*, 1993). Extract *A* is to be translated in written form. Use your dictionary if necessary. Time limit: 45 minutes. Extract *B* is to be translated offhand. No dictionary is allowed.

BLACK HOLES AND THE CENTRIFUGAL FORCE PARADOX

An object orbiting close to a black hole feels a centrifugal force pushing inward rather than outward. This paradoxical effect has important implications for astrophysics

by Marek Artur Abramowicz

A

If you have ever traveled in a car, bus or train as it sped around a bend, you have experienced the centrifugal force: the outward push, away from the center of the curve that grows stronger as the vehicle's speed increases. You can therefore imagine how surprised my colleague A. R. Prasanna of the Physical Research Laboratory in Ahmedabad, India, and I were when we realized recently that

Einstein's general theory of relativity predicts that in certain circumstances the centrifugal force may be directed toward, not away from, the center of a circular motion. We demonstrated that if an astronaut manages to steer a spacecraft sufficiently close to some extremely massive and compact object, such as a black hole, the astronaut would feel a centrifugal force pushing inward, not outward. Contrary to everyday experience, an increase in the orbital speed of the rocket strengthens the inward push of the centrifugal force.

According to our calculations, in the region close to a black hole not only does the centrifugal force reverse direction but all dynamic effects that depend on the sense of inward and outward are also reversed. This realization is important for understanding some aspects of the physics of black holes, which are believed to be a crucial part of the mysterious central engines that power the brightest galaxies in the cosmos. Investigations of the centrifugal force paradox have provided some tantalizing insights into the behavior of these galactic energy sources.

The reason for the centrifugal force paradox is the fantastically strong gravitational field produced by a black hole. As Albert Einstein predicted in 1915, a gravitational field warps space and bends light rays. In 1919 Sir Arthur Stanley Eddington confirmed this prediction by measuring the minute deflection of rays passing close to the sun. The gravitational field of the sun will bend a light ray less than one thousandth of a degree if the ray grazes the surface. Because a black hole generates a gravitational field far stronger than that of the sun, it can deflect light to a correspondingly greater extent.

Astronomers have not observed black holes directly, but they have gathered enough indirect evidence to convince most scientists that black holes must really exist. During the past two decades, astronomers have identified many objects that seem to contain black holes. These include several bright x-ray sources in our galaxy and many so-called active galactic nuclei, which are unusually bright cores of some distant galaxies.

A black hole traps forever any radiation or matter that gets too close to it. This point of no return defines the size of the black hole, or its gravitational radius. A black hole that has the same mass as the sun should have a gravitational radius of about three kilometers. If a light ray travels parallel to the surface of the black hole at a distance equal to, say, three times the gravitational radius, it will be bent by about 45 degrees. Most remarkably, if a light ray passes the black hole at a distance of exactly 1.5 times the gravitational radius, it will orbit the black hole in a perfect circle. The existence of the circular light ray is a key element in the centrifugal force paradox.

B

Jean-Pierre Lasota (now at the Paris Observatory) and I discovered the first hint of the paradox quite by chance, almost 20 years ago. We were working at the Copernicus Astronomical Center in Warsaw on a rather technical problem in the general theory of relativity. In particular, we were struggling with a complicated formula derived by Bozena Muchotrzeb, one of our students. Something was obviously wrong. The formula yielded a prediction about what force an object would feel if it orbited around a black hole along the same path as a circular light ray. The formula implied that no matter how fast the object moved, it would always feel exactly the same total force pushing inward. In particular, a motionless object would feel exactly the same inward force as a projectile that traveled around the circle at almost the speed of light.

We thought this could be nothing but nonsense. According to elementary dynamics, the centrifugal force depends on the orbital speed, whereas the gravitational force does not. Therefore, the total force—which is just the sum of the centrifugal and gravitational forces—must also depend on the orbital speed. Because the formula did not give the answer we expected, we were firmly convinced that it could not possibly be right. Yet after carefully repeating all the calculations in its derivation, we could find no mistakes. As it turned out, the formula was correct, as well as its paradoxical prediction about how matter behaves when traveling along the path of a circular light ray.

2 Read and translate into Russian two extracts taken from the article *The Language of Fractals* by HARTMUT JÜRGENS, HEINZ-OTTO PEITGEN AND DIETMAR SAUPE (*Scientific American*, 1990). Extract *A* is to be translated in written form. Use your dictionary if necessary. Time limit: 45 minutes. Extract *B* is to be translated offhand. No dictionary is allowed.

THE LANGUAGE OF FRACTALS

These unimaginable detailed structures are more than mathematical curiosities. Fractal geometry succinctly describes complex natural objects and processes

by Hartmut Jürgens, Heinz-Otto Peitgen and Dietmar Saupe

A

"Nature has played a joke on mathematicians. The 19th-century mathematicians may have been lacking in imagination, but nature was not. The same pathological structures that the mathematicians invented to break loose from 19th-century naturalism turned out to be inherent in familiar objects all around us." — FREEMAN DYSON, "Characterizing Irregularity," *Science,* May 12, 1978

The "pathological structures" conjured up by 19th-century mathematicians have, in recent years, taken the form of fractals, mathematical figures that have fractional dimension rather than the integral dimensions of familiar geometric figures (such as one-dimensional lines or two-dimensional planes). The current fascination with fractals is largely a result of the work of Benoit B. Mandelbrot of the IBM Thomas J. Watson Research Center in Yorktown Heights, N.Y. Mandelbrot coined the term fractal in 1975; he derived the word from the Latin *fractus,* the adjectival form of *frangere,* or "to break." The concept of fractals exploded into the consciousness of mathematicians, scientists and the lay public in 1983, when Mandelbrot's ground-breaking book, *The Fractal Geometry of Nature,* was published.

Fractals are much more than a mathematical curiosity. They offer an extremely compact method for describing objects and formations. Many structures have an underlying geometric regularity, known as scale invariance or self-similarity. If one examines these objects at different size scales, one repeatedly encounters the same fundamental elements. The repetitive pattern defines the fractional, or fractal, dimension of the structure. Fractal geometry seems to describe natural shapes and forms more gracefully and succinctly than does Euclidean geometry.

Scale invariance has a noteworthy parallel in contemporary chaos theory, which reveals that many phenomena, even though they follow strict deterministic rules, are in principle unpredictable. Chaotic events, such as turbulence in the atmosphere or the beating of a human heart, show similar patterns of variation on different time scales, much as scale-invariant objects show similar structural patterns on different spatial scales. The correspondence between fractals and chaos is no accident. Rather it is a symptom of a deep-rooted relation: fractal geometry is the geometry of chaos.

Another parallel between fractal geometry and chaos theory lies in the fact that recent discoveries in both fields have been made possible by powerful modern computers. This development challenges the traditional conception of mathematics. Many mathematicians have greeted computers with a sense of rejuvenation and liberation, but others view them as a rejection of pure mathematics.

B

Fractals are first and foremost a language of geometry. Yet their most basic elements cannot be viewed directly. In this aspect they differ fundamentally from the familiar elements of Euclidean geometry, such as the line and circle. Fractals are expressed not in primary shapes but in algorithms, sets of mathematical procedures. These algorithms are translated into geometric forms with the aid of a computer. The supply of algorithmic elements is inexhaustibly large. Once one has a command of the fractal language, one can describe the shape of a cloud as precisely and simply as an architect might describe a house with blueprints that use the language of traditional geometry.

Language is an apt metaphor for the ideas that underlie fractal geometry. Indo-European languages are based on a finite alphabet (the 26 letters from which English words are constructed, for instance). Letters do not carry meaning unless they are strung together into words. Euclidean geometry likewise consists of only a few elements (line, circle and so on) from which complex objects can be constructed. These objects, in a sense, only then have geometric meaning.

Asian languages such as Mandarin Chinese are made up of symbols that themselves embody meaning. The number of possible symbols or elements in these languages is arbitrarily large and can be considered infinite. Fractal geometry is constructed in much the same way. It is made up of infinitely many elements, each complete and unique. The geometric elements are defined by algorithms, which function as units of "meaning" in the fractal language.

3 Read and translate into Russian two extracts taken from the article *Knot Theory and Statistical Mechanics* by VAUGHAN F. R JONES (*Scientific American*, 1990). Extract *A* is to be translated in written form. Use your dictionary if necessary. Time limit: 45 minutes. Extract *B* is to be translated offhand. No dictionary is allowed.

KNOT THEORY AND STATISTICAL MECHANICS

Mathematical theories developed for quantum physics forge a connection between these two disparate fields

by Vaughan F. R Jones

A

In 1984 I stumbled on a set of techniques that linked two of the most apparently disparate fields in mathematics and physics: knot theory and statistical mechanics. Statistical mechanics involves the study of systems with an extremely large number of component parts. Small systems are largely irrelevant. In knot theory, meanwhile, even the smallest knots and links may have subtle properties.

Nevertheless, certain algebraic relations used to solve models in statistical mechanics were key to describing a mathematical property of knots known as a polynomial invariant. This connection, tenuous at first, has since developed into a significant flow of ideas. The appearance of such common ground is not atypical of recent developments in mathematics and physics—ideas from different fields interact and produce unexpected results.

Indeed, the discovery of the connection between knots and statistical mechanics passed through a theory intimately related to the mathematical structure of quantum physics. This theory, called von Neumann algebras, is distinguished by the idea of continuous dimensionality. Spaces typically have dimensions that are natural numbers, such as 2, 3 or 11, but in von Neumann algebras dimensions such as $\sqrt{2}$ or π are equally possible. This possibility for continuous dimension played a key role in joining knot theory and statistical mechanics.

In another direction, the knot theory invariants were soon found to occur in quantum field theory. Indeed, Edward Witten of the Institute for Advanced Study in Princeton, N.J., has shown that "topological" quantum field theory provides a natural way of expressing the new ideas about knots. This advance, in turn, has allowed a beautiful generalization about the invariants of knots in more complicated three-dimensional spaces known as three-manifolds, in which space itself may contain holes and loops.

The new knot theory has already been of use in another, entirely unrelated domain. Molecular biologists have established that the double helices of DNA become knotted and linked in the course of biological process such as recombination and replication. The untying mechanisms used in cells bear an uncanny resemblance to the simplest mathematical method for generating the new polynomial invariants.

Knots have been used for both practical and decorative purposes since time immemorial. Sailors have developed elaborate knots—sometimes with equally elaborate names—to serve their purposes. Mathematicians first became interested in knots only in the 19th century. Lord Kelvin, for example, attempted to deduce the structure of the periodic table of the elements by assuming that atoms were actually knotted vortices in the "ether." (Although the work proved unsuccessful, it did inspire Peter G. Tail to create the first knot tables, listing knots by some order of complexity.)

Since then, knot theory has become a fruitful branch of mathematics. One of the beauties of the discipline is that the main objects of study are so familiar: just take some string and join it at both ends. This serves as a perfectly adequate model for the "smooth non-self-intersecting closed curve" used by the mathematician. A more general version of a knot, called a link, may consist of more than one loop. Two knots or links are the same if they can be made to look exactly alike by pushing and pulling the string but not cutting it.

B

Thus far there seems to be no hint of how knot theory might be linked to statistical mechanics. The connection is not at the surface; it requires some explanation of what statistical mechanics is and what it is good for. That, in turn, requires starting from classical mechanics.

In classical mechanics a system of particles can be specified by giving the position and momentum of each particle at a certain time. The whole future evolution of the system is then determined by physical laws. But because a gram of hydrogen gas contains about 3×10^{23} molecules, it would be unreasonable to try to specify all the positions and momenta of the gas particles. Moreover, the change in the system that results from removing a few molecules would be completely unnoticeable to any observer apprehending the whole system.

The only quantities that are of interest to statistical mechanics are those that are insensitive to microscopic changes—for example, the average energy (temperature) of an ensemble of molecules. If one imagines a large system build by the addition of one atom at a time, those quantities will have a definite limit as the size of the system tends to infinity.

Innocent though it may sound, consideration of aggregate behavior creates some puzzles. One of the most obvious is irreversibility. The laws of motion do not change if the direction of time is reversed. For example, an elastic collision between a ball and an obstacle looks the same whether time runs forward or backward. And yet consider a system of balls bouncing around without friction on a rectangular table, constrained by a wall in one half of the table. If the wall is removed the balls will rapidly spread out over the entire table and will never conspire to go back into the half from which they started. The simple fact that a system contains a large number of particles seems to give time a definite direction.

4 Read and translate into Russian two texts about Godel's theorem. Text *A* — *What is Godel's Theorem?* by MELVIN HENRIKSEN (taken from *Scientific American.com*) — is to be translated in written form. Use your dictionary if necessary. Time limit: 45 minutes. Text *B* — *Diagonalization and Gödel's Theorem* (a part of the article *A Brief History of Infinity* by A. W MOORE (*Scientific American*, 1995) — is to be translated offhand. No dictionary is allowed.

A

WHAT IS GODEL'S THEOREM?

by Melvin Henriksen

Giving a mathematically precise statement of Godel's Incompleteness Theorem would only obscure its important intuitive content from almost anyone who is not a specialist in mathematical logic. So instead, I will rephrase and simplify it in the language of computers.

Imagine that we have access to a very powerful computer called Oracle. As do the computers with which we are familiar, Oracle asks that the user "inputs" instructions that follow precise rules and it supplies the "output" or answer in a way that also follows these rules. The same input will always produce the same output. The input and output are written as integers (or whole numbers) and Oracle performs only the usual operations of addition, subtraction, multiplication and division (when possible). Unlike ordinary computers, there are no concerns regarding efficiency or time. Oracle will carry out properly given instructions no matter how long it takes and it will stop only when they are executed—even if it takes more than a million years.

Let's consider a simple example. Remember that a positive integer (let's call it N) that is bigger than 1 is called a prime number if it is not divisible by any positive integer besides 1 and N. How would you ask Oracle to decide if N is prime? Tell it to divide N by every integer between 1 and $N-1$ and to stop when the division comes out evenly or it reaches $N-1$. (Actually, you can stop if it reaches the square root of N. If there have been no even divisions of N at that point, then N is prime.)

What Godel's theorem says is that there are properly posed questions involving only the arithmetic of integers that Oracle cannot answer. In other words, there are statements that—although inputted properly—Oracle cannot evaluate to decide if they are true or false. Such assertions are called undecidable, and are very complicated. And if you were to bring one to Dr. Godel, he would explain to you that such assertions will always exist.

Even if you were given an "improved" model of Oracle, call it OracleT, in which a particular undecidable statement, UD, is decreed true, another undecidable statement would be generated to take its place. More puzzling yet, you might also be given another "improved" model of Oracle, call it OracleF, in which UD would be decreed false. Regardless, this model too would generate other undecidable statements, and might yield results that differed from OracleT's, but were equally valid.

Do you find this shocking and close to paradoxical? It was even more shocking to the mathematical world in 1931, when Godel unveiled his incompleteness theorem. Godel did not phrase his result in the language of computers. He worked in a definite logical system and mathematicians hoped that his result depended on the peculiarities of that system. But in the next decade or so, a number of mathematicians—including Stephen C. Kleene, Emil Post, J.B. Rosser and Alan Turing—showed that it did not.

Research on the consequences of this great theorem continues to this day. Anyone with Internet access using a search engine like

Alta Vista can find several hundred articles of highly varying quality on Godel's Theorem. Among the best things to read, though, is Godel's Proof by Ernest Nagel and James R. Newman, published in 1958 and released in paperback by New York University Press in 1983.

B

DIAGONALIZATION AND GÖDEL'S THEOREM

by A. W Moore

The diagonalization used in establishing Cantor's theorem also lies at the heart of Austrian mathematician Kurt Gödel's celebrated 1931 theorem. Seeing how offers a particularly perspicuous view of Gödel's result.

Gödel's theorem deals with formal systems of arithmetic. By arithmetic I mean the theory of positive integers and the basic operations that apply to them, such as addition and multiplication. The theorem states that no single system of laws (axioms and rules) can be strong enough to prove all true statements of arithmetic without at the same time being so strong that it "proves" false ones, too. Equivalently, there is no single algorithm for distinguishing true arithmetical statements from false ones. Two definitions and two lemmas, or propositions, are needed to prove Gödel's theorem. Proof of the lemmas is not possible within these confines, although each is fairly plausible.

Definition 1: A set of positive integers is arithmetically definable if it can be defined using standard arithmetical terminology. Examples are the set of squares, the set of primes and the set of positive integers less than, say, 821.

Definition 2: A set of positive integers is decidable if there is an algorithm for determining whether any given positive integer belongs to the set. The same three sets above serve as examples.

Lemma 1: There is an algorithmic way of pairing off positive integers with arithmetically definable sets.

Lemma 2: Every decidable set is arithmetically definable.

Given lemma 1, diagonalization yields a set of positive integers that is not arithmetically definable. Call this set D. Now suppose, contrary to Gödel's theorem, there is an algorithm for distinguishing between true arithmetical statements and false ones. Then D, by virtue of its construction, is decidable. But given lemma 2, this proposition contradicts the fact that D is not arithmetically definable. So Gödel's theorem must hold after all. Q.E.D.

Appendix II: Transcript

FERMAT'S LAST THEOREM [3]

PROF. ANDREW WILES:
Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room and it's dark, completely dark, one stumbles around bumping into the furniture and then gradually you learn where each piece of furniture is, and finally after six months or so you find the light switch, you turn it on suddenly it's all illuminated, you can see exactly where you were.

At the beginning of September I was sitting here at this desk when suddenly, totally unexpectedly, I had this incredible revelation. It was the most, the most important moment of my working life. Nothing I ever do again will . . . I'm sorry.

NARRATOR:
This is the story of one man's obsession with the world's greatest mathematical problem. For seven years Professor Andrew Wiles

[3]The BBC Horizon Programme, "Fermat's Last Theorem" — written and edited by John Lynch, directed by Simon Singh — was broadcast in January, 1996. The BBC Web site for this programme can be found at `http://www.bbc.co.uk/horizon/95-96/960115.html`.

worked in complete secrecy, creating the calculation of the century. It was a calculation which brought him fame, and regret.

ANDREW WILES:

So I came to this. I was a 10-year-old and one day I happened to be looking in my local public library and I found a book on math and it, it told a bit about the history of this problem that someone had resolved this problem 300 years ago, but no-one had ever seen the proof, no-one knew if there was a proof, and people ever since have looked for the proof and here was a problem that I, a 10-year-old, could understand, but none of the great mathematicians in the past had been able to resolve, and from that moment of course I just, just tried to solve it myself. It was such a challenge, such a beautiful problem.

This problem was Fermat's last theorem.

NARRATOR:

Pierre de Fermat was a 17th-century French mathematician who made some of the greatest breakthroughs in the history of numbers. His inspiration came from studying the *Arithmetica,* that Ancient Greek text.

PROF. JOHN CONWAY:

Fermat owned a copy of this book, which is a book about numbers with lots of problems, which presumably Fermat tried to solve. He studied it, he, he wrote notes in the margins.

NARRATOR:

Fermat's original notes were lost, but they can still be read in a book published by his son. It was one of these notes that was Fermat's greatest legacy.

JOHN CONWAY:

And this is the fantastic observation of Master Pierre de Fermat which caused all the trouble. "Cubum autem in duos cubos"

NARRATOR:

This tiny note is the world's hardest mathematical problem. It's been unsolved for centuries, yet it begins with an equation so simple that children know it off by heart.

CHILDREN:
The square of the hypotenuse is equal to the sum of the squares of the other two sides.

JOHN CONWAY:
Yes well, that's Pythagoras's theorem, isn't it, that's what we all did at school. So Pythagoras's theorem, the clever thing about it is that it tells us when three numbers are the sides of a right-angle triangle. That happens just when x squared plus y squared equals z squared.

ANDREW WILES:
X squared plus y squared equals *zee* squared, and you can ask: well what are the whole numbers solutions of this equation? And you quickly find there's a solution 3 squared plus 4 squared equals 5 squared. Another one is 5 squared plus 12 squared is 13 squared, and you go on looking and you find more and more. So then a natural question is, the question Fermat raised: supposing you change from squares, supposing you replace the two by three, by four, by five, by six, by any whole number 'n', and Fermat said simply that you'll never find any solutions, however, however far you look you'll never find a solution.

NARRATOR:
You will never find numbers that fit this equation, if n is greater than 2. That's what Fermat said, and what's more, he said he could prove it. In a moment of brilliance, he scribbled the following mysterious note.

JOHN CONWAY:
Written in Latin, he says he has a truly wonderful proof "Demonstrationem mirabilem" of this fact, and then the last words are: "Hanc marginis exigiutas non caperet" — this margin is too small to contain this.

NARRATOR:
So Fermat said he had a proof, but he never said what it was.

JOHN CONWAY:
Fermat made lots of marginal notes. People took them as challenges

and over the centuries every single one of them has been disposed of, and the last one to be disposed of is this one. That's why it's called the last theorem.

NARRATOR:
Rediscovering Fermat's proof became the ultimate challenge, a challenge which would baffle mathematicians for the next 300 years.

JOHN CONWAY:
Gauss, the greatest mathematician in the world ...

BARRY MAZUR:
Oh yes, Galois ...

JOHN COATES:
Kummer of course ...

KEN RIBET:
Well, in the 18th-century Euler didn't prove it.

JOHN CONWAY:
Well, you know there's only been the one woman really ...

KEN RIBET:
Sophie Germain

BARRY MAZUR:
Oh, there are millions, there are lots of people

PETER SARNAK:
But nobody had any idea where to start.

ANDREW WILES:
Well, mathematicians just love a challenge and this problem, this particular problem just looked so simple, it just looked as if it had to have a solution, and of course it's very special because Fermat said he had a solution.

NARRATOR:
Mathematicians had to prove that no numbers fitted this equation but with the advent of computers, couldn't they check each number one by one and show that none of them fitted?

JOHN CONWAY:
Well, how many numbers are there to beat that with? You've got to do it for infinitely many numbers. So after you've done it for one, how much closer have you got? Well, there's still infinitely many left. After you've done it for 1,000 numbers, how many, how much closer have you got? Well, there's still infinitely many left. After you've done a few million, there's still infinitely many left. In fact, you haven't done very many, have you?

NARRATOR:
A computer can never check every number. Instead, what's needed is a mathematical proof.

PETER SARNAK:
A mathematician is not happy until the proof is complete and considered complete by the standards of mathematics.

NICK KATZ:
In mathematics there's the concept of proving something, of knowing it with absolute certainty.

PETER SARNAK:
Which, well, it's called rigorous proof.

KEN RIBET:
Well, rigorous proof is a series of arguments ...

PETER SARNAK:
... based on logical deductions.

KEN RIBET:
... which just builds one upon another.

PETER SARNAK:
Step by step.

KEN RIBET:
Until you get to ...

PETER SARNAK:
A complete proof.

NICK KATZ:
That's what mathematics is about.

NARRATOR:
A proof is a sort of reason. It explains why no numbers fit the equation without having to check every number. After centuries of failing to find a proof, mathematicians began to abandon Fermat in favour of more serious maths.

In the 70s Fermat was no longer in fashion. At the same time Andrew Wiles was just beginning his career as a mathematician. He went to Cambridge as a research student under the supervision of Professor John Coates.

JOHN COATES:
I've been very fortunate to have Andrew as a student, and even as a research student he, he was a wonderful person to work with. He had very deep ideas then and it, it was always clear he was a mathematician who would do great things.

NARRATOR:
But not with Fermat. Everyone thought Fermat's last theorem was impossible, so Professor Coates encouraged Andrew to forget his childhood dream and work on more mainstream maths.

ANDREW WILES:
The problem with working on Fermat is that you could spend years getting nothing so when I went to Cambridge my advisor, John Coates, was working on Iwasawa theory and elliptic curves and I started working with him.

NARRATOR:
Elliptic curves were the in-thing to study, but perversely, elliptic curves are neither ellipses nor curves.

BARRY MAZUR:
You may never have heard of elliptic curves, but they're extremely important.

JOHN CONWAY:
OK, so what's an elliptic curve?

BARRY MAZUR:
Elliptic curves — they're not ellipses, they're cubic curves whose solution has a shape that looks like a doughnut.

PETER SARNAK:
It looks so simple yet the complexity, especially arithmetic complexity, is immense.

NARRATOR:
Every point on the doughnut is the solution to an equation. Andrew Wiles now studied these elliptic equations and set aside his dream. What he didn't realise was that on the other side of the world elliptic curves and Fermat's last theorem were becoming inextricably linked.

GORO SHIMURA:
I entered the University of Tokyo in 1949 and that was four years after the War, but almost all professors were tired and the lectures were not inspiring.

NARRATOR:
Goro Shimura and his fellow students had to rely on each other for inspiration. In particular, he formed a remarkable partnership with a young man by the name of Utaka Taniyama.

GORO SHIMURA:
That was when I became very close to Taniyama. Taniyama was not a very careful person as a mathematician. He made a lot of mistakes, but he, he made mistakes in a good direction and so eventually he got right answers and I tried to imitate him, but I found out that it is very difficult to make good mistakes.

NARRATOR:
Together, Taniyama and Shimura worked on the complex mathematics of modular functions.

NICK KATZ:
I really can't explain what a modular function is in one sentence. I can try and give you a few sentences to explain it.

PETER SARNAK:
LAUGHS

NICK KATZ:
I really can't put it in one sentence.

PETER SARNAK:
Oh, it's impossible.

ANDREW WILES:
There's a saying attributed to Eichler that there are five fundamental operations of arithmetic: addition, subtraction, multiplication, division and modular forms.

BARRY MAZUR:
Modular forms are functions on the complex plane that are inordinately symmetric. They satisfy so many internal symmetries that their mere existence seems like accidents, but they do exist.

NARRATOR:
This image is merely a shadow of a modular form. To see one properly your TV screen would have to be stretched into something called hyperbolic space. Bizarre modular forms seem to have nothing whatsoever to do with the humdrum world of elliptic curves. But what Taniyama and Shimura suggested shocked everyone.

GORO SHIMURA:
In 1955 there was an international symposium and Taniyama posed two or three problems.

NARRATOR:
The problems posed by Taniyama led to the extraordinary claim that every elliptic curve was really a modular form in disguise. It became known as the Taniyama-Shimura conjecture.

JOHN CONWAY:
The Taniyama-Shimura conjecture says, it says that every rational elliptic curve is modular and that's so hard to explain.

BARRY MAZUR:
So let me explain. Over here you have the elliptic world, the elliptic curve, these doughnuts, and over here you have the modular world, modular forms with their many, many symmetries. The

Shimura-Taniyama conjecture makes a bridge between these two worlds. These worlds live on different planets.

It's a bridge, it's more than a bridge, it's really a dictionary, a dictionary where questions, intuitions, insights, theorems in the one world get translated to questions, intuitions in the other world.

KEN RIBET:
I think that when Shimura and Taniyama first started talking about the relationship between elliptic curves and modular forms people were very incredulous. I wasn't studying mathematics yet. By the time I was a graduate student in 1969 or 1970 people were coming to believe the conjecture.

NARRATOR:
In fact, Taniyama-Shimura became a foundation for other theories which all came to depend on it. But Taniyama-Shimura was only a conjecture, an unproven idea, and until it could be proved, all the maths which relied on it was under threat.

ANDREW WILES:
Built more and more conjectures stretched further and further into the future but they would all be completely ridiculous if Taniyama-Shimura was not true.

NARRATOR:
Proving the conjecture became crucial, but tragically, the man whose idea inspired it didn't live to see the enormous impact of his work. In 1958, Taniyama committed suicide.

GORO SHIMURA:
I was very much puzzled. Puzzlement may be the best word. Of course I was sad that, see, it was so sudden and I was unable to make sense out of this.

NARRATOR:
Taniyama-Shimura went on to become one of the great unproven conjectures. But what did it have to do with Fermat's last theorem?

ANDREW WILES:
At that time no one had any idea that Taniyama-Shimura could

have anything to do with Fermat. Of course in the 80s that all changed completely.

NARRATOR:
Taniyama-Shimura says: every elliptic curve is modular and Fermat says: no numbers fit this equation. What was the connection?

KEN RIBET:
Well, on the face of it the Shimura-Taniyama conjecture, which is about elliptic curves, and Fermat's last theorem have nothing to do with each other because there's no connection between Fermat and elliptic curves. But in 1985 Gerhard Frey had this amazing idea.

NARRATOR:
Frey, a German mathematician, considered the unthinkable: what would happen if Fermat was wrong and there was a solution to this equation after all?

PETER SARNAK:
Frey showed how starting with a fictitious solution to Fermat's last equation if such a horrible, beast existed, he could make an elliptic curve with some very weird properties.

KEN RIBET:
That elliptic curve seems to be not modular, but Shimura-Taniyama says that every elliptic curve is modular.

NARRATOR:
So if there is a solution to this equation it creates such a weird elliptic curve it defies Taniyama-Shimura.

KEN RIBET:
So in other words, if Fermat is false, so is Shimura-Taniyama, or said differently, if Shimura-Taniyama is correct, so is Fermat's last theorem.

NARRATOR:
Fermat and Taniyama-Shimura were now linked, apart from just one thing.

KEN RIBET:
The problem is that Frey didn't really prove that his elliptic curve

was not modular. He gave a plausibility argument which he hoped could be filled in by experts, and then the experts started working on it.

NARRATOR:
In theory, you could prove Fermat by proving Taniyama, but only if Frey was right. Frey's idea became known as the epsilon conjecture and everyone tried to check it. One year later, in San Francisco, there was a breakthrough.

KEN RIBET:
I saw Barry Mazur on the campus and I said let's go for a cup of coffee and we sat down for cappuccinos at this café and I looked at Barry and I said you know, I'm trying to generalise what I've done so that we can prove the full strength of Serre's epsilon conjecture and Barry looked at me and said well you've done it already, all you have to do is add on some extra gamma zero of m structure and run through your argument and it still works, and that gives everything you need, and this had never occurred to me as simple as it sounds. I looked at Barry, I looked to my cappuccino, I looked back at Barry and said my God, you're absolutely right.

BARRY MAZUR:
Ken's idea was brilliant.

ANDREW WILES:
I was at a friend's house sipping iced tea early in the evening and he just mentioned casually in the middle of a conversation: by the way, do you hear that Ken has proved the epsilon conjecture? And I was just electrified. I, I knew that moment the course of my life was changing because this meant that to prove Fermat's last theorem I just had to prove Taniyama-Shimura conjecture. From that moment that was what I was working on. I just knew I would go home and work on the Taniyama-Shimura conjecture.

NARRATOR:
Andrew abandoned all his other research. He cut himself off from the rest of the world and for the next seven years he concentrated solely on his childhood passion.

ANDREW WILES:
I never use a computer. I sometimes might scribble, I do doodles I start trying to, to find patterns really, so I'm doing calculations which try to explain some little piece of mathematics and I'm trying to fit it in with some previous broad conceptual understanding of some branch of mathematics. Sometimes that'll involve going and looking up in a book to see how it's done there, sometimes it's a question of modifying things a bit, sometimes doing a little extra calculation, and sometimes you realise that nothing that's ever been done before is any use at all, and you, you just have to find something completely new and it's a mystery where it comes from.

JOHN COATES:
I must confess I did not think that the Shimura-Taniyama conjecture was accessible to proof at present. I thought I probably wouldn't see a proof in my lifetime.

KEN RIBET:
I was one of the vast majority of people who believe that the Shimura-Taniyama conjecture was just completely inaccessible, and I didn't bother to prove it, even think about trying to prove it. Andrew Wiles is probably one of the few people on earth who had the audacity to dream that you can actually go and prove this conjecture.

ANDREW WILES:
In this case certainly for the first several years I had no fear of competition. I simply didn't think I or any one else had any real idea how to do it. But I realised after a while that talking to people casually about Fermat was, was impossible because it just generates too much interest and you can't really focus yourself for years unless you have this kind of undivided concentration which too many spectators will have destroyed.

NARRATOR:
Andrew decided that he would work in secrecy and isolation.

PETER SARNAK:
I often wondered myself what he was working on.

NICK KATZ:
Didn't have an inkling.

JOHN CONWAY:
No, I suspected nothing.

KEN RIBET:
This is probably the only case I know where someone worked for such a long time without divulging what he was doing, without talking about the progress he had made. It's just unprecedented.

NARRATOR:
Andrew was embarking on one of the most complex calculations in history. For the first two years, he did nothing but immerse himself in the problem, trying to find a strategy which might work.

ANDREW WILES:
So it was now known that Taniyama-Shimura implied Fermat's last theorem. What does Taniyama-Shimura say? It, it says that all elliptic curves should be modular. Well, this was an old problem been around for 20 years and lots of people would try to solve it.

KEN RIBET:
Now one way of looking at it is that you have all elliptic curves and then you have the modular elliptic curves and you want to prove that there are the same number of each. Now of course you're talking about infinite sets, so you can't, just can't count them per se, but you can divide them into packets and you could try to count each packet and see how things go, and this proves to be a very attractive idea for about 30 seconds, but you can't really get much further than that, and the big question on the subject was how you could possibly count, and in effect, Wiles introduced the correct technique.

NARRATOR:
Andrew's trick was to transform the elliptic curves into something called Galois representations which would make counting easier. Now it was a question of comparing modular forms with Galois representations, not elliptic curves.

ANDREW WILES:
Now you might ask and it's an obvious question, why can't you do this with elliptic curves and modular forms, why couldn't you count elliptic curves, count modular forms, show they're the same number? Well, the answer is people tried and they never found a way of counting, and this was why this is the key breakthrough, that I found a way to count not the original problem, but the modified problem. I found a way to count modular forms and Galois representations.

NARRATOR:
This was only the first step, and already it had taken three years of Andrew's life.

ANDREW WILES:
My wife's only known me while I've been working on Fermat. I told her a few days after we got married. I decided that I really only had time for my problem and my family and when I was concentrating very hard and I found that with young children that's the best possible way to relax. When you're talking to young children they simply aren't interested in Fermat, at least at this age, they want to hear a children's story and they're not going to let you do anything else.

So I'd found this wonderful counting mechanism and I started thinking about this concrete problem in terms of Iwasawa theory. Iwasawa theory was the subject I'd studied as a graduate student and in fact with my advisor, John Coates, I'd used it to analyse elliptic curves.

NARRATOR:
Andrew hopes that Iwasawa theory would complete his counting strategy.

ANDREW WILES:
Now I tried to use Iwasawa theory in this context, but I ran into trouble. I seemed to be up against a wall. I just didn't seem to be able to get past it. Well sometimes when I can't see what to do next I often come here by the lake. Walking has a very good effect

in that you're in this state of concentration, but at the same time you're relaxing, you're allowing the subconscious to work on you.

NARRATOR:
Iwasawa theory was supposed to help create something called a class number formula, but several months passed and the class number formula remained out of reach.

ANDREW WILES:
So at the end of the summer of '91 I was at a conference. John Coates told me about a wonderful new paper of Matthias Flach, a student of his, in which he had tackled a class number formula, in fact exactly the class number formula I needed, so Flach using ideas of Kolyvagin had made a very significant first step in actually producing the class number formula. So at that point I thought this is just what I need, this is tailor-made for the problem. I put aside completely the old approach I'd been trying and I devoted myself day and night to extending his result.

NARRATOR:
Andrew was almost there, but this breakthrough was risky and complicated. After six years of secrecy, he needed to confide in someone.

NICK KATZ:
In January of 1993 Andrew came up to me one day at tea, asked me if I could come up to his office, there was something he wanted to talk to me about. I had no idea what, what this could be. Went up to his office. He closed the door, he said he thought he would be able to prove Taniyama-Shimura. I was just amazed, this was fantastic.

ANDREW WILES:
It involved a kind of mathematics that Nick Katz is an expert in.

NICK KATZ:
I think another reason he asked me was that he was sure I would not tell other people, I would keep my mouth shut, which I did.

JOHN CONWAY:
Andrew Wiles and Nick Katz had been spending rather a lot of

time huddled over a coffee table at the far end of the common room working on some problem or other. We never knew what it was.

NARRATOR:
In order not to arouse any more suspicion, Andrew decided to check his proof by disguising it in a course of lectures which Nick Katz could then attend.

ANDREW WILES:
Well, I explained at the beginning of the course that Flach had written this beautiful paper and I wanted to try to extend it to prove the full class number formula. The only thing I didn't explain was that proving the class number formula was most of the way to Fermat's last theorem.

NICK KATZ:
So this course was announced. It said calculations on elliptic curves, which could mean anything. Didn't mention Fermat, didn't mention Taniyama-Shimura, there was no way in the world anyone could have guessed that it was about that, if you didn't already know. None of the graduate students knew and in a few weeks they just drifted off because it's impossible to follow stuff if you don't know what it's for, pretty much. It's pretty hard even if you do know what's it for, but after a few weeks I was the only guy in the audience.

NARRATOR:
The lectures revealed no errors and still none of his colleagues suspected why Andrew was being so secretive.

PETER SARNAK:
Maybe he's run out of ideas. That's why he's quiet, you never know why they're quiet.

NARRATOR:
The proof was still missing a vital ingredient, but Andrew now felt confident. It was time to tell one more person.

ANDREW WILES:

So I called up Peter and asked him if I could come round and talk to him about something.

PETER SARNAK:
I got a phone call from Andrew saying that he had something very important he wanted to chat to me about, and sure enough he had some very exciting news.

ANDREW WILES:
Said I, I think you better sit down for this. He sat down. I said I think I'm about to prove Fermat's last theorem.

PETER SARNAK:
I was flabbergasted, excited, disturbed. I mean I remember that night finding it quite difficult to sleep.

ANDREW WILES:
But there was still a problem. Late in the spring of '93 I was in this very awkward position and I thought I'd got most of the curves to be modular, so that was nearly enough to be content to have Fermat's last theorem, but there was this, these few families of elliptic curves that had escaped the net and I was sitting here at my desk in May of '93 still wondering about this problem and I was casually glancing at a paper of Barry Mazur's and there was just one sentence which made a reference to actually what's a 19th-century construction and I just instantly realised that there was a trick that I could use, that I could switch from the families of elliptic curves I'd been using, I'd been studying them using the prime three, I could switch and study them using the prime five. It looked more complicated, but I could switch from these awkward curves that I couldn't prove were modular to a different set of curves which I'd already proved were modular and use that information to just go that one last step and I just kept working out the details and time went by and I forgot to go down to lunch and it got to about teatime and I went down and Nada was very surprised that I'd arrived so late and then, then she, I told her that I, I believed I'd solved Fermat's last theorem.

I was convinced that I had Fermat in my hands and there was a conference in Cambridge organised by my advisor, John Coates.

I thought that would be a wonderful place. It's my old home town, I'd been a graduate student there, ... be a wonderful place to talk about it if I could get it in good shape.

JOHN COATES:
The name of the lectures that he announced was simply 'Elliptic curves and modular forms. There was no mention of Fermat's last theorem.

KEN RIBET:
Well, I was at this conference on L functions and elliptic curves and it was kind of a standard conference and all of the people were there, it didn't seem to be anything out of the ordinary, until people started telling me that they'd been hearing weird rumours about Andrew Wiles's proposed series of lectures.

I started talking to people and I got more and more precise information. I've no idea how it was spread.

PETER SARNAK:
Not from me, not from me.

JOHN CONWAY:
Whenever any piece of mathematical news had been in the air, Peter would say, oh, that's nothing, wait until you hear the big news, there's something big going to break.

PETER SARNAK:
Maybe some hints, yeah.

ANDREW WILES:
People would ask me leading up to my lectures what exactly I was going to say and I said well, come to my lecture and see.

KEN RIBET:
It's a very charged atmosphere a lot of the major figures of arithmetical, algebraic geometry were there. Richard Taylor and John Coates, Barry Mazur.

BARRY MAZUR:
Well, I'd never seen a lecture series in mathematics like that before. What was unique about those lectures were the glorious ideas how

many new ideas were presented, and the constancy of his dramatic build-up that was suspenseful until the end.

KEN RIBET:
There was this marvellous moment when we were coming close to a proof of Fermat's last theorem, the tension had built up and there was only one possible punch line.

ANDREW WILES:
So after I'd explained the 3/5 switch on the blackboard, I then just wrote up a statement of Fermat's last theorem, said I'd proved it, said I think I'll stop there.

JOHN COATES:
The next day what was totally unexpected was that we were deluged by enquiries from newspapers, journalists from all around the world.

ANDREW WILES:
It was a wonderful feeling after seven years to have really solved my problem, I've finally done it. Only later did it come out that there was a, a problem at the end.

NICK KATZ:
Now it was time for it to be refereed which is to say for people appointed by the journal to go through and make sure that the thing was really correct.

So for, for two months, July and August, I literally did nothing but go through this manuscript, line by line and what, what this meant concretely was that essentially every day, sometimes twice a day, I would E-mail Andrew with a question: I don't understand what you say on this page on this line. It seems to be wrong or I just don't understand.

ANDREW WILES:
So Nick was sending me E-mails and at the end of the summer he sent one that seemed innocent at first. I tried to resolve it.

NICK KATZ:
It's a little bit complicated so he sends me a fax, but the fax doesn't seem to answer the question, so I E-mail him back and I get another

fax which I'm still not satisfied with, and this in fact turned into the error that turned out to be a fundamental error and that we had completely missed when he was lecturing in the spring.

ANDREW WILES:
That's where the problem was in the method of Flach and Kolyvagin that I'd extended, so once I realised that at the end of September, that there was really a, a problem with the way I'd made the construction I spent the fall trying to think what kind of modifications could be made to the construction. There are lots of simple and rather natural modifications that any one of which might work.

PETER SARNAK:
And every time he would try and fix it in one corner it would ... sort of some other difficulty would add up in another corner. It was like he was trying to put a carpet in a room where the carpet had more size than the room, but he could put it in, in any corner and then when he ran to the other corner it would pop up in this corner and whether you could not put the carpet in the room was not something that he was able to decide.

NICK KATZ:
I think he externally appeared normal but at this point he was keeping a secret from the world and I think he must have been in fact pretty uncomfortable about it.

JOHN CONWAY:
Well, you know we were behaving a little bit like Kremlinologists. Nobody actually liked to come out and ask him how he's getting on with, with the proof, so somebody would say I saw Andrew this morning. Did he smile? Well yes, but he didn't look too happy.

ANDREW WILES:
The first seven years I'd worked on this problem. I loved every minute of it. However hard it had been there'd been, there'd been setbacks often, there'd been things that had seemed insurmountable but it was a kind of private and very personal battle I was engaged in.

And then after there was a problem with it doing mathematics in that kind of rather over-exposed way is certainly not my style and I have no wish to repeat it.

NARRATOR:
Other mathematicians, including his former student Richard Taylor, tried to help fix the mistake. But after a year of failure, Andrew was ready to abandon his flawed proof.

ANDREW WILES:
In September, I decided to go back and look one more time at the original structure of Flach and Kolyvagin to try and pinpoint exactly why it wasn't working, try and formulate it precisely. One can never really do that in mathematics but I just wanted to set my mind at rest that it really couldn't be made to work. And I was sitting here at this desk. It was a Monday morning, September 19th and I was trying convincing myself that it didn't work, just seeing exactly what the problem was when suddenly, totally unexpectedly, I had this incredible revelation. I, I realised what was holding me up was exactly what would resolve the problem I'd had in my Iwasawa theory attempt three years earlier, it was the most, the most important moment of my working life. It was so indescribably beautiful, it was so simple and so elegant and I just stared in disbelief for twenty minutes. Then during the day I walked round the department, I'd keep coming back to my desk and looking to see it was still there, it was still there. Almost what seemed to be stopping the method of Flach and Kolyvagin was exactly what would make horizontally Iwasawa theory. My original approach to the problem from three years before would make exactly that work, so out of the ashes seemed to rise the true answer to the problem. So the first night I went back and slept on it, I checked through it again the next morning and by 11 o'clock I satisfied and I went down, told my wife I've got it, I think I've got it, I've found it, and it was so unexpected, she, I think she thought I was talking about a children's toy or something, and said, 'Got what?' I said, 'I've fixed my proof, I've got it.'

JOHN COATES:
I think it will always stand as, as one of the high achievements of number theory.

BARRY MAZUR:
It was magnificent.

JOHN CONWAY:
It's not every day that you hear the proof of the century.

GORO SHIMURA:
Well, my first reaction was: I told you so.

NARRATOR:
The Taniyama-Shimura conjecture is no longer a conjecture, and as a result Fermat's last theorem has been proved. But is Andrew's proof the same as Fermat's?

ANDREW WILES:
Fermat couldn't possibly have had this proof. It's a 20th-century proof. There's no way this could have been done before the 20th-century.

JOHN CONWAY:
I'm relieved that this result is now settled. But I'm sad in some ways because Fermat's last theorem has been responsible for so much. What will we find to take its place?

ANDREW WILES:
There's no other problem that will mean the same to me. I had this very rare privilege of being able to pursue in my adult life what had been my childhood dream. I know it's a rare privilege but if, if one can do this it's more rewarding than anything I could imagine.

BARRY MAZUR:
One of the great things about this work is it embraces the ideas of so many mathematicians. I've made a partial list: Klein, Fricke, Hurwitz, Hecke, Dirichlet, Dedekind ...

KEN RIBET:
The proof by Langlands and Tunnell ...

JOHN COATES:
Deligne, Rapoport, Katz ...

NICK KATZ:
Mazur's idea of using the deformation theory of Galois representations …

BARRY MAZUR:
Igusa, Eichler, Shimura, Taniyama …

PETER SARNAK:
Frey's reduction …

NICK KATZ:
The list goes on and on …

BARRY MAZUR:
Bloch, Kato, Selmer, Frey, Fermat.

Л. Н. Выгонская

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