## On Borsuk's problem

by A. Lukina, group 305, English teacher: A.A.Savchenko

- 1. The problem under consideration is an open question in combinatorial geometry, the branch of mathematics that appeared only in 19<sup>th</sup> century. It studies combinatorial properties of finite or discrete bodies or bodies with some particular characteristics, for instance, of a certain diameter.
- 2. Borsuk suggested that every set of diameter one in  $R^d$  can be partitioned into d+1 pieces of diameter smaller than one.
- 3. Kahn and Kalai proved that the conjecture does not hold for every *d*. So it is quite interesting to understand for which *d* and for which kinds of bodies Borsuk was right.
- 4. A number of questions arise from this problem. For instance, we can partition bodies not only into d+1 pieces, but into some number of sets of smaller diameter.
- 5. It is useful to deal with not just a body of diameter one, but with its approximation, called universal cover system. The better approximation is, the better bounds can be obtained.
- 6. There are some common ways to amend universal covers. One of them was used by Grünbaum to prove Borsuk's conjecture for d=3.
- 7. A number of results are known in  $R^2$ , but there is almost nothing in  $R^3$ , so new interesting relations can be discovered without using complex techniques.

English teacher: \_\_\_\_\_/ A.A.Savchenko