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WRITING MATHS

QUOTATIONS

Методическая разработка для аспирантов

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Москва
2017

Introduction

Everybody wants their paper to be read by someone—but who is going to read it? Mathematical papers are read by specialists in a given domain. Tip: think of a (specific) reader—someone whom you'd like to see reading your paper. This should be a mathematician who can understand your text completely, but does not work on exactly the same subject (and is not necessarily a Fields' medallist; your choice should be realistic!). When writing your paper, try to interest your reader; give the definitions he/she does not (or may not) know; explain why the next fragment is important or worth reading—your reader has no time to waste; leave out all that the reader considers trivial. Finally, prepare your paper carefully, so that your reader does not become bored or irritated. [1]

As you write a mathematics paper remember that, unlike you, the reader has not been thinking intensely about the material for an extended period of time. Therefore, provide the reader with references, include useful comments, and give additional explication so that someone unfamiliar with the work can follow it. [2]

Write a paper that you yourself would want to read. Make it accessible. Bear in mind that the referee for your paper will be a busy person who has no patience for a tract that he cannot fathom. Lay out the material so that it is rapidly apparent what your main result is, what the background for that result is, and how you are going to go about proving it. If the proof is long and complicated, then break it up into digestible pieces. Tell the reader what is going to happen before it happens. Tell the reader what has just happened before you go on to the next step. At the end of a long argument, summarize it. [3]

Another critical feature is the layout or architecture of the page. If it looks like solid prose, it will have a forbidding, sermony aspect; if it looks like computational hash, with a page full of symbols, it will have a frightening, complicated aspect. The golden mean is golden. Break it up, but not too small; use prose, but not too much. Intersperse enough displays to give the eye a chance to help the brain; use symbols, but in the middle of enough prose to keep the mind from drowning in a morass of suffixes. [4]

Aim at positive writing, consider ethics, copyright, avoid biased language and plagiarism. Verify appropriate word choice, affectation and jargon, do not use vague words (*real, nice, important, good, bad*), and clichés (*last but not least*). Minimize the problems with grammar. Review punctuation, abbreviations, capitalization, contractions, dates, italics, number and measurement units, proofreading, and spelling. [5]

Organization

Most short technical papers are divided up into about a half dozen sections, which are numbered and titled. (The pages too should be numbered for easy reference.) Most papers have an abstract, an introduction, a number of sections of discussion, and a list of references, but no formal table of contents or index. On occasion, papers have appendices, which give special detailed information or provide necessary general background to secondary audiences. In some fields, papers routinely have a conclusion . This section is not present simply to balance the introduction and to close the paper. Rather, the conclusion discusses the results from an overall perspective, brings together the loose ends, and makes recommendations for further research. In mathematics, these issues are almost always treated in the introduction, where they reach more readers; so a conclusion is rare. [6]

The ideal is to make an outline in which every preliminary heuristic discussion, every lemma, every theorem, every corollary, every remark, and every proof are mentioned, and in which all these pieces occur in an order that is both logically correct and psychologically digestible. [4]

In organization of a piece of writing, the question of what to put in is hardly more important than what to leave out; too much detail can be as discouraging as none. [4]

The moral is that it’s best to organize your work around the central, crucial examples and counterexamples. [4]

Sectioning involves more than merely dividing up the material; you have to decide about what to put where, about what to leave out, and about what to emphasize. If you make the wrong decisions, you will lose your readers. There is no simple formula for deciding, because the decisions depend heavily on the subject and the audience. However, you must structure your paper in a way that is easy for your readers to follow, and you must emphasize the key results. [6]

If your paper runs to several pages, divide it into sections with informative titles. This will help clarify the logical structure of the paper and make it easier to follow. Introduce one idea at the time. [7]

Title

Every paper should have one. It should be informative without being too long. Choosing a good title in a mathematics paper is not so easy. Often a paper hinges on a concept that is defined only within the paper itself, so using the name of that concept in the title will convey no meaning at all. [8]

The title is very important. If it is inexact or unclear, it will not attract all the intended readers. A strong title identifies the general area of the subject and its most distinctive features. A strong title contains no distracting secondary details and no formulas. A strong title is concise. [6]

Your typical reader will find out your text by browsing the internet. Here the role of the title cannot be overestimated.

Here are some conditions a good title has to satisfy:

- Should indicate the branch of mathematics ("On a theorem of Kuratowski" is no good).
- Should not be too long.
- Should not be too general.
- Should not contain abbreviations or complicated symbols, especially in special typefaces (such titles are often misquoted); when typesetting a title, do not use macros: the title is often extracted from the paper by the publisher for various purposes. [1]

Capitalize the initial letters of the words of the title, except articles (a, an, the), coordinating conjunctions (and, but), or prepositions (to, at, into) unless they begin or end the title. [5]

Stylistic devices in titles:

- Question: "Can B(L p) ever be amenable?"
- Complete sentence, statement of the main result: "Every weak L p space has the Radon–Nikodym property"
- Verbal element (gerund or participle): "Computing the eigenvalues of M-matrices", "The complemented subspace problem revisited"

Instead of decorating the author's name or the title with asterisks, it is preferable to place all thanks, grant acknowledgements etc. at the end of the paper, just before "References", and put them into "Acknowledgements".

This also saves space on the first and second pages of the paper—and these pages will decide whether the reader will want to read more. [1]

Abstract

The abstract is the most important section. First it identifies the subject; it repeats words and phrases from the title to corroborate a reader's first impression, and it gives details that didn't fit into the title. Then it lays out the central issues, and summarizes the discussion to come. The abstract includes *no general background* material. It is essentially a table of contents in a paragraph of prose. The abstract allows readers to decide quickly about reading on. While many will decide to stop there, the potentially interested will continue. The goal is not to entice all, but to inform the interested efficiently. Remember, readers are busy. They have to decide quickly whether your paper is worth their time. They have to decide whether the subject matter is of interest to them, and whether the presentation will bog them down. A well-written abstract will increase the readership. [6]

Do use one paragraph, roughly 5-6 sentences (as a general rule of thumb).
Do start from scratch, using words that everyone knows: "To every knot in a 3-manifold, one can associate a number..." Of course, what everyone knows depends on the situation. For a conference on a specialized topic, you can assume more.
Do give context for your work - where does it fit into the big picture?
Do de-symbolize your abstract as much as possible, just for the sake of readability. Symbols (except for the most basic ones, like $\mathbb{R}^3$) just inherently take longer to process.
Do get feedback from a trusted advisor.
Don’t cram too many technical words into the title or abstract. [9]

I will share an example of one of my own abstracts. The first version is not bad. In the second version, I got rid of the symbols, I started out with basic mathematical terms, I abandoned the attempt to explain the definition of "pseudo-Anosov" (even suppressing this term to a parenthetical), I shortened to one paragraph, I added some signposts as to where this fits into math: topology, geometry, dynamics, and last but not least the second title has a better ring to it. Big difference!
BEFORE:
Group-theoretical, geometrical, and dynamical aspects of surfaces
A pseudo-Anosov map of a surface is a homeomorphism that locally looks like a hyperbolic matrix (two distinct real eigenvalues) acting on the plane. Like its linear counterpart, a pseudo-Anosov map stretches in one direction by a factor K, called the dilatation, and contracts in

another direction by *1/K*. The dilatation of a pseudo-Anosov map is an algebraic integer that determines the entropy of the map, the growth rate of lengths of curves under iteration by the map, and the length of the corresponding loop in moduli space, for example. The set of possible dilatations is quite mysterious. For a fixed surface, this set is known to be discrete in *R*, and so the small dilatation pseudo-Anosov maps are of particular interest. In joint work with Benson Farb and Chris Leininger, we introduce two universality phenomena concerning small dilatations. The first can be described as "algebraic complexity implies dynamical complexity", and the second can be described as "geometric complexity implies dynamical complexity".

AFTER:

Group theory, geometry and dynamics of surface homeomorphisms

Attached to every homeomorphism of a surface is a real number called its dilatation. For a generic (i.e. pseudo-Anosov) homeomorphism, the dilatation is an algebraic integer that records various properties of the map. For instance, it determines the entropy (dynamics), the growth rate of lengths of geodesics under iteration (geometry), the growth rate of intersection numbers under iteration (topology), and the length of the corresponding loop in moduli space (complex analysis). The set of possible dilatations is quite mysterious. In this paper I will explain the discovery, joint with Benson Farb and Chris Leininger, of two universality phenomena. The first can be described as "algebraic complexity implies dynamical complexity", and the second as "geometric complexity implies dynamical complexity" [9]

Key Words

Some journalists list key words supplied by the author, usually after abstract. The number of key words is usually ten or less. Since the key words may be used in computer searches, you should try to anticipate words for which reader might search and make them specific enough to give a good indication of paper’s content. [10]

Introduction

Write a good introduction. Most people who read a mathematics paper will only read the introduction and skim the theorems. [2]

The introduction is the place where readers settle into the "story," and often make the final decision about reading the whole paper. It identifies the subject precisely and instills interest in it by giving details that did not fit into the title or abstract, such as how the subject arose and where it is headed, how it relates to other subjects and why it is important. A strong introduction touches on all the significant points, and no more. A strong introduction gives enough background material for understanding the paper as a whole, and no more. Put background material pertinent to a particular section in that section, weaving it unobtrusively into the text. A strong introduction discusses the relevant literature, citing a good survey or two. Finally, a strong introduction describes the organization of the paper, making explicit references to the section numbers. It summarizes the contents in more detail than the abstract, and it says what can be found in each section. [6]

Amusingly, this is the last part of the paper that your typical reader will read (statistically speaking); the rest will be read by very few individuals. So what you write here will determine the impression the paper will make on most readers.
What are the criteria for including something in (or omitting something from) the introduction?

- Write only what you consider interesting.
- Two elements should always appear: your theorems and discussion of the relevant literature. The theorems presented in the introduction may be (but do not have to be) repeated later—literally or with some modifications; with the same numbering (e.g. as **Theorems 1, 2, 3**) or with other labels (e.g. **Theorem 5.1** etc.). [1]

Remember that in an introduction you are often trying to "sell" your work and convince others of its importance. Also be aware that many people write the introduction to a paper after they have finished the body. This gives them the advantage of knowing exactly what will be done in the paper as they compose the introduction. [2]

How to begin? One possibility is to formulate the fundamental problem your paper is concerned with. Alternatively, you might provide some historical information. First target: interest your reader with the first paragraph.
How not to begin? Whatever else you do, do not start by giving a long and precise list of definitions and notations. There will be time for precision later; first, arrest the reader’s attention. [1]

An introductory section listing every term and every symbol that are going to come up may serve as handy reference, but will overwhelm the reader. Again, it is better to introduce such things in small doses. If you still feel a strong need to include glossary, put it at the end. [7]

The first paragraph of the introduction should be comprehensible to any mathematicians. Describe in general terms what the paper is about; and do it in a way that entices the reader to continue reading. [7]

Besides your results and historical comments, you can put into the introduction any other interesting elements, like:

- comments about proofs, or even sketches and/or heuristical proofs;
- schemes of logical dependence of sections;
- suggestions for further research, etc. [1]

Your very first sentence, in particular, must command the reader's immediate interest.
Would you guess that most continuous functions are nowhere differentiable?
is excellent.
This paper describes an unusual application of the Mean-Value Theorem
is also good; it could say more about the application, but can stand as is because the Mean-Value Theorem is so familiar.
Consider a sheaf of germs of holomorphic functions
is too technical (even in the late 1980s). [7]

Body

The body discusses the various aspects of the subject individually.
First, present the material in small digestible portions. Second, beware of jumping haphazardly from one detail to another, and of illogically making some details specific and others generic. Third, if possible, follow a sequential path through the subject. If such a path simply doesn't exist, then break the subject down into logical units, and present them in the order most conducive to understanding. If the units are independent, then order them according to their importance to the primary audience. [6]

If your manuscript is more than 8 types pages (double spaced), divide it into numbered sections and number theorems serially within each section: 1.1, 1.2, …, 2.1, 2.2, … (unless there are very few theorems. [7]

Citation

Rightly or wrongly, mathematics is regarded as having an existence independent of the words used to describe it. Thus your text may describe the Chain Rule theorem in slightly different words than anyone else, but that doesn’t give its authors any special credit. If you use the Chain Rule, and you learned about it from your text, you should not reference your text at the point where you introduce the Chain Rule — the authors would never claim credit for the Chain Rule themselves. This same principle applies to definitions; don’t reference your text for the definition of derivative.

Note. Theorems have two sorts of names: descriptive names, such as "Chain Rule"; and sequential names, such as "Theorem 6" and "Limit Rule II". A text gives a descriptive name only if it is widely used by others, so you can use the name "Chain Rule" too. But sequential names are specific to an individual text. Thus, if you must talk about Limit Rule II in your text, you have to reference your text and give a page number — otherwise readers won’t have a clue what rule you are referring to. But why make your readers look this up when you could simply state the rule in your paper? [8]

Briefly put, direct quotes and almost direct quotes must be credited, but paraphrase generally need not be. Furthermore, mathematics papers rarely include direct quotes. [8]

The original discoverers of mathematical results are given credit, and if a result is fairly recent, the paper in which it is first published must be referenced. But all the theorems in your calculus course are classical (some of them over 300 years old). They are so much considered a common heritage that people’s names are only occasionally associated with individual results, and references to original publications are never made (except in a history of mathematics paper). I’ve discussed giving credit for definitions and theorems. In your papers you are more likely to use examples (for instance, a max-min problem) taken from or based on your text. Should these be credited? Again, the answer is "usually no". If you use the exact words or even just the exact numbers of an example from some book, then (and only then) give credit; but in general you shouldn’t be using problems verbatim anyway. [8]

Finally, if your whole line of thinking comes from one source, it is appropriate to give one reference to it. Somewhere near the beginning of your paper, say something like "In writing this paper, I have drawn heavily on Goldstein et al. [150]" (where Goldstein is reference 1 in your bibliography) or "Much of my material is adapted from lectures by Smith [160]" [8]

The list of **references** contains bibliographical information about each source cited.

The citation is treated somewhat like a parenthetical remark within a sentence. Footnotes are not used; neither are the abbreviations "loc. cit.," "op. cit.," and "ibid." The reference key, usually a numeral, is enclosed in square brackets. Within the brackets and after the reference key, place – as a service – specific page numbers, section numbers, or equation numbers, preceded by a comma [**5**, Section 3.7, p. 9]. If the citation comes at the end of a sentence, put the period after the citation, *not* before the brackets or inside them. In the list of reference, give the full page numbers of each article appearing in a journal, a proceedings volume, or other collection; do not give the numbers of the particular pages cited in the text. [6]

Please follow our bibliographic format carefully, based on the examples below. Entries may appear either in alphabetical order or in order of citation (but choose one order and stick to it). Journal titles are abbreviated as in *Mathematical Reviews*, for instance, *Amer. Math. Monthly*; volume numbers of journals are set in bold. Authors' names are not inverted: Frank A. Farris, not Farris, Frank A. [190]. The abbreviation pp. is used for books, but not journal articles. [11]

Definitions

"Global" definitions apply throughout your paper. Most global definitions introduce words or notation. [8]

In definitions, emphasize the term being defined:

> *We define the **convex hull** of E to be the smallest convex set containing E.*

The word order is different if "call" is used:

> *We call the smallest convex set containing E the **convex hull** of E.*

When defining symbols, the notation := or =: is useful, as it shows which side of the equality is being defined (the one next to the colon):

> *Then* $F = abcde + fghi \coloneqq A + B$.

[1]

In contrast, "local" definitions apply only briefly, say, to the current example or current paragraph. Usually local definitions are definitions of symbols, for instance, "Let $f(x) = x^2$." A few paragraphs later it is acceptable to say "Now let $f(x) = e^x$." Local definitions don't require special highlighting, although they should be displayed if they involve complicated formulas. [8]

When you give a definition, you can do it "in-line" (within a paragraph), but the word or phrase being defined should be highlighted, by underlining, by italic print, or by boldface. Boldface is now the most common format, since underlining is not common in typeset material and italic already has a use in both mathematical and ordinary writing to indicate emphasis. Here’s an example of a definition: "A **prime number** is a positive integer with no positive integer divisors other than 1 and itself." Another format is to display definitions just like theorems are displayed. The display format should be reserved for the most important definitions. For instance, in calculus the definition of derivative might be displayed, but the definition of polynomial (if you need it at all) can be done in-line. [8]

Proof

When writing a proof, do think about your (intended) reader. A person who is reading a proof has to be 1) competent and 2) concentrated. [1]

Write all necessary hypotheses in statements of theorems. A person should be able to open your paper to any theorem, read it, and know what you are talking about without having to refer to earlier portions of the document. In fact, it is likely that this is how most people will use your papers. If at all possible, make the statements of your theorems completely self-contained so that the reader does not have to look throughout your paper to decipher notation or find definitions of special terminology. [2]

The first question is where the theorem should be stated, and my answer is: first. Don’t ramble on a leisurely way, not telling the reader where you are going, and then suddenly announce "Thus we have proved that...". The reader can pay closer attention to the proof if he knows what you are proving, and he can see better where the hypotheses are used if he knows in advance what they are. [4]

It is often good to illustrate the theorem before proving it – or even instead of proving it, in case the illustration contains the essence of the proof, or the proof is too hard.
Strive for proofs that are conceptual rather than computational – the way you would describe the result to a colleague (or student) during a walk across the campus. If a proof is at all involved, begin by explaining the underlying idea. Leave all purely routine computations (i.e., not involving an unexpected trick) to the reader. [7]

Use counter-examples to demonstrate the necessity of conditions on theorem. [4]

Beware of any proof by contradiction; often there's a simpler direct argument. Finally, when the proof has ended, say so outright; for instance, say, "The proof is now complete," or use the Halmos symbol □. In addition, surround the proof – and the statement as well – with some extra white space. [6]

Do not give arguments that are obvious to every mathematician. [1]

If the proof requires analysing several cases, maybe some of them can be omitted:

The analysis of case (b) is similar and left to the reader. [1]

Make sure that, for each statement you write, the reader knows whether it has already been proved, will be proved, or whether the reader is supposed to supply the proof.
The combination "By (12), we have $A = B$. To see this, ..." may be misleading. It is then preferable to announce the proof before the statement: "We now prove that (12) implies $\mathrm{A} = \mathrm{B}$. To see this, ..." [1]

A proof should give enough information to make the theorem believable and leave the reader with the confidence (as well as the ability) to fill in details should it be necessary.
Whatever format or style you choose to adopt, especially if it deviates from the publisher's style, make sure that it is consistent. This is mostly a difficulty with books. If one proof ends with a "QED," then they all should, etc. [12]

Be clear about the status of every assertion; indicate whether it is a conjecture, the previous theorem, or the next corollary. If it is not a standard result and you omit its proof, then give a reference, preferably in the text just before the statement. (If you give the reference in the statement, then do so after the heading like this: **Theorem 5-1** [**2**, p. 202].) Tell whether the omitted proof is hard or easy to help readers decide whether to try to work it out for themselves. If the theorem has a name, use it: say "by the First Fundamental Theorem," not "by Theorem 5-1. [6]

Examples of proof

Every odd integer can be expressed in exactly one of the forms $4n + 1$ *or* $4n + 3$*, where n is an integer.*

Here is a poor proof:

Proof. An odd integer leaves a remainder of 1 when divided by 2. Now, if we divide the integer by 4, it will leave a remainder of 1 or 3. So the number is of the form $4n + 1$ or $4n + 3$.

Why is this bad? There are several reasons. First, the "proof" is heavy-handed: it says that something will happen without really explaining why it will happen. Second, it is confused; it talks about "an odd integer" (in the first sentence) and "the integer" (in the second sentence). Third, the n comes out of nowhere at the end; there is no mention of n until its sudden appearance in the last sentence.

Let's try again.
Proof. An odd integer is an integer of the form $2m + 1$, where m is an integer which may be even or odd. In the first case, the original integer is of the form $4n + 1$, while in the second case it is of the form $4n + 3$.

But this is another wrong turn. What is wrong? We lapsed into an "English-only" mode and forgot to use the notation that we set up. Good mathematical writing is a blend of words and mathematical notation. More precisely, it consists of grammatically correct sentences in which some of the terms are represented by notation.
Notice also how imprecise the above "proof" is. The connection between m and n is unclear. A good proof explains every step clearly.

Proof. An odd integer can be expressed in the form $2m + 1$, where m is an integer. If m is even, then $m = 2n$, for some integer n, so that the given integer equals $2(2n) + 1 = 4n + 1$. If m is odd, then $m = 2n + 1$, so that the given integer equals $2(2n + 1) + 1 = 4n + 3$.

This is a good proof. Every step is succinctly and correctly explained, and we proved what we set out to prove. Notice that we don't belabor the obvious at the end of proof. It is not necessary to say something like, "Since we started with an arbitrary odd integer, and we showed that it is of the form $4n + 1$ or $4n + 3$, we are done." Such a statement, while true, is tedious and useless. [13]

Fonts for Math

A font is a style of type, e.g., roman or italic. In books and journals, letters representing mathematical quantities, such as a, y and $f(x)$, are set in italic, as just done, to distinguish them from ordinary roman text. [8]

Whatever font you use for a mathematical symbol, make sure you use that font all the time. [8]

Whether you use italic or not, you have to think about spacing within mathematics expressions too. "Tight spacing" like *Δy=f(x+h)−f(x)* makes already complicated expressions even harder to grasp. "Wide spacing" like $\Delta y = f(x + h) - f(x)$ is much preferred — see the difference? My personal preference is for tight spacing within inner groups (e.g., expressions within parentheses) and wide spacing otherwise: *Δy = f(x+h) − f(x)*. [8]

Mathematics is case-sensitive too, that is, upper and lower case mean different things. For instance, that statistics paper probably uses X for the "random variable" of which x is a value. [8]

Displays

Any long expression with mathematical symbols is displayed — centered on a line by itself, with extra vertical space around it. [8]

Long passages without displayed formulas are hard to read; on the other hand, not every formula ought to be displayed.
What should be displayed?

- a formula longer than 3/4 of text width;
- a formula with high elements, like fractions, matrices, sums, integrals (unless they are very simple);
- a formula containing a definition that will be used after a few pages;
- two analogous formulas in the proof.
- What should not be displayed?
- short formulas, to which the reader will not have to return later;
- twice the same formula, especially not far apart. [1]

A display should be numbered if you find that you refer to it anywhere other than within a few lines of it. [8]

In most cases, these should be lined up so that the main connectives (usually an =, but maybe ≤ or ⇒) line up vertically:

$$\begin{aligned} 2x + 1 &= 2 + 3 \\ 2x &= 4 \qquad\qquad\qquad (*) \\ x &= 2 \end{aligned}$$

Similarly, one writes

$$\begin{aligned} x^2 + 2x &< x^2 + 2x + 1 \\ &= (x + 1)^2 \qquad\qquad\qquad (**) \end{aligned}$$

Notice that display (*) has expressions on both sides of the equal sign on each line, but display (**) has only a right-hand side after the first line [8]

Notice the commas and the period in display (*). Mathematics is written in sentences, and this display is a sentence consisting of a sequence of clauses, the equations. Thus the clauses are separated by commas (some writers would use semicolons) until the sentence ends with a period. [8]

Punctuate the display with commas, a period, and so forth as you would if you had not displayed it [6]

Formulas are difficult to read because readers have to stop and work through the meaning of each term. Don't merely list a sequence of formulas with no discernible goal, but give a running commentary. Define all the terms as they are introduced, state any assumptions about their validity, and give examples to provide a feeling for them. [6]

Very elementary algebra, as in (*), need not be explained, but more complicated calculations should be, especially if the calculations are justified by mathematics that the reader is just learning. There are two ways to provide the explanation. One way is to put it before, between and after the lines of calculation. For instance, suppose you have defined $f(x) = x^2$, and want to show that $[f(x+h) - f(x)]/h = 2x + h$. You could write: Substituting the definition $f(x) = x^2$,

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= 2x + h$$

Then expanding the first square, combining like terms, and finally canceling the common factor h, we obtain

$$\frac{f(x+h) - f(x)}{h} = \frac{(x^2 + 2xh + h^2) - x^2}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= 2x + h$$

[8]

Examples, figures, illustrations

Examples really help to make abstract concepts clear, so a good expository paper contains many – more examples than definitions and theorems. Examples are like definitions, in that they can appear in-line or be highlighted by indentation and extra spacing. For a very brief example, the in-line method is fine. However, a lengthier example should be displayed and numbered, especially if it is a key item of your work. [8]

Figures can be extremely helpful in an expository paper, just as they are in books. Each figure should be numbered (for easy reference later) and inserted shortly after the first reference to it. Another convention is to put all figures at the end. If you use this alternative, please say so the first time you reference a figure. Usually, each figure should have a caption as well; e.g., "The steeper the line, the greater the slope". [8]

Illustrations cannot stand alone; they must be introduced in the text. Assign them titles, like **Figure 5-1** or **Table 5-1**, and refer to them as Figure 5-1 or Table 5-1 (Note that the references are capitalized and set in roman. Assign them captions that tell, independently of the text, what they are and how they differ from one another, without being overly specific. [6]

Smooth the transitions between your words and pictures. First of all, match the information in your text and illustrations. Secondly, place the illustrations closely after – never before – their references in the text. [6]

Numbers only those expressions you refer to; the numbers then serve as clear signals (This is likely to require checking and revising.) [7]

Notation

Use consistent notation. This may take some planning. No one would call the angles of a triangle δ, L and t_1. So don't get stuck with

BAD: $a_1x + a_2y, \text{or } ax_1 + bx_2$:

prepare the notation so that you end up with

GOOD: $ax + by$ or $a_1x_1 + a_2x_2$.

[7]

The first mistake is not to rely on mathematical symbols enough. If, for instance, you refer to the same function more than once, give it a name (let $f(x) = x^2$). Even if you are just discussing functions in general, you need to give the generic function a name if you are going to say anything about it. (e.g., for any function $f(t)$, $f'(t)$ measures the rate at which. . .).

The second mistake is to rely on mathematical symbols too much. This happens when you present several lines of computation without any commentary. [8]

Notation that hasn't been used in several pages (or even paragraphs) should carry a reference or a reminder of the meaning.

The same symbol should never be used for more than one thing; if you have used n as a counter in one proof, use m in the next proof, unless the two play a similar role in each

All notation should be meaningful (no free variables).

The letters that are used to denote the concepts you'll discuss are worthy of thought and careful design. A good, consistent notation can be a tremendous help, and I urge that it be designed at the beginning. [4]

The statement of a theorem is generally the wrong place to introduce notation for the proof:

BAD: *A differential function f is continuous.*

GOOD: *A differential function is continuous.*

[7]

Bad notation can make good exposition bad and bad exposition worse; ad hoc decisions about notation, made mid-sentence in the heat of composition, are almost certain to result in bad notation.

Good notation has a kind of alphabetical harmony and avoids dissonance. Example: either $ax + by$ or $a_1x_1 + a_2x_2$ is preferable to $ax_1 + bx_2$. Or: if you must use Σ for an index set, make sure you don't run into $\sum_{\delta\in\Sigma} a_\delta$. Along the same lines: perhaps most readers wouldn't notice that you used $|z| < \varepsilon$ at the top of the page and $z \varepsilon U$ at the bottom, but that's the sort of near dissonance that causes a vague non-localized feeling of malaise. The remedy is easy and is getting more and more nearly universally accepted: ∈ is reserved for membership and ε for ad hoc use. [4]

Use standard or familiar or suggestive notation that your readers can assimilate easily, allowing them to devote their energies to the mathematics. [7]

Mathematics has access to potentially infinite alphabet (e.g. x, x', x'', x''', ...), but, in practice, only a small finite fragment of it is usable. One reason is that a human being's ability to distinguish between symbols is very much more limited than his ability to conceive of new ones; another reason is the bad habit of freezing letters. Some old-fashioned analysts would speak of "xyz-space", meaning, I think, 3-dimensional Euclidean space, plus the convention that a point of that space shall always be denoted by "(x, y, z)". This is bad: it "freezes" x, and y, and z, i.e., prohibits their use in another context, and, at the same time, it makes it impossible (or, in any case, inconsistent) to use, say, "(a, b, c)" when "(x, y, z)" has been temporarily exhausted. Modern versions of the custom exist, and are no better. Examples: matrices with "property L" — a frozen and unsuggestive designation. [4]

Whenever it is possible to avoid the use of a complicated alphabetic apparatus, avoid it. A good attitude to the preparation of written mathematical exposition is to pretend that it is spoken. Pretend that you are explaining the subject to a friend on a long walk in the woods, with no paper available; fall back on symbolism only when it is really necessary [4]

Avoid the use of irrelevant symbols. Example: "On a compact space every real-valued continuous function f is bounded." What does the symbol "f" contribute to the clarity of that statement? [4]

Devote some thoughts to how you use symbols within a line of text.
BAD: *Then every number on the left < every number on the right*
and
BAD: *We conclude that the two expressions are =*
are grammatically impeccable, but most people would agree that they are inelegant at best. Editors will reject both. [7]

Style

Good English style implies correct grammar, correct choice of words, correct punctuation, and, perhaps above all, common sense. [4]

Your article does not have to qualify as literary masterpiece, but it should adhere to the accepted principles of correct diction, grammar, punctuation, and spelling. [7]

Mathematical papers ought to be written in grammatical (but not necessarily idiomatical) English. You should not think that a text without articles or with many language mistakes is still understandable—there are many examples to the contrary. Caring about the language of your paper is also a form of respecting your reader.

Gather a collection of books and articles by British and American mathematical writers in your area of mathematics; look them up as often as possible when writing your paper; they should be your main source of information on what is and what is not correct mathematical English.

When writing, if at all possible, do not translate from your native tongue, but write in English at once, borrowing phrases and sentences from English-speaking authors. It is much better to copy a definition or theorem from a text by a native speaker than write it in your own words, with your own mistakes.

Do not copy fragments of your previous papers—with each consecutive article, your command of English improves (with high probability). Also, do not follow the style of your thesis adviser (if he is not a native speaker of English) or other non-native speakers.

If you find a new expression or word in some text, and you'd like to use it, do not rely on your intuition: first check its meaning in a (thick) dictionary.

Do not use fancy and/or witty words, complicated expressions, inverted commas etc. It is best to use simple and direct language, otherwise you can open yourself up to ridicule. [1]

A common mistake is to use overly complex prose. Don't string adjectives before nouns, lest they lose their strength and precision; instead, use prepositional phrases and dependent clauses, or use two sentences. Keep your sentences simple and to the point. It may help to keep most of them short, but you need some longer ones to keep your writing from sounding choppy and to provide variety and emphasis. [6]

Some papers stagnate because they lack variety. The sentences begin the same way, run the same length, and are of the same type. The paragraphs have the same length and structure.

Don't worry about varying your sentences and paragraphs at first; wait until you polish your writing. [6]

Whatever the usage adopted by an author in a given paper, that usage should be maintained consistently throughout the paper. [14]

Strong writing does not require using synonyms, contrary to popular belief. Indeed, by repeating a word, you often strengthen the bond between two thoughts. Moreover, few words are exact synonyms, and often, using an exact synonym adds nothing to the discussion. [6]

To achieve a natural and effective writing style, you should adhere to the following principles:

1. Write simply
2. Use the active voice.
3. Use plain English words rather than nonstandard technical jargon or foreign phrases.
4. Use standard technical terms correctly.
5. Avoid long sentences and extremely long (or short) paragraphs.
6. Avoid slavish adherence to any set of rules for technical writing, including the rules enumerated here.
7. Remember that the main objective is to communicate your ideas clearly to your audience. [15]

Do not use common blackboard abbreviations. For example, write "if and only if" rather than "iff", and "without loss of generality" rather than "WLOG". This also applies to symbols such as $\forall$ and $\exists$. Unless one is writing a paper in mathematical logic, one should write out "for all" and "there exists".

Punctuate equations and mathematical symbols. Mathematical expressions are no different than the words they represent, and they should be punctuated accordingly. This applies even to displayed equations so that, for example, if a displayed equation is at the end of a sentence it should end with a period.

Do not use contractions in formal writing. Thus words such as "don’t", "can’t", "I’m", and "we’ve" should be written out.

Do not start a sentence with a variable or symbol. Although it is perhaps technically correct, it is considered bad style to do so. Usually this can be avoided by simply rewording the sentence; e.g., rather than "n points are on the interior" one would write "The interior contains n points". (There are, of course, some exceptions to this rule. In particular, most people would consider it acceptable to start a sentence with a mathematical term that contains a symbol, especially if it is an uppercase symbol. For example, one should feel free to start a sentence with the word C^*-algebra or the word *K*-theory.) [2]

Good style involves structuring phrases, sentences and paragraphs so as to make the essay's argument easy to follow. More than this, good style involves organizing the content in such a way that the structure of the essay actually mirrors that of the argument itself. [16]

Avoid ponderous nouns. Instead of long nouns ending in "tion", use verbs:
NOT VERY GOOD: *for the preparation of*
GOOD: *for preparing*
GOOD: *to prepare*
NOT VERY GOOD: *for the acquisition of*
GOOD: *to acquire*
GOOD: *to get*
[7]

1. Give priority to clarity over style. Avoid long and involved sentences; break long sentences into shorter ones.
BAD: *We note the fact that the polynomial* $2x^2 - x - 1$ *has the coefficient of the* x^2 *term positive.*
GOOD: *The polynomial* $2x^2 - x - 1$ *has positive leading coefficient.* [17]

BAD: *The inverse of the matrix A requires the determinant of A to be non-zero in order to exist, but the matrix A has zero determinant, and so its inverse does not exist.*
GOOD: *The matrix A has zero determinant, and hence it has no inverse.*

2. Prefer the active to the passive voice.
BAD: *The convergence of the above series will now be established.*
GOOD: *We establish the convergence of the above series.* [17]

3. Vary the choice of words to avoid monotony. Use a thesaurus.
BAD: *The function defined above is a function of both x and y.*
GOOD: *The function defined above depends on both x and y.* [17]

4. Do not use unfamiliar words unless you know their exact meaning.
BAD: *A simplistic argument shows that our polynomial is irreducible.*

GOOD: *A simple argument shows that our polynomial is irreducible.* [17]

5. Do not use vague, general statements, to lend credibility to your writing. Avoid emphatic statements.
BAD: *Differential equations are extremely important in modern mathematics.*
BAD: *The proof is very easy, as it makes a quite elementary use of the triangle inequality.*
GOOD: *The proof uses the triangle inequality.* [17]

6. Do not use jargon, or text messages abbreviations: it looks immature rather than 'cool'.
BAD: *Spse U subs x into T eq. Wot R T soltns?* [17]

7. Enclose side remarks within commas, which is very effective, or parentheses (it gets out of the way). To isolate a phrase, use hyphenation —it really sticks out— or, if you have a word processor, change font (but don't overdo it). [17]

Strive for accuracy and clarify above all else. [6]

Choice of words

In many treatments of advanced mathematics, the key results are stated formally as theorems, propositions, corollaries, and lemmas. However, these four terms are often used carelessly, robbing them of some useful information they have to convey: the nature of the result. A theorem is a major result, one of the main goals of the work. Use the term "theorem" sparingly. Call a minor result a proposition if it is of independent interest. Call a minor result a corollary if it follows with relatively little proof from a theorem, a proposition, or another corollary. Sometimes a result could properly be called either a proposition or a corollary. If so, then call it a proposition if it is relatively important, and call it a corollary if it is relatively unimportant. Call a subsidiary statement a lemma if it is used in the proof of a theorem, a proposition, or another lemma. Thus a lemma never has a corollary, although a lemma may be used, on occasion, in deriving a corollary. Normally, a lemma is stated and proved before it is used. [6]

The terms "definition" and "remark" are also often abused. A formal *definition* should simply introduce some terminology or notation; there should be no accompanying discussion of the new terms or symbols. It is traditional to use "if" instead of "if and only if"; for example, a matrix is called *symmetric* if it is equal to its transpose. A formal *remark* should be a brief comment made in passing; the main discussion should be logically independent of the content of the remark. Often it is better to weave definitions and remarks into the general discussion rather than setting them apart formally. [6]

I recommend *then* with *if* in all mathematical contexts. The presence of *then* can never confuse; its absence can.

In everyday English *any* is an ambiguous word; depending on context that may hint at an existential quantifier ("have you any wool?", "if anyone can do it, he can") or a universal one ("any number can play"). Conclusion: never use *any* in mathematical writing. Replace it by *each* or *every*, or recast the whole sentence. [4]

Other offenders, charged with lesser crimes, are *where*, and *equivalent*, and *if... then...if...then*. *Where* is usually a sign of a lazy afterthought that should have been thought through before. "If n is sufficiently large, then $|an| < e$, where e is a preassigned positive number"; both disease and cure are clear. "Equivalent" for theorems is logical nonsense. (By "theorem" I mean a mathematical truth, something that has been proved. A meaningful statement can be false, but a theorem cannot; "a false theorem" is self-contradictory). As for *if...then...if...then*, that is just a frequent stylistic bobble committed by quick writers and rued by slow readers. "If *p*, then if *q*, then *r*." Logically, all is well, but psychologically it is just another pebble to stumble over, unnecessarily. Usually all that is needed to avoid it is to recast the sentence, but no universally good recasting exists; what is best depends on what is important in the case at hand. It could be "If *p* and *q* then *r*", or "In the presence of *p*, the hypothesis *q* implies the conclusion *r*", or many other versions. [4]

Use words correctly: distinguish between *function* and *value*. [4]

Let sets forth a convention, usually temporary, usually for a symbol. A related but not mathematically synonymous word is *suppose*. You could say "Suppose $f(x) = x^2$", but you really shouldn't. *Suppose* is best used for temporary hypotheses, not temporary definitions.

It wouldn't really be correct to say instead "Let $f'(x) > 0$ for $x < a$" because we don't really have control over the sign of $f'(x)$, and the word *let* means that the matter is a convention that is up to us.

As for *thus* and *so*, they mean that the next sentence or clause is a logical consequence of the previous sentence or clause. Therefore, if the next sentence would still make sense and be true even if you had not included the previous sentence, then the next sentence may not begin with *thus* or *so*. [8]

Prove has a very strict meaning; *show* is looser, and *illustrate* is looser still and refers to examples. Although illustration is the loosest, it is very important. Sometimes good examples will do more to help the reader understand and believe a result than a complete proof will. [8]

Comprise vs. *compose*. A set *comprises* (*embraces*, *consists of*, *is composed of*) its elements; the elements *compose* (or *constitute*) the set. If you are not sure about *comprise*, try *embrace* (or *include*); if it's wrong, use *compose*.

$\mathbb{R}$ *comprises the rationals and irrationals.*

$\mathbb{R}$ *is comprised [embraced] of the rationals and the irrationals.*

$\mathbb{R}$ *is composed of the rationals and the irrationals*

Comprise is all-inclusive: $\mathbb{R}$ *does not comprise the rationals.* [7]

I.e. vs. *e.g.* Don't use *i.e.* when you mean *e.g.* The first stands for *id est, = that is*; the second is *exempli gratia*: "of example for the sake" – i.e., *for the sake of example*. [7]

Use *then*, not *therefore*, after an assumption:

BAD: *Suppose I lend you $10. Therefore you owe me $10.*

BAD: Assume $\mathrm{x} = 3$. Therefore $2\mathrm{x} = 6$.

GOOD: *Assume* $x = 3$. *Then* $2x = 6$ [7]

As we learn how to write mathematics, our first aim is to achieve total accuracy. We analyse some typical mistakes and imprecisions which result from a poor choice of words.

BAD: *the equation* $x - 3 < 0$

GOOD: *the inequality* $x - 3 < 0$ [17]

BAD: *the equation* $x^2 - 1 = (x-1)(x+1)$

GOOD: *the identity* $x^2 - 1 = (x-1)(x+1)$ [17]

BAD: *the interval* $[0, \infty)$

GOOD: *the ray* $[0, \infty)$ *(the infinite interval* $[0, \infty)$) [17]

BAD: *the solution of* $x^k = x$

GOOD: *a solution of* $x^k = x$ [17]

BAD: *the function* $f(x)$

GOOD: *the function* f [17]

BAD: *the area of the unit circle*

GOOD: *the area of the unit* disc [17]

BAD: *the function* $g(A)$ *of the set A*

GOOD: *the image* $g(A)$ *of the set A* [17]

BAD: *the absolute value is positive*

GOOD: *the absolute value is non-negative* [17]

BAD: *the coordinates of a complex number*

GOOD: *the real and imaginary parts of a complex number* [17]

BAD: *the exponential function crosses the vertical axis at a positive point*

GOOD: *the graph of the exponential function intersects the ordinate axis at a positive point* [17]

The construction

Since . . ., then . . .

is discordant. Instead of "then", say "it follows that", or "we have", or nothing:

BAD: *Since this limit exists, then the series converges.*

GOOD: *Since this limit exists, the series converges*

[7]

Don't use *if and only if* in a definition (except in formal logic) – it's too pompous. By tradition, *if* is sufficient:

An integer > 1 is said to be prime if its only positive divisors are itself and 1.

Let after the title DEFINITION is redundant and sounds gauche:

GOOD: *Let* $|S|$ *denote the cardinal number of S.*

BAD: DEFINITION: *Let* $|S|$ *denote the cardinal number of S.*

GOOD: DEFINITION: $|S|$ *denotes the cardinal number of S.*

[7]

Grammar

In constructing each sentence, place old and new information in the respective positions where readers generally expect to find these types of information.

Place in the topic position (that is, at the beginning of the sentence) the old information linking backward to the previous discussion.
Place at the stress position (that is, at the end of the sentence) the new information you want to emphasize.
Place the subject of the sentence in the topic position, and follow the subject with the verb as soon as possible.
Express the action of each sentence in its verb. [15]

A pronoun normally refers to the first preceding noun, but sometimes it refers broadly to a preceding phrase, topic, or idea. Make sure the reference is immediately clear, especially with *it*, *this*, and *which*. [6]

Begin each paragraph with a sentence that summarizes the topic to be discussed or with a sentence that helps the transition from previous paragraph.
Avoid paragraphs of extreme length – that is, one-sentence paragraphs and those exceeding 200 words.
Place the important conclusions in the stress position at the end of the paragraph. [15]

Make sure your writing flows. Avoid writing a succession of loose sentences. Particularly when writing proofs, it is easy to become so engrossed in the mathematics that one forgets to pay attention to English style. The result is often a proof that reads " . . . and then . . . and then . . . and then . . .". Try to use a variety of words in proofs, such as "therefore", "consequently", "it follows that", "we see", "hence", or "thus". [2]

After you have written your solution, re-read it and ask yourself the following questions:

– Is each paragraph about a single topic?
– Is each paragraph made up of complete, properly punctuated sentences?
– Does each phrase follow logically from the one before it?
– Is each logical leap justified?

Does the progression from one phrase to the next match the argument you are trying to make? [16]

Be forthright : write in an unhesitating, straightforward, and friendly style, ridding your language of needless and bewildering formality. Be wary of awkward and inefficient passive constructions. Often the passive voice is used simply to avoid the first person. However, the pronoun "we" is now generally considered acceptable in contexts where it means the author and reader together, or the author with the reader looking on. (Still, "we" should not be used as a formal equivalent of "I," and "I" should be used rarely, if at all.) For instance, don't write, "By solving the equation, it is found that the roots are real." Instead write, "Solving the equation, we find the roots are real," or "Solving the equation yields *real* roots." It is acceptable, but less desirable, to write, "Solving the equation, one finds the roots are real"; the personal pronoun "one" is a sign of formality, and is uncommon in mathematical writing today. [6]

1. If you are unfamiliar with the basic terminology of grammar (adjective, adverb, noun, pronoun, verb, etc.), look it up in a book. [17]

2. Write in complete sentences. Every sentence should begin with a capital letter, end with a full stop, and contain a subject and a verb. The expression "A cubic polynomial" is not a sentence, because it doesn’t have a verb. It would be appropriate as a caption, or a title, but you can’t insert it as it is in the middle of a paragraph. [17]

3. Make sure that the nouns match the verbs grammatically.
BAD: *The set of primes are infinite.*
GOOD: *The set of primes is infinite.*
(The verb refers to "the set", which is singular.)
Make a pronoun agree with its antecedent.
BAD: *Each function should be greater than their derivative.*
GOOD: *Each function should be greater than its derivative.*
(The pronoun "its" refers to "function", which is singular.)

Do not split infinitives.
BAD: *We have to again eliminate a variable.*
GOOD: *We have to eliminate a variable again.*
(The infinitive is "to eliminate".)[17]

4. Check the spelling: no point in crafting a document carefully, if you then spoil it with spelling mistakes. If you use a word processor, take advantage of a spell checker. These are some frequently misspelled words:
BAD: *auxillary, catagory, consistant, correspondance, impliment, indispensible, ocurrence, preceeding, refering, seperate.*

These are misspelled mathematical words that I found in mathematics examination papers:

BAD: *arithmatic, arithmatric, divisable, infinaty, matrics, orthoganal, orthoginal, othogonal, reciprical, scalor, theorom.* [17]

5. Be careful about distinctions in meaning.

Do not confuse *it's* (abbreviation for *it is*) with *its* (possessive pronoun).

BAD: *Its an equilateral triangle: it's sides all have the same length.*

GOOD: *It's an equilateral triangle: its sides all have the same length.*

Do not confuse the noun *principle* (general law, primary element) with the adjective *principal* (main, first in rank of importance).

BAD: *the principal of induction*

BAD: *the principle branch of the logarithm*

Do not use *less* (of smaller amount, quantity) when you should be using *fewer* (not as many as).

BAD: *There are less primes between 100 and 200 than between 1 and 100.* [17]

6. Do not use *where* inappropriately. As a relative adverb, *where* stands for *in which* or *to which*; it does not stand for *of which*.

BAD: *We consider the logarithmic function, where the derivative is positive.*

GOOD: *We consider the logarithmic function, whose derivative is positive.*

The adverb *when* is subject to similar misuse.

BAD: *A prime number is when there are no proper divisors.*

GOOD: *A prime number is an integer with no proper divisors.* [17]

7. Do not say *which* when *that* sounds better. Experiment to decide which is better, and if you can substitute *that* for *which*, do it.2 The general rule is to use *which* only when it is preceded by a comma or by a preposition, or *when* it is used interrogatively. In some cases both pronouns are correct, but have different meaning. *That* is the defining pronoun —it is used to identify an object uniquely— while *which* is non-defining —it adds information to an object already identified.

> *The argument that was used above is based on induction.*
>
> [Specifies which argument.]
>
> *The following argument, which will be used in subsequent proofs, is based on induction.*
>
> [Adds a fact about the argument in question.]

[17]

Which vs. *that*. This Distinction might seem a high-flown affair, but it rests on being able to recognize what a definition is, and mathematicians are good at that. "That", present or understood, is the defining pronoun; "which", which invariably follows a comma, is nondefining one:

Here's the calculus book that I bought yesterday.

[Defines which one.]

Here's the calculus book I bought yesterday.

[Defines which one.]

Here's the calculus book, which I bought yesterday

[Adds information about the one already under discussion.] [7]

8. In presence of parentheses, the punctuation follows strict rules. The punctuation outside parentheses should be correct if the statement in parentheses is removed; the punctuation within parentheses should be correct independently of the outside.

BAD: *This is bad.* (Superficially, it looks good).

GOOD: *This is good.* (Superficially, it looks the same as the BAD one.)

BAD: *This is bad,* (on two accounts.)

GOOD: *This is good* (as you would expect). [17]

Use the active voice. All writing experts agreed on this. Passive constructions leave the reader wondering who is doing what to whom; moreover, they encourage a verbose style, which makes things worse.

BAD: *It has been noticed that*

GOOD: *We have noticed that*

BAD: *Occurrences were observed in which*

GOOD: *I saw*

[7]

Avoid mixing voices and tenses, e.g., "Let *X* be compact; we shall define *H(X)*, and then *K(X)* can be defined."

Avoid the subjunctive, the future, and the future conditional; they are overused. Generally the present indicative and imperative moods clearer and stronger writing. [18]

Beware of dangling participles. It is wrong to write, "Solving the equation, the roots are real," because "the roots" cannot solve the equation. [6]

Plurals. *Data,* like *curricula, extrema, maxima*, and *minima*, are plural, as are *bacteria, media*, and *symposia*, and, from the Greek, *criteria* (sing. *criterion*) and *phenomena* (sing. *phenomenon*).

BAD: *The data is interesting.*

GOOD: *These data are interesting.*

BAD: *This criteria is the one.*

GOOD: *That criterion is the one.* [7]

Numbers and symbols

1. A sentence containing numbers and symbols must still be a correct English sentence, including punctuation.

BAD: $a < b \ \ a \neq 0$

GOOD: *Let* $a < b$*, with* $a \neq 0$*.*

GOOD: *We find that* $a < b$ *and* $a \neq 0$*.* [17]

BAD: $x^2 - 11^2 = 0.\ x = \pm 11.$

GOOD: *Let* $x^2 - 11^2 = 0$*; then* $x = \pm 11$.

GOOD: *The equation* $x^2 - 11^2 = 0$ *has two solutions:* $x = \pm 11$.

2. Omit unnecessary symbols.

BAD: *Every differentiable real function f is continuous.*

GOOD: *Every differentiable real function is continuous.* [17]

3. If you use small numbers for counting, write them out in full; if you refer to specific numbers, use numerals.

BAD: *The equation has 4 solutions.*

GOOD: *The equation has four solutions.*

GOOD: *The equation has 127 solutions.*

BAD: *Both three and five are prime numbers.*

GOOD: *Both 3 and 5 are prime numbers.* [17]

4. If at all possible, do not begin a sentence with a numeral or a symbol.

BAD: *ρ is a rational number with odd denominator.*

GOOD: *The rational number ρ has odd denominator.* [17]

5. Do not combine operators ($+$, $\neq$, $\Longrightarrow$, etc.) with words.

BAD: *The number* $\sqrt{2} - 3/2$ *is* < 0

GOOD: *The number* $\sqrt{2} - 3/2$ *is negative.*

BAD: *If a is an integer ⇒ a is a rational number.*

GOOD: *If a is an integer, then a is a rational number.* [17]

6. Within a sentence, adjacent formulae or symbols must be separated by words.

BAD: *Consider* $A_n, n < 5$.

GOOD: *Consider* A_n, *where* $n < 5$. [17]

BAD: *Add p k times to c.*

BAD: *Add p to c k times.*

GOOD: *Add p to c, repeating this process k times.* [17]

For displayed equations the rules are a bit different, because the spacing between symbols becomes a syntactic element. Thus an expression of the type

$$A_n = B_n, n < 5$$

is quite acceptable. [17]

In informal writing, it is common to use shorthand for quantifiers and logical connectives: symbols such as ∀, ∃, $\Longrightarrow$, $\Longleftrightarrow$, or abbreviations like "iff" and "s.t.".

However, in formal writing such shorthand should generally be avoided. You should write out "for all", "there exists", "implies", "if and only if", and "such that". Most other symbols are acceptable in formal writing, after defining them where needed. The membership symbol ∈ ("is an element of") is traditionally acceptable in formal writing, as are relations (e.g.,$<$,$>$,$\times$,$+$,∪,∩, etc.), variable names (e.g., x, y, z, etc.), and symbols for sets ($\mathbb{Q}, \mathbb{R}, \mathbb{Z}$, etc.). Here is an acceptable use of symbols in formal mathematical writing:

Let A, B be two subsets of $\mathbb{R}$. *We say A dominates B if for every* $x \in A$ *there exists* $y \in B$ *such that* $y > x$. [19]

Beware of using symbols to convey too much information all at once.

VERY BAD: *If* $\Delta = b^2 - 4ac \geq 0$, *then the roots are real.*

BAD: *If* $\Delta = b^2 - 4ac$ *is nonnegative, then the roots are real.*

GOOD: *Set* $\Delta = b^2 - 4ac$. *If* $\Delta \geq 0$*, then the roots are real.* [17]

Use consistent notation. Don't say "A_j, where $1 \leq j \leq n$ " one place and "A_k, where $1 \leq k \leq n$ " another.

Keep the notation simple. For example, don't write "x_i is an element of X" if "*x* is an element of X" will do. [6]

Word division

– A word should be divided only between syllables (*com•pu•ter*). Dictionaries indicate how to divide words at syllable breaks
– If a vowel stands alone as a single syllable, it must remain on the same line as the first part of the word (*experi•ment*)
– A word is generally divided between double consonants (*neces•sary*), unless it means breaking up the root of the word (*process•ing*)
– If a word contains a natural hyphen, divide only at that point (*sixty-five*)
– If a word contains a prefix or suffix, it is best to divide at that point (*auto•correlation*)
– Do not divide one-syllable words (*length*), no matter how long the word may be
– Do not divide a word in the first or last line on a page
– Do not divide a word in the first line of a paragraph
– Do not divide words on two consecutive lines
– Do not divide a proper name or number (*Kolmogorov, 1931*)
– Do not separate two letters from the rest of the word
– Do not separate contractions or abbreviations (*wouldn't, ATSC*)
– Do not use a hyphen to break a URL or an e-mail address
– Do not use excessive word division
– Do not separate the unit of measurement from the number (*550 kHz*) [5]

Rules for writing numbers

Write out all numbers below ten:

– zero deviations from the expected value
– nine devices to count for

The exception to this rule are numbers used with:

- units of measurement (*3 meters*)
- age (*15 years old*)
- dates (*October 11, 1957*)
- time (*2 seconds*)
- page numbers (*page 4*)
- percentages (*5 percent*)
- money (*$8*)
- proportions (*30:1 or 30 to 1*)

Write the numbers as numerals if two or more are in the same section:

the transmitter has 5 audio amplifiers, 2 pass-band filters and a net gain of 60 dB

Place a hyphen between a number and a unit of measurement when they modify a noun:

3-month-old experiment

Use the singular for fractions and decimals that are used as adjectives:

0.5 kilogram

0.1 centimeter

Write decimals and fractions as numerals:

zero point two five – 0.25

Do not begin a sentence with numerals [5]

Punctuation

The comma (,)

- The comma is used to indicate pauses and to separate entries in lists
- It is also used to set off a word, phrase, or clause that is in apposition to a noun and that is non-restrictive

The semi-colon (;)

- The semi-colon separates closely-related ideas
- In most cases a period should be used instead
- When a list includes items that have commas within them, use a semi-colon to separate the items

The colon (:)

- The colon is used in headings, to announce that more is to follow
- It is used to introduce a list of things (words, equations)

– It introduces a quotation
– When used between two clauses, it indicates that the second one provides an explanation of what was said in the first
– Never use the colon after the main verb in a sentence

The hyphen (-)

– This mark may be used to separate two parts of a compound noun (*light-year*)
– It is also used to break up words at the end of a line, and should always be placed between syllables. Proper nouns should not be broken up by hyphens

Parentheses ( )

– Parentheses are used to set off an interruption in the middle of a sentence, including references to pages, diagrams, illustrations, chapters
– They are also used to make a point which is not part of the main flow of the sentence
– Use them to enclose acronyms: "Moscow State University (MSU)."

The apostrophe (’)

– The apostrophe shows possession (*Marcelo’s book on scientific style in English*)
– It shortens certain word combinations (*can’t, he’s*)
– Contractions should be avoided in formal written work. (*cannot, he is*)

Brackets [ ]

– Brackets are used for citations or to enclose a word inserted into a quotation

The diagonal or dash (–)

– A dash may be used in place of a colon, to set off a word or phrase at the end of a sentence, or an appositive to be emphasized
– A dash is used to summarize a thought added to the end of a sentence

Quotation marks (" ")

– The main use of quotation marks is to show that the exact words written are being repeated
– They are also used to enclose titles of articles, chapter names, and short stories
– Single quotation marks (‘ ’) can be used within a quotation
– A comma should precede a quotation and other punctuation should be placed inside the quotation marks [5]

The revision process

Perhaps the most important element of a mathematical writing endeavour is the revision process. Nobody writes perfect mathematical explanations the first time. We must review our work and rewrite it, striving for greater accuracy, precision, and clarity. [13]

After your paper is finished and draft typed, then you must challenge every single word, sentence; phrase, paragraph, and section. Is the order right? Why this choice of words? Can excess verbiage be trimmed? Can explanations and proofs be made clearer? [18]

When reading your work, ask yourself the following questions: Does each sentence make sense? If not, then fix the sentences that don't make sense. Does the logic make common sense? If not, then revise the parts that are illogical. [13]

The best way to start writing, perhaps the only way, is to write on the spiral plan. According to the spiral plan the chapters get written and re-written in the order 1, 2, 1, 2, 3, 1, 2, 3, 4, etc. You think you know how to write Chapter 1, but after you've done it and gone on to Chapter 2, you'll realize that you could have done a better job on Chapter 2 if you had done Chapter 1 differently. There is no help for it but to go back, do Chapter 1 differently, do a better job on Chapter 2, and then dive into Chapter 3.
The same phenomenon will occur not only for chapters, but for sections, for paragraphs, for sentences, and even for words.
In the first draft of each chapter I recommend that you spill your heart, write quickly, violate all rules, write with hate or with pride, be snide, confused, be "funny" if you must, be unclear, be ungrammatical — just keep on writing. When you come to rewrite, however, and however often that may be necessary, do not edit but rewrite. It is tempting to use a red pencil to indicate insertions, deletions, and permutations, but in my experience it leads to catastrophic blunders.
The spiral plan of writing goes hand in hand with the spiral plan of organization, a plan that is frequently (perhaps always) applicable to mathematical writing. It goes like this. Begin with whatever you have chosen as your basic concept—vector spaces, say—and do right by it: motivate it, define it, give examples, and give counterexamples. That's Section 1. In Section 2 introduce the first related concept that you propose to study—linear dependence, say—and do right by it: motivate it, define it, give examples, and give counterexamples, and then, this is the important point, review Section 1, as nearly completely as possible, from the point of view of

Section 2. For instance: what examples of linearly dependent and independent sets are easily accessible within the very examples of vector spaces that Section 1 introduced? (Here, by the way, is another clear reason why the spiral plan of writing is necessary: you may think, in Section 2, that you forgot to give as examples in Section 1.)

When you've written everything you can think of, take a day or two to read over the manuscript quickly and to test it for the obvious major points that would first strike a stranger's eye. Is the mathematics good, is the exposition interesting, is the language clear, is the format pleasant and easy to read? Then proofread and check the computations; that's an obvious piece of advice, and no one needs to be told how to do it. "Ripening" is easy to explain but not always easy to do: it means to put the manuscript out of sight and try to forget it for a few months. When you have done all that, and then re-read the whole work from a rested point of view, you have done all you can. Don't wait and hope for one more result, and don't keep on polishing. Even if you do get that result or do remove that sharp corner, you'll only discover another mirage just ahead. [4]

A Draft Article for Improvement

Below is a shortened version of an article that I wrote for an undergraduate mathematics magazine. I have introduced over twenty errors of various kinds, though most are relatively minor. How many can you spot? If you are an inexperienced writer, criticizing this "draft," will be a valuable exercise. [10]

Numerical Linear Algebra in the Sky

In aerospace computations, transformations between different co-ordinate systems are accomplished using the *direction cosine matrix* (DCM), which is defined as the solution to a time dependent matrix differential equation. The DCM is 3x3 and exactly orthogonal, but errors in computing it lead to a loss of orthogonality. A simple remedy, first suggested in a research paper in 1969 is to replace the computed DCM by the nearest orthogonal matrix every few steps. These computations are done in real-time by an aircrafts on-board computer so it is important that the amount of computation be kept to a minimum. One suitable method for computing a nearest orthogonal matrix is described in this Article. We begin with the case of 1×1 matrices—scalars.

(a) Let x_1 be a nonzero real number and define the sequence:

$$x_{k+1} = \frac{1}{2}(x_k + 1/x_k), \quad k = 1,2,\ldots$$

If you compute the first few terms on your calculator for different x_1 (e.g. $x_1 = 5, x_1 = -3$) you'll find that x_k converges to ± 1; the sign depending on the sign of x_1. Prove that this will always be the case (*Hint*: relate $x_{k+1} \pm 1$ to $x_k \pm 1$ and then divide this two relations). This result can be interpreted as saying that the iteration computes the nearest real number of modulus one to x_1.

(b) This scalar iteration can be generalized to matrices without loosing it's best approximation property. For a given nonsingular $X_1 \in \mathbb{R}^{n\times n}$ define

$$X_{k+1} = \tfrac{1}{2}(X_k + X_k^{-T})$$

(This is one of those very rare situations where it really is necessary to compute a matrix inverse!) Here, X_k^{-T} denotes the transpose of the inverse of X_k. Natural questions to ask are: Is the iteration well defined (i.e., is X_k always nonsingular)? Does it converge? If so, to what matrix?

To investigate the last question suppose that $X_k \to X$. Then X will satisfy $X = \frac{X+X^{-T}}{2}$, or $X = X^{-T}$, or

$$X^T X = I,$$

thus X is orthogonal! Moreover, X is not just any orthogonal matrix. It is the nearest one to X_1 as shown by the following

Theorem 1

$$\|X - X_1\| = \min\{\|Q - X_1\| : Q \in R^{nxn}, Q^T Q = I_n\}$$

where the norm is denoted by

$$\|X\| = \left(\sum_{i}^{n}\sum_{j=1}^{n} x_{ij}^2\right)^{1/2}.$$

This is the matrix analogue of the property stated in (a).

Returning to the aerospace application, the attractive feature of iteration (1) is that if we don't wait to long before "re-orthogonalising" our computed iterates then just one or two applications of the iteration (1) will yield the desired accuracy with relatively little work. □ [10]

Here are the corrections I would make to the article. (In repeating the exercise myself some time after preparing this section, I could not find all the errors!)

1. First paragraph: hyphenate *time-dependent*; comma after 1969; *aircraft's*; comma after *on-board computer*; *article* in lower case.

2. Second paragraph: no colon after *sequence*. In display, $\frac{1}{x_k}$ instead of $1/x_k$, and replace "...,." by "....". Comma after *e.g.* and instead of semicolon after ± 1. *These two relations*; *modulus* 1.

3. Third paragraph: *losing*; *its*. Right parenthesis in display (1) is too large and full stop needed at end of display.

4. Fourth paragraph: third X should be in mathematics font, not roman; $(X + X^{-T})/2$. The equation $X^T X = I$ should not be displayed and it should be followed by a semicolon instead of a comma. (The spacing in $X^T X$ should be tightened up — see page 192 for how to do this in $LATEX$.) Comma after X_1; *following theorem*.

5. No need to number the theorem as it's the only one. It should begin with words: *The matrix* X_1 *satisfies*. $\mathbb{R}^{n\times n}$ instead of R^{nxn} (two changes). Comma at end of first display. "I_n" is inconsistent with "I" earlier: make both I. *Denoted* should be *defined*. In first sum of second display, $i = 1$. The parentheses are too large.

6. Last paragraph: *too long*. Wrong opening quotes. For consistency with *generalized* (earlier in article), spell as *re-orthogonalizing*. Logical omission: I haven't shown that the iteration converges, or given or referred to a proof of the theorem. [10]

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[7] Leonard Gillman. *Writing Mathematics Well. A Manual for Authors.* The Mathematical Association of America, Washington, D.C., 1987.
Краткое, но полезное пособие, содержащее много практических советов.

[8] *Advice for Undergraduates on Special Aspects of Writing Mathematics*
http://www.swarthmore.edu/Documents/WRITE_PRIMUS.pdf

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[13] Martin Erickson. *How to Write Mathematics*.
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[14] *A Manual for Authors of Mathematical Papers.* http://www.ams.org/journals/bull/1943-49-03/S0002-9904-1943-07884-6/S0002-9904-1943-07884-6.pdf

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[16] Adrian Butscher, Charles Shepherd and Lindsey Shorser. *Style Guide for Writing Mathematical Proofs*. http://www.math.toronto.edu/writing/MathStyleGuide.pdf

[17] Franco Vivaldi. *Mathematical Writing*. Springer-Verlag, London, 2014.

[18] Harley Flanders, Manual for Monthly Authors, *Amer. Math. Monthly*, 78, 1971.

[19] Francis Edward Su. *Guidelines for Good Mathematical Writing*. http://www.math.uh.edu/~tomforde/Images/Writing-Guidelines-Su.pdf

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